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Possibility Neutrosophic Vague Soft Set for Medical Diagnosis Decision under Uncertainty

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Abstract This paper presents a novel concept based on the idea that each element of the initial universe has a possibility degree related to each element of the parameter set. It is an improved hybrid model that combines the key features of soft set, vague set and neutrosophic set, thus making it highly suitable for use in decision-making problems that involve uncertain and indeterminate data. Based on this new concept we define some related notions as well as some basic operations namely the complement, subset, equality, union, intersection, AND and OR along with illustrative examples. A similarity measure of two possibility neutrosophic vague soft sets is introduced and an application of this similarity measure in medical diagnosis is illustrated. Finally, a comparison between different existing methods and possibility neutrosophic vague soft set to show the ascendancy of our proposed method is provided.

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1. Introduction

Fuzzy set was introduced by Zadeh [1] as a mathematical tool to solve problems and vagueness in everyday life. Since then many research have been undertaken on fuzzy sets and fuzzy logic [2–5]. Later on several researches present a number of results using different direction of fuzzy set such as interval fuzzy set [6], intuitionistic fuzzy set [7] and vague set [8]. However, all these theories have their inherent difficulties and weaknesses. This led to the introduction of the theory of soft sets by Molodtsov [9]. Molodtsov mentioned a soft set as a mathematical way to represent and solve these problems with uncertainties which traditional mathematical tools cannot handle. Alkhazaleh et al. [10] introduced the concept of soft multiset as a generalization of Molodtsov's soft set. They

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also introduced possibility fuzzy soft set [11] and a similarity measure of two possibility fuzzy soft sets. Chen [12] provides measure of similarity between vague sets. Vague soft set theory was provided by Xu [13]. Alhazaymeh and Hassan then introduced the concepts of possibility vague soft set [14], possibility interval-valued vague soft set [15] and generalized vague soft set [16] as a generalization of vague soft set. Smarandache [17] firstly proposed the theory of neutrosophic set as a generalization of the intuitionistic set, classical set, fuzzy set, paraconsistent set, dialetheist set, paradoxist set, tautological set based on "neutrosophy". The words "neutrosophy" and "neutrosophic" were introduced by Smarandache in his 1998 book [18]. "neutrosophy" (noun) means knowledge of neutral thought, while "neutrosophic" (adjective), means having the nature of, or having the characteristic of neutrosophy. Neutrosophic set can deal uncertain, indeterminate and incongruous information where the indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are completely independent. Maji [19] introduced neutrosophic soft set which can be viewed as a new path of thinking for engineers, mathematicians, computer scientists and others. Karaaslan [20] proposed the theory of possibility neutrosophic soft sets as a generalization of possibility fuzzy soft set and possibility intuitionistic fuzzy soft set [21]. As a combination of neutrosophic set and vague set, Alkhazaleh [22] introduced the concept of neutrosophic vague set which is an effective tool to process incomplete, indeterminate and inconsistent information.

We will extend the study on neutrosophic vague set by establishing a novel notion called possibility neutrosophic vague soft set (PNVSS) which provides a more adequate parameterization tool that can represent the problem parameters in a more comprehensive manner. Further, each element of the universal set has a possibility degree related to each element of the parameter set, unlike other methods such as fuzzy soft set, intuitionistic fuzzy soft set, vague soft set and neutrosophic soft set where the possibility of each element of the initial universe related to each parameter is considered as 1. Therefore, our proposed method presents a more general perspective than the former mentioned methods.

To facilitate our discussion, we first review some background on neutrosophic vague set and possibility fuzzy soft set in Section 2. In Section 3, we introduce the concept of possibility neutrosophic vague soft set and give the set theoretic operations on this concept. In Section 4, we introduce a similarity measure between two possibility neutrosophic vague soft sets. In Section 5, we give an application of the introduced similarity measure in medical diagnosis. In Section 6, a comparison among possibility neutrosophic vague soft set and other existing methods is made to reveal the dominance of our proposed method. Finally, conclusions are pointed out in Section 7.

2. Preliminaries

In this section, we recall some basic notions in neutrosophic vague set, neutrosophic vague soft set and possibility fuzzy soft set.

Definition 2.1 ([22]). A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as

$$A_{NV} = \{< x; \widehat{T}_{A_{NV}}(x); \widehat{I}_{A_{NV}}(x); \widehat{F}_{A_{NV}}(x) >; x \in X\}$$

whose truth-membership, indeterminacy-membership and falsity-membership functions is defined as $\widehat{T}_{A_{NV}}(x) = [T^-, T^+]$, $\widehat{I}_{A_{NV}}(x) = [I^-, I^+]$ and $\widehat{F}_{A_{NV}}(x) = [F^-, F^+]$ where

$$(1)\ \ T^+ = 1 - F^-, \quad (2)\ \ F^+ = 1 - T^- \text{ and } (3)\ \ {}^-0 \le T^- + I^- + F^- \le 2^+.$$

Definition 2.2 ([22]). Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, (1) $\widehat{T}_{A_{NV}}(u_i) \le \widehat{T}_{B_{NV}}(u_i)$, (2) $\widehat{I}_{A_{NV}}(u_i) \ge \widehat{I}_{B_{NV}}(u_i)$ and (3) $\widehat{F}_{A_{NV}}(u_i) \ge \widehat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where $1 \le i \le n$.

Definition 2.3 ([22]). The complement of a NVS A_{NV} is denoted by A^c and is defined by $\widehat{T}^c_{A_{NV}}(x) = [1 - T^+, 1 - T^-]$, $\widehat{I}^c_{A_{NV}}(x) = [1 - I^+, 1 - I^-]$ and $\widehat{F}^c_{A_{NV}}(x) = [1 - F^+, 1 - F^-]$.

Definition 2.4 ([22]). The union of two NVSs A_{NV} and B_{NV} is a NVS C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$$\widehat{T}_{C_{NV}}(x) = \left[\max\left(T^-_{A_{NV\,x}}, T^-_{B_{NV\,x}}\right), \max\left(T^+_{A_{NV\,x}}, T^+_{B_{NV\,x}}\right)\right],$$
$$\widehat{I}_{C_{NV}}(x) = \left[\min\left(I^-_{A_{NV\,x}}, I^-_{B_{NV\,x}}\right), \min\left(I^+_{A_{NV\,x}}, I^+_{B_{NV\,x}}\right)\right] \text{ and}$$
$$\widehat{F}_{C_{NV}}(x) = \left[\min\left(F^-_{A_{NV\,x}}, F^-_{B_{NV\,x}}\right), \min\left(F^+_{A_{NV\,x}}, F^+_{B_{NV\,x}}\right)\right]$$

Definition 2.5 ([22]). The intersection of two NVSs A_{NV} and B_{NV} is a NVS C_{NV}, written as $H_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$$\widehat{T}_{H_{NV}}(x) = \left[\min\left(T^-_{A_{NV\,x}}, T^-_{B_{NV\,x}}\right), \min\left(T^+_{A_{NV\,x}}, T^+_{B_{NV\,x}}\right)\right],$$
$$\widehat{I}_{H_{NV}}(x) = \left[\max\left(I^-_{A_{NV\,x}}, I^-_{B_{NV\,x}}\right), \max\left(I^+_{A_{NV\,x}}, I^+_{B_{NV\,x}}\right)\right] \text{ and}$$
$$\widehat{F}_{H_{NV}}(x) = \left[\max\left(F^-_{A_{NV\,x}}, F^-_{B_{NV\,x}}\right), \max\left(F^+_{A_{NV\,x}}, F^+_{B_{NV\,x}}\right)\right]$$

We will now give a definition of neutrosophic vague soft set below.

Definition 2.6. Let U be an initial universal set and let E be a set of parameters. Let $NV(U)$ denote the power set of all neutrosophic vague subsets of U and let $A \subseteq E$. A collection of pairs $(\widehat{F}, E)$ is called a *neutrosophic vague soft set* (NVSS) over U where $\widehat{F}$ is a mapping given by

$$\widehat{F} : A \to NV(U).$$

Definition 2.7 ([11]). Let $U = \{x_1, x_2, ..., x_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ be a fuzzy subset of E, i.e. $\mu : E \to I^U$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \to I^U \times I^U$ be a function defined as follows:

$$F_\mu(e) = (F(e)(x),\ \ \mu(e)(x)), \forall x \in U.$$

Then F_μ is called a *possibility fuzzy soft set* (PFSS in short) over the soft universe (U, E).

Definition 2.8 ([11])**.** Let F_μ and G_δ be two PFSSs over (U, E). *Similarity* between F_μ and G_δ, denoted by $S(F_\mu, G_\delta)$, is defined as follows:

$S(F_\mu, G_\delta) = M(F(e), G(e)) \cdot M(\mu(e), \delta(e))$ such that
$M(F(e), G(e)) = \max_i M_i(F(e), G(e))$ and $M(\mu(e), \delta(e)) = \max_i M_i(\mu(e), \delta(e))$ where
$M_i(F(e), G(e)) = 1 - \frac{\sum_{j=1}^{n} |F_{ij}(e) - G_{ij}(e)|}{\sum_{j=1}^{n} |F_{ij}(e) + G_{ij}(e)|}$,
and $M_i(\mu(e), \delta(e)) = 1 - \frac{\sum_{j=1}^{n} |\mu_{ij}(e) - \delta_{ij}(e)|}{\sum_{j=1}^{n} |\mu_{ij}(e) + \delta_{ij}(e)|}$.

3. Possibility Neutrosophic Vague Soft Set

In this section we introduce the concept of possibility neutrosophic vague soft sets as an extension of the neutrosophic vague soft sets and define some operations on a possibility neutrosophic vague soft set, namely subset, equality, null, absolute, complement, union, intersection, AND and OR .

Now we propose the definition of the possibility neutrosophic vague soft set and we give an illustrative example of it.

Definition 3.1. Let $U = \{x_1, x_2, ..., x_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. Let $F : E \to$ NV(U), where $NV(U)$ is the collection of all neutrosophic vague subsets of U and μ be a fuzzy subset of E, that is $\mu : E \to I^U$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \to$ NV(U) $\times I^U$ be a function defined as follows:

$$F_\mu(e) = (F(e)(x),\ \mu(e)(x)), \forall x \in U.$$

Then F_μ is called a *possibility neutrosophic vague soft set* (PNVSS in short) over the soft universe (U, E).

For each parameter e_i , $F_\mu(e_i) = (F(e_i)(x)\ ,\ \mu(e_i)(x))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $\mu(e_i)$. So we can write $F_\mu(e_i)$ as follows:
$F_\mu(e_i) = \left\{ \left(\frac{x_1}{F(e_i)(x_1)}, \mu(e_i)(x_1) \right), \left(\frac{x_2}{F(e_i)(x_2)}, \mu(e_i)(x_2) \right), ..., \left(\frac{x_n}{F(e_i)(x_n)}, \mu(e_i)(x_n) \right) \right\}.$

We can write F_μ as (F_μ, E). If $A \subseteq E$ we can also have a PNVSS (F_μ, A).

Example 3.2. Let $U = \{x_1, x_2\}$ be a universe set. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters and let $\mu : E \to I^U$.
We define a function $F_\mu : E \to$ NV(U) $\times I^U$ as follows:

$$F_\mu(e_1) = \left\{ \left(\frac{x_1}{([0.2, 0.5], [0.5, 0.9], [0.5, 0.8])},\ 0.7 \right), \left(\frac{x_2}{([0.7, 0.8], [0.2, 0.5], [0.2, 0.3])},\ 0.5 \right) \right\},$$

$$F_\mu(e_2) = \left\{ \left(\frac{x_1}{([0.4, 0.7], [0.1, 0.2], [0.3, 0.6])},\ 0.2 \right), \left(\frac{x_2}{([0.1, 0.2], [0.8, 0.9], [0.8, 0.9])},\ 0.3 \right) \right\},$$

$$F_\mu(e_3)=\left\{\left(\frac{x_1}{([0.8,0.9],[0.5,0.6],[0.1,0.2])},\ 0.9\right),\left(\frac{x_2}{([0.3,0.4],[0.4,0.5],[0.6,0.7])},\ 0.6\right)\right\}.$$

Then F_μ is a PNVSS over (U,E). In matrix notation we can write

$$F_\mu=\begin{pmatrix} ([0.2,0.5],[0.5,0.9],[0.5,0.8])\,,\ 0.7 & ([0.7,0.8],[0.2,0.5],[0.2,0.3])\,,\ 0.5 \\ ([0.4,0.7],[0.1,0.2],[0.3,0.6])\,,\ 0.2 & ([0.1,0.2],[0.8,0.9],[0.8,0.9])\,,\ 0.3 \\ ([0.8,0.9],[0.5,0.6],[0.1,0.2])\,,\ 0.9 & ([0.3,0.4],[0.4,0.5],[0.6,0.7])\,,\ 0.6 \end{pmatrix}.$$

In the following, we introduce the concept of the subset of two PNVSSs and give an illustrative example.

Definition 3.3. Let F_μ and G_δ be two PNVSSs over (U,E). Then F_μ is said to be a *possibility neutrosophic vague soft subset* of G_δ and we write $F_\mu\subseteq G_\delta$ if

(1) $\mu(e)$ is a fuzzy subset of $\delta(e)$, $\forall\, e\in E$,
(2) $F(e)$ is a neutrosophic vague subset of $G(e)$, $\forall\, e\in E$.

Example 3.4. Let $U=\{x_1,x_2\}$ be a set of two cars and let $E=\{e_1,e_2,e_3\}$ be a set of parameters where $e_1=$ cheap, $e_2=$ expensive, $e_3=$ red.

Let F_μ be a PNVSS over (U,E) defined as follows:

$$F_\mu(e_1)=\left\{\left(\frac{x_1}{([0.1,0.5],[0.5,0.9],[0.5,0.9])},\ 0.4\right),\left(\frac{x_2}{([0.6,0.8],[0.2,0.9],[0.2,0.4])},\ 0.4\right)\right\},$$
$$F_\mu(e_2)=\left\{\left(\frac{x_1}{([0.6,0.7],[0.1,0.3],[0.3,0.4])},\ 0.6\right),\left(\frac{x_2}{([0.2,0.2],[0.8,0.9],[0.8,0.8])},\ 0.5\right)\right\},$$
$$F_\mu(e_3)=\left\{\left(\frac{x_1}{([0.7,0.9],[0.5,0.6],[0.1,0.3])},\ 0.8\right),\left(\frac{x_2}{([0.2,0.4],[0.4,0.5],[0.6,0.8])},\ 0.5\right)\right\}.$$

Let $G_\delta: E\rightarrow \mathrm{NV(U)}\times I^U$ be another PNVSS over (U,E) defined as follows:

$$G_\mu(e_1)=\left\{\left(\frac{x_1}{([0.1,0.6],[0.4,0.8],[0.4,0.9])},\ 0.5\right),\left(\frac{x_2}{([0.7,0.8],[0.1,0.5],[0.2,0.3])},\ 0.6\right)\right\},$$
$$G_\mu(e_2)=\left\{\left(\frac{x_1}{([0.8,0.9],[0.1,0.2],[0.1,0.2])},\ 0.8\right),\left(\frac{x_2}{([0.3,0.5],[0.7,0.8],[0.5,0.7])},\ 0.6\right)\right\},$$
$$G_\mu(e_3)=\left\{\left(\frac{x_1}{([0.8,0.9],[0.4,0.6],[0.1,0.2])},\ 0.9\right),\left(\frac{x_2}{([0.3,0.5],[0.2,0.4],[0.5,0.7])},\ 0.9\right)\right\}.$$

It is clear that F_μ is a possibility neutrosophic vague soft subset of G_δ.

In the following, we introduce the concept of the equality of two PNVSSs.

Definition 3.5. Let F_μ and G_δ be two PNVSSs over (U,E). Then F_μ and G_δ are said to be *equal* and we write $F_\mu=G_\delta$ if F_μ is a PNVSS subset of G_δ and G_δ is a PNVSS subset of F_μ. In other words, $F_\mu=G_\delta$ if the following conditions are satisfied:

(1) $\mu(e)$ is equal to $\delta(e)$, $\forall\, e\in E$ and (2) $F(e)$ is equal to $G(e)$, $\forall\, e\in E$.

In the following, we introduce the definition of the possibility null neutrosophic vague soft set and the definition of the possibility absolute neutrosophic vague soft set.

Definition 3.6. A PNVSS is said to be a *possibility null neutrosophic vague soft set*, denoted by ϕ_0, if $\phi_0 : E \to NV(U) \times I^U$ such that

$$\phi_0(e) = (F(e)\,(x)\ ,\ \mu(e)\,(x))\,,\ \forall\ e \in E,$$

where $F(e) = ([0,0],[1,1],[1,1])$, and $\mu(e) = 0\,,\ \forall\ e \in E$.

Definition 3.7. A PNVSS is said to be a *possibility absolute neutrosophic vague soft set*, denoted by ψ_1, if $\psi_1 : E \to NV(U) \times I^U$ such that

$$\psi_1(e) = (F(e)\,(x)\ ,\ \mu(e)\,(x))\,,\ \forall\ e \in E,$$

where $F(e) = ([1,1],[0,0],[0,0])$, and $\mu(e) = 1\,,\ \forall\ e \in E$.

In the following, we introduce the definition of the complement of a PNVSS and give an illustrative example.

Definition 3.8. Let F_μ be a PNVSS over (U,E). Then the complement of F_μ, denoted by ${F_\mu}^c$, is defined by

$${F_\mu}^c = (\tilde{c}(F(e)), c(\mu(e)))\quad ,\forall e \in E,$$

where $\tilde{c}$ is a neutrosophic vague complement and c is a fuzzy complement.

Example 3.9. Consider the matrix notation in Example 3.2.

$$F_\mu = \begin{pmatrix} ([0.2,0.5],[0.5,0.9],[0.5,0.8])\,,\ 0.7 & ([0.7,0.8],[0.2,0.5],[0.2,0.3])\,,\ 0.5 \\ ([0.4,0.7],[0.1,0.2],[0.3,0.6])\,,\ 0.2 & ([0.1,0.2],[0.8,0.9],[0.8,0.9])\,,\ 0.3 \\ ([0.8,0.9],[0.5,0.6],[0.1,0.2])\,,\ 0.9 & ([0.3,0.4],[0.4,0.5],[0.6,0.7])\,,\ 0.6 \end{pmatrix}.$$

By using the basic neutrosophic vague complement and fuzzy complement, we have

$$F_\mu^c = \begin{pmatrix} ([0.5,0.8],[0.1,0.5],[0.2,0.5])\,,\ 0.3 & ([0.2,0.3],[0.5,0.8],[0.7,0.8])\,,\ 0.5 \\ ([0.3,0.6],[0.8,0.9],[0.4,0.7])\,,\ 0.8 & ([0.8,0.9],[0.1,0.2],[0.1,0.2])\,,\ 0.7 \\ ([0.1,0.2],[0.4,0.5],[0.8,0.9])\,,\ 0.1 & ([0.6,0.7],[0.5,0.6],[0.3,0.4])\,,\ 0.4 \end{pmatrix}.$$

In the following, we introduce the definition of the union of two PNVSSs and give an illustrative example.

Definition 3.10. *Union* of two PNVSSs F_μ and G_δ denoted by $F_\mu \tilde{\cup} G_\delta$ is a PNVSS $H_\nu : E \to$ NV(U) $\times I^U$ defined by

$$H_\nu(e) = (H\,(e)\,(x)\,,\ \nu\,(e)\,(x))\,,\ \forall e \in E,$$

such that $\nu(e) = max\,(\mu(e)\,,\delta(e))$, $\forall e \in E$, $H(e) = F(e)\,\tilde{\cup}\,G(e)$, $\forall e \in E$, where $\tilde{\cup}$ denote the neutrosophic vague set union.

Example 3.11. Consider Example 3.4 where F_μ and G_δ are PNVSSs defined as follows:

$$F_\mu(e_1) = \left\{ \left(\frac{x_1}{([0.3, 0.5], [0.5, 0.9], [0.5, 0.7])},\ 0.5 \right), \left(\frac{x_2}{([0.4, 0.8], [0.2, 0.9], [0.2, 0.6])},\ 0.6 \right) \right\},$$
$$F_\mu(e_2) = \left\{ \left(\frac{x_1}{([0.7, 0.7], [0.1, 0.3], [0.3, 0.3])},\ 0.6 \right), \left(\frac{x_2}{([0.2, 0.2], [0.8, 0.9], [0.8, 0.8])},\ 0.5 \right) \right\},$$
$$F_\mu(e_3) = \left\{ \left(\frac{x_1}{([0.8, 0.9], [0.5, 0.6], [0.1, 0.2])},\ 0.8 \right), \left(\frac{x_2}{([0.3, 0.4], [0.4, 0.5], [0.6, 0.7])},\ 0.5 \right) \right\},$$

$$G_\mu(e_1) = \left\{ \left(\frac{x_1}{([0.3, 0.6], [0.4, 0.8], [0.4, 0.7])},\ 0.2 \right), \left(\frac{x_2}{([0.5, 0.8], [0.2, 0.5], [0.2, 0.5])},\ 0.8 \right) \right\},$$
$$G_\mu(e_2) = \left\{ \left(\frac{x_1}{([0.1, 0.5], [0.1, 0.2], [0.5, 0.9])},\ 0.7 \right), \left(\frac{x_2}{([0.4, 0.8], [0.7, 0.8], [0.2, 0.6])},\ 0.5 \right) \right\},$$
$$G_\mu(e_3) = \left\{ \left(\frac{x_1}{([0.8, 0.9], [0.4, 0.6], [0.1, 0.2])},\ 0.9 \right), \left(\frac{x_2}{([0.3, 0.5], [0.2, 0.4], [0.5, 0.7])},\ 0.9 \right) \right\}.$$

By using the basic fuzzy union and the basic neutrosophic vague union, we have $F_\mu \tilde{\cup} G_\delta = H_\nu$, where

$$H_\nu(e_1) = \left\{ \left(\frac{x_1}{([0.3, 0.6], [0.4, 0.8], [0.4, 0.7])},\ 0.5 \right), \left(\frac{x_2}{([0.5, 0.8], [0.2, 0.5], [0.2, 0.5])},\ 0.8 \right) \right\},$$
$$H_\nu(e_2) = \left\{ \left(\frac{x_1}{([0.7, 0.7], [0.1, 0.2], [0.3, 0.3])},\ 0.7 \right), \left(\frac{x_2}{([0.4, 0.8], [0.7, 0.8], [0.2, 0.6])},\ 0.5 \right) \right\},$$
$$H_\nu(e_3) = \left\{ \left(\frac{x_1}{([0.8, 0.9], [0.4, 0.6], [0.1, 0.2])},\ 0.9 \right), \left(\frac{x_2}{([0.3, 0.5], [0.2, 0.4], [0.5, 0.7])},\ 0.9 \right) \right\}.$$

In matrix notation, we can write

$$H_\nu(e) = \begin{pmatrix} ([0.3, 0.6], [0.4, 0.8], [0.4, 0.7]),\ 0.5 & ([0.5, 0.8], [0.2, 0.5], [0.2, 0.5]),\ 0.8 \\ ([0.7, 0.7], [0.1, 0.2], [0.3, 0.3]),\ 0.7 & ([0.4, 0.8], [0.7, 0.8], [0.2, 0.6]),\ 0.5 \\ ([0.8, 0.9], [0.4, 0.6], [0.1, 0.2]),\ 0.9 & ([0.3, 0.5], [0.2, 0.4], [0.5, 0.7]),\ 0.9 \end{pmatrix}.$$

In the following, we introduce the definition of the intersection of two PNVSSs and give an illustrative example.

Definition 3.12. *Intersection* of two PNVSSs F_μ and G_δ denoted by $F_\mu \tilde{\cap} G_\delta$ is a PNVSS $H_\nu : E \to \mathrm{NV(U)} \times I^U$ defined by

$$H_\nu(e) = (H\,(e)\,(x)\,,\ \nu\,(e)\,(x))\,, \forall e \in E,$$

such that $\nu(e) = min\,(\mu(e)\,, \delta(e))$, $\forall e \in E$, $H(e) = F(e) \tilde{\cap} G(e)$, $\forall e \in E$, where $\tilde{\cap}$ denotes the neutrosophic vague set intersection.

Example 3.13. Consider Example 3.11. By using the basic fuzzy intersection and the basic neutrosophic vague intersection, we have $F_\mu \tilde{\cap} G_\delta = H_\nu$, where

$$H_\nu(e_1)=\left\{\left(\frac{x_1}{([0.3,0.5],[0.5,0.9],[0.5,0.7])},\ 0.2\right),\left(\frac{x_2}{([0.4,0.8],[0.2,0.9],[0.2,0.6])},\ 0.6\right)\right\},$$

$$H_\nu(e_2)=\left\{\left(\frac{x_1}{([0.1,0.5],[0.1,0.3],[0.5,0.9])},\ 0.6\right),\left(\frac{x_2}{([0.2,0.2],[0.8,0.9],[0.8,0.8])},\ 0.5\right)\right\},$$

$$H_\nu(e_3)=\left\{\left(\frac{x_1}{([0.8,0.9],[0.5,0.6],[0.1,0.2])},\ 0.8\right),\left(\frac{x_2}{([0.3,0.4],[0.4,0.5],[0.6,0.7])},\ 0.5\right)\right\}.$$

In matrix notation, we can write

$$H_\nu(e)=\begin{pmatrix} ([0.3,0.5],[0.5,0.9],[0.5,0.7])\,,\ 0.2 & ([0.4,0.8],[0.2,0.9],[0.2,0.6])\,,\ 0.6 \\ ([0.1,0.5],[0.1,0.3],[0.5,0.9])\,,\ 0.6 & ([0.2,0.2],[0.8,0.9],[0.8,0.8])\,,\ 0.5 \\ ([0.8,0.9],[0.5,0.6],[0.1,0.2])\,,\ 0.8 & ([0.3,0.4],[0.4,0.5],[0.6,0.7])\,,\ 0.5 \end{pmatrix}.$$

Now we give a proposition on the union and intersection of two PNVSSs.

Proposition 3.14. *Let F_μ, G_δ and H_ν be any three PNVSSs over (U,E). Then the following results hold:*

(1) $F_\mu \tilde{\cup} G_\delta = G_\delta \tilde{\cup} F_\mu$,
(2) $F_\mu \tilde{\cap} G_\delta = G_\delta \tilde{\cap} F_\mu$,
(3) $F_\mu \tilde{\cup} (G_\delta \tilde{\cup} H_v) = (F_\mu \tilde{\cup} G_\delta) \tilde{\cup} H_v$,
(4) $F_\mu \tilde{\cap} (G_\delta \tilde{\cap} H_v) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cap} H_v$.

Proof. The proofs are straightforward by using the fact that neutrosophic vague sets are commutative and associative. ∎

In the following, we introduce the definition of AND and OR operations for PNVSS.

Definition 3.15. If (F_μ, A) and (G_δ, B) are two PNVSSs then "(F_μ, A) AND (G_δ, B)", denoted by $(F_\mu, A)\ \wedge (G_\delta, B)$ is defined by

$$(F_\mu, A)\wedge(G_\delta, B) = (H_\lambda, A\times B)\,,$$

where $H_\lambda(\alpha,\beta) = (H(\alpha,\beta)(x)\,,\ \lambda(\alpha,\beta)(x))\,,\ \forall(\alpha,\beta)\in A\times B$, such that $H(\alpha,\beta) = F(\alpha)\cap G(\beta)$ and $\lambda(\alpha,\beta) = \min(\mu(\alpha),\delta(\beta))$, $\forall(\alpha,\beta)\in A\times B$, and $\cap$ represents the basic neutrosophic vague intersection.

Definition 3.16. If (F_μ, A) and (G_δ, B) are two PNVSSs then "(F_μ, A) OR (G_δ, B)", denoted by $(F_\mu, A)\vee(G_\delta, B)$ is defined by

$$(F_\mu, A)\vee(G_\delta, B) = (H_\lambda, A\times B)\,,$$

where $H_\lambda(\alpha,\beta) = (H(\alpha,\beta)(x)\,,\ \lambda(\alpha,\beta)(x))\,,\ \forall(\alpha,\beta)\in A\times B$, such that $H(\alpha,\beta) = F(\alpha)\cup G(\beta)$ and $\lambda(\alpha,\beta) = \max(\mu(\alpha),\delta(\beta))$, $\forall(\alpha,\beta)\in A\times B$, and $\cup$ represents the basic neutrosophic vague union.

4. Similarity between Two Possibility Neutrosophic Vague Soft Set

In this section we introduce a measure of similarity between two PNVSSs and give an illustrative example.

Several researchers have studied the problem of similarity measurement between fuzzy sets, fuzzy numbers and vague sets. Majumdar and Samanta [23–25] have studied the similarity measure of soft sets, fuzzy soft sets and generalised fuzzy soft sets.

It is well known that similarity measures can be generated from distance measures [26–29], and Ye [30] used the proposed distance measures in his paper to define some similarity measures between interval neutrosophic sets A and B. In this paper we will use the same similarity measure that Ye [30] used, since neutrosophic vague value is an interval value and similarity measures can be generated from distance measures. We define similarity measures between two PNVSSs as follows:

Definition 4.1. Let F_μ and G_δ be two PNVSSs over (U, E). *Similarity* between F_μ and G_δ, denoted by $S(F_\mu, G_\delta)$, is defined as follows:
$S(F_\mu, G_\delta) = M(F(e), G(e)) \cdot M(\mu(e), \delta(e))$ such that
$M(F(e), G(e)) = \max_i M_i(F(e), G(e))$ and $M(\mu(e), \delta(e)) = \max_i M_i(\mu(e), \delta(e))$ where
$M_i(F(e), G(e)) = 1 - \frac{1}{6}\sum_{j=1}^{n}[\,|\Delta_j\ t(x_j)| + |\Delta_j\ f^*(x_j)| + |\Delta_j\ \inf I(x_j)| + |\Delta_j\ \sup I(x_j)| + |\Delta_j\ f(x_j)| + |\Delta_j\ t^*(x_j)|\,]$,

$M_i(\mu(e), \delta(e)) = 1 - \frac{\sum_{j=1}^{n}|\mu_{ij}(e) - \delta_{ij}(e)|}{\sum_{j=1}^{n}|\mu_{ij}(e) + \delta_{ij}(e)|}$, where $\Delta_j\ t(x_j) = t_{F(e)}(x_j) - t_{G(e)}(x_j)$, $\Delta_j\ f^*(x_j) = f^*_{F(e)}(x_j) - f^*_{G(e)}(x_j)$, $\Delta_j\ \inf I(x_j) = \inf I_{F(e)}(x_j) - \inf I_{G(e)}(x_j)$, $\Delta_j\ \sup I(x_j) = \sup I_{F(e)}(x_j) - \sup I_{G(e)}(x_j)$, $\Delta_j\ f(x_j) = f_{F(e)}(x_j) - f_{G(e)}(x_j)$, $\Delta_j\ t^*(x_j) = t^*_{F(e)}(x_j) - t^*_{G(e)}(x_j)$, where $f^*(x_j) = 1 - f(x_j)$ and $t^*(x_j) = 1 - t(x_j)$.

Definition 4.2. Let F_μ and G_δ be two PNVSSs over (U, E). We say that F_μ and G_δ are *significantly similar* if $S(F_\mu, G_\delta) \geq \frac{1}{2}$.

Proposition 4.3. *Let F_μ and G_δ be any two PNVSSs over (U, E) such that F_μ or G_δ is a non-zero PNVSS. Then the following holds:*

(1) $S(F_\mu, G_\delta) = S(G_\delta, F_\mu)$,
(2) $0 \leq S(F_\mu, G_\delta) \leq 1$,
(3) $F_\mu = G_\delta \Rightarrow S(F_\mu, G_\delta) = 1$,
(4) $F_\mu \subseteq G_\delta \subseteq H_\lambda \Rightarrow S(F_\mu, H_\lambda) \leq S(G_\delta, H_\lambda)$,
(5) $F_\mu \tilde{\cap} G_\delta = \emptyset \Rightarrow S(F_\mu, G_\delta) = 0$.

Proof. The proof is straightforward and follows from Definition 4.1. ∎

Example 4.4. Consider Example 3.4 where F_μ and G_δ are defined as follows:

$$F_\mu(e_1) = \left\{\left(\frac{x_1}{([0.1, 0.5], [0.5, 0.9], [0.5, 0.9])}, 0.4\right), \left(\frac{x_2}{([0.6, 0.8], [0.2, 0.9], [0.2, 0.4])}, 0.4\right)\right\},$$

$$F_\mu(e_2)=\left\{\left(\frac{x_1}{([0.6,0.7],[0.1,0.3],[0.3,0.4])},\ 0.6\right),\left(\frac{x_2}{([0.2,0.2],[0.8,0.9],[0.8,0.8])},\ 0.5\right)\right\},$$
$$F_\mu(e_3)=\left\{\left(\frac{x_1}{([0.7,0.9],[0.5,0.6],[0.1,0.3])},\ 0.8\right),\left(\frac{x_2}{([0.2,0.4],[0.4,0.5],[0.6,0.8])},\ 0.5\right)\right\},$$

$$G_\mu(e_1)=\left\{\left(\frac{x_1}{([0.1,0.6],[0.4,0.8],[0.4,0.9])},\ 0.5\right),\left(\frac{x_2}{([0.7,0.8],[0.1,0.5],[0.2,0.3])},\ 0.6\right)\right\},$$
$$G_\mu(e_2)=\left\{\left(\frac{x_1}{([0.8,0.9],[0.1,0.2],[0.1,0.2])},\ 0.8\right),\left(\frac{x_2}{([0.3,0.5],[0.7,0.8],[0.5,0.7])},\ 0.6\right)\right\},$$
$$G_\mu(e_3)=\left\{\left(\frac{x_1}{([0.8,0.9],[0.4,0.6],[0.1,0.2])},\ 0.9\right),\left(\frac{x_2}{([0.3,0.5],[0.2,0.4],[0.5,0.7])},\ 0.9\right)\right\}.$$

Here,

$$M_1\left(\mu(e),\,\delta(e)\right)=1-\frac{\sum_{j=1}^{2}|\mu_{1j}(e)-\delta_{1j}(e)|}{\sum_{j=1}^{2}|\mu_{1j}(e)+\delta_{1j}(e)|}=\ 1-\frac{|(0.4-0.5)|+|(0.4-0.6)|}{|(0.4+0.5)|+|(0.4+0.6)|}=0.84.$$

Similarly we get $M_2\left(\mu(e),\,\delta(e)\right)=0.88$ and $M_3\left(\mu(e),\,\delta(e)\right)=0.84$. Then

$$M\left(\mu(e),\,\delta(e)\right)=\max\left(M_1\left(\mu(e),\,\delta(e)\right),M_2\left(\mu(e),\,\delta(e)\right),M_3\left(\mu(e),\,\delta(e)\right)\right)=0.88$$

$$M_1\left(F(e),\,G(e)\right)=1-\tfrac{1}{6}\sum_{j=1}^{2}[\ |\ \Delta_j\ t(x_j)\ |+|\ \Delta_j\ f^*(x_j)\ |+|\ \Delta_j\inf\ I(x_j)\ |$$
$$+\ |\ \Delta_j\sup I(x_j)\ |+|\ \Delta_j\ f(x_j)\ |+|\ \Delta_j\ t^*(x_j)\ |\ ]$$

$$=1-\tfrac{1}{6}[(|\ 0.1-0.1\ |+|\ 0.5-0.6\ |+|\ 0.5{-}0.4\ |+|\ 0.9{-}0.8\ |+|\ 0.5-0.4\ |$$
$$+\ |\ 0.9{-}0.9\ |)+(|\ 0.6-0.7\ |+|\ 0.8-0.8\ |+|\ 0.2-0.1\ |+|\ 0.9-0.5\ |$$
$$+\ |\ 0.2-0.2\ |+|\ 0.4-0.3\ |)]=0.82.$$

Similarly we get $M_2\left(F(e),\,G(e)\right)=0.68$ and $M_3\left(F(e),\,G(e)\right)=0.83$. Then,

$$M\left(F(e),\,G(e)\right)=\max\left(M_1\left(F(e),\,G(e)\right),\,M_2\left(F(e),\,G(e)\right),M_3\left(F(e),\,G(e)\right)\right)=0.83$$

Hence the similarity between the two PNVSSs F_μ and G_δ is given by

$$S\left(F_\mu,\,G_\delta\right)=M\left(F(e),\,G(e)\right)\ .M\left(\mu(e),\,\delta(e)\right)=0.83\times 0.88\cong 0.73.$$

5. Application of Similarity Measure in Medical Diagnosis

In the following example we will try to estimate the possibility that a sick person having certain visible symptoms is suffering from heart attack. For this we first construct a model possibility neutrosophic vague soft set for heart attack and the possibility neutrosophic vague soft set of symptoms for the sick person. Next we find the similarity measure of these two sets. If they are significantly similar then we conclude that the person is possibly suffering from heart attack.

Let our universal set contain only two elements “yes” and “no”, that is, $U=\{y,n\}$. Here the set of parameters E is the set of certain visible symptoms. Let $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$, where e_1 = chest discomfort, e_2 = upper body discomfort, e_3 = fatigue, e_4 = shortness of breath, e_5 = sweating, e_6 = nausea and vomiting.

Our model possibility neutrosophic vague soft set for heart attack F_μ is given in Table 1 and this can be prepared with the help of a physician. After talking to the sick person we can construct his PNVSS G_δ as in Table 2.

TABLE 1. model PNVSS for heart attack

F_μ	e_1	e_2	e_3
(y)	$\langle[1,1];[0,0];[0,0]\rangle$	$\langle[1,1];[0,0];[0,0]\rangle$	$\langle[0,0];[1,1];[1,1]\rangle$
(μ_y)	1	1	1
(n)	$\langle[0,0];[1,1];[1,1]\rangle$	$\langle[0,0];[1,1];[1,1]\rangle$	$\langle[1,1];[0,0];[0,0]\rangle$
(μ_n)	1	1	1
(F_μ)	e_4	e_5	e_6
(y)	$\langle[1,1];[0,0];[0,0]\rangle$	$\langle[1,1];[0,0];[0,0]\rangle$	$\langle[0,0];[1,1];[1,1]\rangle$
(μ_y)	1	1	1
(n)	$\langle[0,0];[1,1];[1,1]\rangle$	$\langle[0,0];[1,1];[1,1]\rangle$	$\langle[1,1];[0,0];[0,0]\rangle$
(μ_n)	1	1	1

TABLE 2. PNVSS for the sick person

G_δ	e_1	e_2	e_3
(y)	$\langle[0.6,0.8];[0.1,0.2];[0.2,0.4]\rangle$	$\langle[.8,0.9];[0.2,0.4];[0.1,0.2]\rangle$	$\langle[0,0.2];[0.5,0.6];[0.8,1]\rangle$
(δ_y)	0.8	0.6	0.7
(n)	$\langle[0.5,0.6];[0.4,0.7];[0.4,0.5]\rangle$	$\langle[0.3,0.5];[0.4,0.6];[0.5,0.7]\rangle$	$\langle[0.9,1];[0.2,0.6];[0,0.1]\rangle$
(δ_n)	0.3	0.6	0.2
G_δ	e_4	e_5	e_6
(y)	$\langle[0.7,0.9];[0.2,0.6];[0.1,0.3]\rangle$	$\langle[0.4,0.8];[0.2,0.5];[0.2,0.6]\rangle$	$\langle[0.2,0.3];[0.1,0.5];[0.7,0.8]\rangle$
(δ_y)	0.9	0.6	0.1
(n)	$\langle[0.8,0.9];[0.5,0.6];[0.1,0.2]\rangle$	$\langle[0,0.4];[0.3,0.5];[0.6,1]\rangle$	$\langle[0.5,0.6];[0.4,0.6];[0.4,0.5]\rangle$
(δ_n)	0.6	0.9	0.5

Now we find the similarity measure of these two sets (as in Example 4.4), here $S(F_\mu, G_\delta) \cong 0.54 > \frac{1}{2}$. Hence the two PNVSSs are significantly similar. Therefore, we conclude that the person is suffering from heart attack.

6. Comparison Between PNVSS to Other Existing Methods

In this section, we will compare our proposed PNVSS model to two other existing models that are closest in structure to the PNVSS. These models are the possibility vague soft set [14] and possibility neutrosophic soft set [20].

The possibility vague soft set [14] is actually a generalization of possibility fuzzy soft set [11], while the possibility neutrosophic soft set [20] is actually a generalization of possibility fuzzy soft set [11] and possibility intuitionistic fuzzy soft set [21].

To reveal the significance of our proposed PNVSS compared to possibility vague soft set [14], note that the PNVSS is a generalization of possibility vague soft set. The PNVSS can explain the universal set U in more detail with three membership functions, whereas possibility vague soft set can tell us a limited information about the universal set U. It can only handle the incomplete information considering both the truth-membership and falsity-membership values, while PNVSS can handle problems involving imprecise, indeterminacy and inconsistent data, which makes it more accurate and realistic than possibility vague soft set [14].

It is worthy to note that PNVSS is more advantageous than possibility neutrosophic soft set [20] by virtue of vague set which allows using interval-based membership instead of using point-based membership as in neutrosophic set. This enables PNVSS to better capture the vagueness and uncertainties of the data which is prevalent in most real-life situations.

7. Conclusions

In this paper, we reviewed the basic concepts of neutrosophic vague set and possibility fuzzy soft set, and gave some basic operations on both neutrosophic vague set and possibility fuzzy soft set, before establishing the concept of possibility neutrosophic vague soft set. The basic operations on possibility neutrosophic vague soft set, namely complement, subset, equality, union, intersection, AND, and OR operations, were defined. A similarity measure of two possibility neutrosophic vague sets is introduced and discussed. Also an application of this similarity measure in medical diagnosis has been shown. We intend to further explore the applications of possibility neutrosophic vague soft set approach to solve certain decision making problems in order to provide a significant addition to existing theories for handling uncertainties, especially in the area of neutrosophic vague soft expert set [31], Q-fuzzy [32–34], vague soft expert sets [35–37], interval convergence [38–41] and point convergence [42, 43].

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