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On 0-Minimal (0,2)-Bi-Hyperideal of Ordered Semihypergroups

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Abstract Focusing on the ordered semihypergroup, the goal is to find conditions of minimality of left (right) hyperideal, bi-hyperideal and $(0,2)$-hyperideal in ordered semihypergroups. The study begins by examining basic properties of $(0,2)$-hyperideal and bi-hyperideal. Using such knowledge demonstrates that if A is a 0-minimal $(0,2)$-bi-hyperideal of an ordered semihypergroup H with zero, then either $(A^2] = \{0\}$ or A is a left 0-simple.

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1. Introduction and Preliminaries

Algebraic hyperstructures were introduced in 1934 by the French mathematician F. Marty [1]. He defined hypergroups as a generalization of groups. There are many significant results regarding semihypergroups, hypergroups, hyperrings and hyperfields. D. N. Krgović ([2] and [3]) studied minimality of bi-ideal in semigroups and S. Hobanthad and W. Jantanan [4] extended the findings to semihypergroups. The main purpose of this paper seeks conditions of minimality of left (right) hyperideal, hyperideal, bi-hyperideal, $(0,2)$-hyperideal and $(1,2)$-hyperideal of ordered semihypergroups. This paper extend the result of W. Jantana and T. Changphas [5] to ordered semihypergroups. The author starting recalls the terminologies of semihypergroups with zero from P. Corsini and V. Leoreanu ([6] and [7]) as follows:

A *hyperoperation* on a nonempty set H is a map $\circ : H \times H \to P^*(H)$ where $P^*(H)$ is the family of the nonempty subset of H. If A and B are nonempty subsets of H and $x \in H$, then

$$A \circ B = \bigcup_{a\in A, b\in B} a \circ b; \; x \circ A = \{x\} \circ A \text{ and } A \circ x = A \circ \{x\}.$$

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A *semihypergroup* is a system $(H, \circ)$ where H is a nonempty set, $\circ$ is a hyperoperation on H and $(x \circ y) \circ z = x \circ (y \circ z)$, for all $x, y, z \in H$. An element e of a semihypergroup H is called an *identity* of $(H, \circ)$ if $x \in (x \circ e) \cap (e \circ x)$ for all $x \in H$, and it is called a *scalar identity* of $(H, \circ)$ if $(x \circ e) \cap (e \circ x) = \{x\}$, for all $x \in H$. A semihypergroup H with an element 0 such that $0 \circ x = x \circ 0 = \{0\}$ for all x in H; then, 0 is a zero element of H, and H is called a *semihypergroup with zero*.

Definition 1.1 ([8])**.** An algebraic hyperstructure $(H, \circ, \leq)$ is called an *ordered semihypergroup* if $(H, \circ)$ is a semihypergroup and $\leq$ is an order relation on H such that the monotone condition holds as follows:

$$x \leq y \Rightarrow a \circ x \leq a \circ y, \text{ for all } x, y, a \in H.$$

Where, let A and B be nonempty subsets of H. If for every $a \in A$, there exist $b \in B$ such that $a \leq b$, then $A \leq B$.

A nonempty subset A of a ordered semihypergroup H is called a *subsemihypergroup* of H if $A \circ A \subseteq A$.

Definition 1.2 ([8])**.** A nonempty subset A of an ordered semihypergroup $(H, \circ, \leq)$ is called a *left (right) hyperideal* of H if the following conditions hold:

1. $H \circ A \subseteq A (A \circ H \subseteq A)$;
2. If $a \in A$ and $b \leq a$; then, $b \in A$ for every $b \in H$.

A is called a *hyperideal* of H if it is a left and right hyperideal. If $(H, \circ, \leq)$ is an ordered semihypergroup and $A \subseteq H$; then, $(A]$ is the subset of H defined as follows:

$$(A] = \{t \in H : t \leq a \text{ for some } a \in A\}$$

Proposition 1.3 ([8])**.** *Let $(H, \circ, \leq)$ be an ordered semihypergroup then the following holds:*

1. *$A \subseteq (A]$ for every $A \subseteq H$.*
2. *If $A \subseteq B$; then, $(A] \subseteq (B]$ for every $A, B \subseteq H$.*
3. *$(A] \circ (B] \subseteq (A \circ B]$ for every $A, B \subseteq H$.*
4. *$((A]] = (A]$ for every $A \subseteq H$.*
5. *If A and B are hyperideals of H; then, $(A \circ B]$ and $A \cup B$ are hyperideals of H.*
6. *For every $a \in H$, $(H \circ a \circ H]$ is a hyperideal of H.*
7. *If $A, B, C \subseteq H$ such that $A \subseteq B$; then, $C \circ A \subseteq C \circ B$ and $A \circ C \subseteq B \circ C$.*

Definition 1.4 ([9])**.** Let $(H, \circ, \leq)$ be an ordered semihypergroup and let m, n be nonnegative integer. A subsemihypergroup A of H is called a *(m, n)-hyperideal* of H if the following hold:

1. $A^m \circ H \circ A^n \subseteq A$;
2. If $a \in A$ and $b \leq a$, then $b \in A$ for every $b \in H$ or $(A] = A$.

From Definition 1.4, if $m = n = 1$; then, A is called a *bi-hyperideal* of H. If $m = 0$ and $n = 2$; then, A is called a *$(0, 2)$-hyperideal* of H.

Definition 1.5 ([9])**.** A subsemihypergroup A of an ordered semihypergroup $(H, \circ, \leq)$ is called *$(0, 2)$-bi-hyperideal* of H if A is both a bi-hyperideal and $(0, 2)$-hyperideal of H.

Let H be a semihypergroup with zero and L is a left hyperideal of H. Since $H \circ L^2 \subseteq H \circ L \subseteq L$; then, L is a $(0, 2)$-hyperideal of H. Therefore, every left hyperideal of H is a $(0, 2)$-hyperideal of H.

2. Main Results

If A is a subsemihypergroup of the ordered semihypergroup $(H, \circ, \leq)$; then, $H \circ (A \cup H \circ A] \subseteq (H \circ A \cup H^2 \circ A] \subseteq (H \circ A] \subseteq (A \cup H \circ A]$. Thus, $(A \cup H \circ A]$ is a left hyperideal of H. Since,

$$\begin{aligned}(A^2 \cup A \circ H \circ A^2] \circ H \circ (A^2 \cup A \circ H \circ A^2] &\subseteq (A^2 \circ H \circ A^2 \cup A^2 \circ H \circ A \circ H \circ A^2 \\ &\quad \cup A \circ H \circ A^2 \circ H \circ A^2 \\ &\quad \cup A \circ H \circ A^2 \circ H \circ A \circ H \circ A^2] \\ &\subseteq (A \circ H \circ A^2] \\ &\subseteq (A^2 \cup A \circ H \circ A^2];\end{aligned}$$

then, $(A^2 \cup A \circ H \circ A^2]$ is a bi-hyperideal of H. Since

$$(A \cup A \circ H] \circ H \subseteq (A \circ H \cup A \circ H^2] \subseteq (A \circ H] \subseteq (A \cup A \circ H],$$

$(A \cup A \circ H]$ is a right hyperideal of H. Moreover, $(A \cup H \circ A^2]$ is a $(0,2)$-hyperideal of H, because

$$\begin{aligned}H \circ (A \cup H \circ A^2]^2 &= H \circ (A \cup H \circ A^2] \circ (A \cup H \circ A^2] \\ &\subseteq (H \circ A^2 \cup H \circ A \circ H \circ A^2 \cup H^2 \circ A^3 \cup H^2 \circ A^2 \circ H \circ A^2] \\ &\subseteq (H \circ A^2] \\ &\subseteq (A \cup H \circ A^2].\end{aligned}$$

Lemma 2.1. *Let $(H, \circ, \leq)$ be an ordered semihypergroup . Then, A is a $(0,2)$-hyperideal of H if and only if A is a left hyperideal of some left hyperideal of H.*

Proof. If A is a $(0,2)$-hyperideal of H; then,

$$(A \cup H \circ A] \circ A \subseteq (A^2 \cup H \circ A^2] \subseteq (A] = A.$$

Thus, A is a left hyperideal of left hyperideal $(A \cup H \circ A]$ of H. Conversely, assume that A is a left hyperideal of left hyperideal L of H. Then, $H \circ A^2 \subseteq H \circ L \circ A \subseteq L \circ A \subseteq A$. Let $a \in A$ and $b \in H$ be such that $b \leq a$. Since $a \in L$, so $b \in L$. The assumption implies $b \in A$. Therefore, A is $(0,2)$-hyperideal of H. ∎

Theorem 2.2. *Let $(H, \circ, \leq)$ be an ordered semihypergroup . The following statements are equivalent:*

1. *A is a $(1,2)$-hyperideal of H;*
2. *A is a left hyperideal of some bi-hyperideal of H;*
3. *A is a bi-hyperideal of some left hyperideal of H;*
4. *A is a $(0,2)$-hyperideal of some right hyperideal of H;*
5. *A is a right hyperideal of some $(0,2)$-hyperideal of H.*

Proof. $(1 \Rightarrow 2)$ If A is a $(1,2)$-hyperideal of H; then, $(A^2 \cup A \circ H \circ A^2] \circ A = (A^2 \cup A \circ H \circ A^2] \circ (A] \subseteq (A^3 \cup A \circ H \circ A^3] \subseteq (A^2 \cup A \circ H \circ A^2] \subseteq (A] = A$. Clearly, if $a \in A, b \in (A^2 \cup A \circ H \circ A^2]$ such that $b \leq a$; then, $b \in A$. Hence, A is a left hyperideal of the bi-hyperideal $(A^2 \cup A \circ H \circ A^2]$ of H.

$(2 \Rightarrow 3)$ If A is a left hyperideal of a bi-hyperideal B of H; then, $A^2 \subseteq B \circ A \subseteq A$ and $A \circ (A \cup H \circ A] \circ A = (A] \circ (A \cup H \circ A] \circ (A] \subseteq (A^3 \cup A \circ H \circ A^2] \subseteq (A \cup B \circ H \circ B \circ A] \subseteq$

$(A \cup B \circ A] \subseteq (A] = A$. Let $a \in A, b \in (A \cup H \circ A]$ such that $b \leq a$. Since $a \in A$, so $a \in B$. Thus, $b \in B$. Hence, $b \in A$. Therefore, A is a bi-hyperideal of the left hyperideal $(A \cup H \circ A]$ of H.

$(3 \Rightarrow 4)$ If A is a bi-hyperideal of some left hyperideal L of H; then $(A \cup A \circ H] \circ A^2 \subseteq (A \cup A \circ H] \circ (A^2] \subseteq (A^3 \cup A \circ H \circ A^2] \subseteq (A \cup A \circ H \circ L \circ A] \subseteq (A \cup A \circ L \circ A] \subseteq (A] = A$. Let $a \in A, b \in (A \cup A \circ H]$ such that $b \leq a$; then, $a \in L$. Thus $b \in L$. Thus $b \in A$. Hence, A is a $(0,2)$-hyperideal of the right hyperideal $(A \cup A \circ H]$ of H.

$(4 \Rightarrow 5)$ If A is a $(0,2)$-hyperideal of some right hyperideal R of H; then, $A \circ (A \cup H \circ A^2] \subseteq (A^2 \cup A \circ H \circ A^2] \subseteq (A \cup R \circ H \circ A^2] \subseteq (A \cup R \circ A^2] \subseteq (A] = A$. Assume that $a \in A, b \in (A \cup H \circ A^2]$ such that $b \leq a$. Since $a \in R$, so $b \in R$. Thus, $b \in A$. Hence, A is a right hyperideal of the $(0,2)$-hyperideal $(A \cup H \circ A^2]$ of H.

$(5 \Rightarrow 1)$ If A is a right hyperideal of a $(0,2)$-hyperideal R of H; then, $A \circ H \circ A^2 \subseteq A \circ H \circ R^2 \subseteq A \circ R \subseteq A$. Assume that $a \in A, b \in H$ such that $b \leq a$. Since $a \in R$, so $b \in R$. Thus, $b \in A$. Hence, A is a $(1,2)$-hyperideal of H. ∎

Lemma 2.3. *Let $(H, \circ, \leq)$ be an ordered semihypergroup and let A be a subsemihypergroup of H such that $A = (A]$. Then, A is a $(1,2)$-hyperideal of H if and only if there exist a $(0,2)$-hyperideal L of H and a right hyperideal R of H such that $R \circ L^2 \subseteq A \subseteq R \cap L$.*

Proof. Assume that A is a $(1,2)$-hyperideal of H. Since $(A \cup H \circ A^2]$ and $(A \cup A \circ H]$ are $(0,2)$-hyperideal and right hyperideal of H, respectively.
Setting $L = (A \cup H \circ A^2]$ and $R = (A \cup A \circ H]$; so,

$$\begin{aligned} R \circ L^2 &\subseteq (A \cup A \circ H] \circ (A \cup H \circ A^2] \circ (A \cup H \circ A^2] \\ &\subseteq (A^3 \cup A^2 \circ H \circ A^2 \cup A \circ H \circ A^3 \cup A \circ H \circ A^2 \circ H \circ A^2 \\ &\quad \cup A \circ H \circ A^2 \cup A \circ H \circ A \circ H \circ A^2 \cup A \circ H^2 \circ A^3 \\ &\quad \cup A \circ H^2 \circ A^2 \circ H \circ A^2] \\ &\subseteq (A^3 \cup A \circ H \circ A^2] \\ &\subseteq (A] = A. \end{aligned}$$

Clearly, it is $A \subseteq R \cap L$. Hence, $R \circ L^2 \subseteq A \subseteq R \cap L$. Conversely, let R be a right hyperideal of H and L be a $(0,2)$-hyperideal of H such that $R \circ L^2 \subseteq A \subseteq R \cap L$. Then, $A \circ H \circ A^2 \subseteq (R \cap L) \circ H \circ (R \cap L) \circ (R \cap L) \subseteq R \circ H \circ L^2 \subseteq R \circ L^2 \subseteq A$. Hence, A is a $(1,2)$-hyperideal of H. ∎

A left hyperideal, right hyperideal, hyperideal, $(0,2)$-hyperideal and $(0,2)$-bi-hyperideal A of an ordered semihypergroup $(H, \circ, \leq)$ with zero will be said to be *0-minimal* if $A \neq \{0\}$ and $\{0\}$ is the only left hyperideal, right hyperideal, hyperideal, $(0,2)$-hyperideal, $(0,2)$-bi-hyperideal, respectively of H properly contained in A. From every left hyperideal of H is a $(0,2)$-hyperideal of H. Hence, if L is a 0-minimal $(0,2)$-hyperideal of H and A is a left hyperideal of H contained in L; then, $A = \{0\}$ or $A = L$.

Lemma 2.4. *Let $(H, \circ, \leq)$ be an ordered semihypergroup with zero. If L is a 0-minimal left hyperideal of H and A is a subsemihypergroup with zero of L such that $A = (A]$; then, A is a $(0,2)$-hyperideal of H contained in L if and only if $(A^2] = \{0\}$ or $A = L$.*

Proof. Assume that A is a $(0,2)$-hyperideal of H contained in L, then $(H \circ A^2] \subseteq L$. Since $(H \circ A^2]$ is a left hyperideal of H, so $(H \circ A^2] = \{0\}$ or $(H \circ A^2] = L$. If $(H \circ A^2] = L$.

Then, $L = (H \circ A^2] \subseteq (A] = A$. Hence, $A = L$. If $(H \circ A^2] = \{0\}$. Thus, $H \circ (A^2] \subseteq (H \circ A^2] = \{0\} \subseteq (A^2]$. Therefore, $(A^2]$ is a left hyperideal of H contained in L. By the minimality of L, $(A^2] = \{0\}$ or $(A^2] = L$. If $(A^2] = L$; then, $L = (A^2] \subseteq (A] = A$. Hence, $A = L$. The opposite direction is clear. ∎

Lemma 2.5. *Let $(H, \circ, \leq)$ be an ordered semihypergroup with zero. If L is a 0-minimal $(0, 2)$-hyperideal of H; then, $(L^2] = \{0\}$ or L is a 0-minimal left hyperideal of H.*

Proof. Assume that L is a 0-minimal $(0, 2)$-hyperideal of H. Consider $H \circ (L^2]^2 = H \circ (L^2] \circ (L^2] \subseteq (H \circ L^2] \circ (L^2] \subseteq (L^2]$. Then, $(L^2]$ is a $(0, 2)$-hyperideal of H contained in L. Hence, $(L^2] = \{0\}$ or $(L^2] = L$. Suppose that $(L^2] = L$. Since $H \circ L = H \circ (L^2] \subseteq (H \circ L^2] \subseteq (L] = L$. Thus, L is left hyperideal of H. Let B be a left hyperideal of H contained in L . Therefore, B is a $(0, 2)$-hyperideal of H contained in L. Then, $B = \{0\}$ or $B = L$. Thus, L is a 0-minimal left hyperideal of H. ∎

The following corollary follows from Lemma 2.4 and Lemma 2.5.

Corollary 2.6. *Let $(H, \circ, \leq)$ be an ordered semihypergroup without zero. Then, L is a minimal $(0, 2)$-hyperideal of H if and only if L is a minimal left hyperideal of H.*

Lemma 2.7. *Let $(H, \circ, \leq)$ be an ordered semihypergroup without zero and let A be a nonempty subset of H. Then, A is a minimal $(2, 1)$-hyperideal of H if and only if A is a minimal bi-hyperideal of H.*

Proof. Assume that A is a minimal $(2, 1)$-hyperideal of H. Since, $(A^2 \circ H \circ A]^2 \circ H \circ (A^2 \circ H \circ A] \subseteq (A^2 \circ H \circ A]$ and $(A^2 \circ H \circ A] \subseteq (A] = A$; then, $(A^2 \circ H \circ A]$ is a $(2, 1)$-hyperideal of H contained in A. Therefore, $(A^2 \circ H \circ A] = A$. Since $A \circ H \circ A = (A^2 \circ H \circ A] \circ H \circ A \subseteq (A^2 \circ H \circ A \circ H \circ A] \subseteq (A^2 \circ H \circ A] = A$; then, A is a bi-hyperideal of H. Suppose that there exist a bi-hyperideal B of H contained in A. Then, $B^2 \circ H \circ B \subseteq B^2 \subseteq B \subseteq A$. Thus, B is a $(2, 1)$-hyperideal of H contained in A. Using the minimality of A, so $B = A$. Conversely, assume that A is a minimal bi-hyperideal of H. Then, A is a $(2, 1)$-hyperideal of H. Let C be a $(2, 1)$-hyperideal of H contained in A. Since

$$\begin{aligned}(C^2 \circ H \circ C] \circ H \circ (C^2 \circ H \circ C] &\subseteq (C^2 \circ H \circ C \circ H \circ C^2 \circ H \circ C] \\ &\subseteq (C^2 \circ H \circ C],\end{aligned}$$

so $(C^2 \circ H \circ C]$ is a bi-hyperideal of H. This implies that $(C^2 \circ H \circ C] = A$. Since $A = (C^2 \circ H \circ C] \subseteq (C] = C$, so $A = C$. Therefore, A is a minimal $(2, 1)$-hyperideal of H. ∎

Lemma 2.8. *Let $(H, \circ, \leq)$ be an ordered semihypergroup with zero. Then, A is a $(0, 2)$-bi-hyperideal of H if and only if A is a hyperideal of some left hyperideal of H.*

Proof. Assume that A is a $(0, 2)$-bi-hyperideal of H. Then, $H \circ (A^2 \cup H \circ A^2] \subseteq (H \circ A^2 \cup H^2 \circ A^2] \subseteq (H \circ A^2] \subseteq (A^2 \cup H \circ A^2]$. Hence, $(A^2 \cup H \circ A^2]$ is a left hyperideal of H. Since $A \circ (A^2 \cup H \circ A^2] \subseteq (A^3 \cup A \circ H \circ A^2] \subseteq (A] = A$ and $(A^2 \cup H \circ A^2] \circ A \subseteq (A^3 \cup H \circ A^3] \subseteq (A] = A$; so, A is a hyperideal of $(A^2 \cup H \circ A^2]$. Conversely, if A is a hyperideal of a left hyperideal L of H; by Lemma 2.1, A is a $(0, 2)$-hyperideal of H. Since, $A \circ H \circ A \subseteq A \circ H \circ L \subseteq A \circ L \subseteq A$, hence A is a bi-hyperideal of H. Therefore, A is a $(0, 2)$-bi-hyperideal of H. ∎

Theorem 2.9. *Let $(H, \circ, \leq)$ be an ordered semihypergroup with zero scalar element 0. If A is a 0-minimal $(0,2)$-bi-hyperideal of H and $a \in A$; then, exactly one of the following cases occurs:*

1. $A = (\{0, a\}], a^2 = \{0\}, (a \circ H \circ a] = \{0\}$
2. $A = (\{0, a\}], a^2 = \{0\}, (a \circ H \circ a] = A$
3. $\forall a \in A \setminus \{0\}, (H \circ a^2] = A$.

Proof. Assume that A is a 0-minimal $(0,2)$-bi-hyperideal of H. Let $a \in A \setminus \{0\}$, so $(H \circ a^2] \subseteq A$. Moreover, $(H \circ a^2]$ is a $(0,2)$-bi-hyperideal of H. Hence, $(H \circ a^2] = \{0\}$ or $(H \circ a^2] = A$. If $(H \circ a^2] = \{0\}$, hence either $a \circ a = \{0\}$ or $a \circ a = \{a\}$ or $a \circ a = \{0, a\}$ or there exist $x \in a^2$ such that $x \notin \{0, a\}$. If $a \circ a = \{a\}$ this is impossible, because $a \in a \circ a \circ a \subseteq H \circ a^2 = \{0\}$. If $a \circ a = \{0, a\}$, so $(a \circ a) \circ a = \{0, a\} \circ a = 0 \circ a \cup a \circ a = \{0, a\}$. This is a contradiction, because $a \in a \circ a \circ a \subseteq H \circ a^2 = \{0\}$. If there exists $x \in a^2$ such that $x \notin \{0, a\}$, so $x \in A$. Then, $\{0, x\} \subseteq \{0, x, a\} \subseteq A$. Since $H \circ x \subseteq H \circ a^2 = \{0\}$, so $H \circ x = \{0\}$. Thus, $H \circ x^2 = (H \circ x) \circ x = \{0\}$, consider

$$
\begin{aligned}
H \circ (\{0, x\}]^2 &= H \circ (\{0, x\}] \circ (\{0, x\}] \\
&= (H \circ 0^2 \cup H \circ 0 \circ x \cup H \circ x \circ 0 \cup H \circ x^2] \\
&= (\{0\}] \subseteq (\{0, x\}],
\end{aligned}
$$

so $(\{0, x\}]$ is a $(0,2)$-hyperideal of H. Since

$$
\begin{aligned}
(\{0, x\}] \circ H \circ (\{0, x\}] &= (x \circ H \circ x] \\
&= (x \circ \{0\}] \\
&= (\{0\}] \subseteq (\{0, x\}],
\end{aligned}
$$

hence $(\{0, x\}]$ is a $(0,2)$-bi-hyperideal of H contained in A. If $(\{0, x\}] = A$; then,

$$
\begin{aligned}
a \circ a \subseteq A \circ A &\subseteq H \circ (\{0, x\}] \\
&\subseteq (H \circ x] \\
&\subseteq (H \circ a^2] = (\{0\}].
\end{aligned}
$$

This is a contradiction, because $\{0, x\} \subseteq a \circ a$. Hence, $(\{0, x\}] \neq A$. Using the minimality of A, so $a^2 = \{0\}$ and $A = (\{0, a\}]$. It is clear that $a \circ H \circ a$ is a bi-hyperideal of H contained in A and

$$
\begin{aligned}
H \circ (a \circ H \circ a]^2 &\subseteq (H \circ a \circ H \circ a^2 \circ H \circ a] \\
&= (H \circ a \circ H \circ \{0\} \circ H \circ a] \\
&= (\{0\}] \subseteq (a \circ H \circ a].
\end{aligned}
$$

Then, $a \circ H \circ a$ is a $(0,2)$-bi-hyperideal of H contained in A. Thus, $a \circ H \circ a = \{0\}$ or $a \circ H \circ a = A$. ∎

The following corollary follows from Theorem 2.9.

Corollary 2.10. *Let A be a 0-minimal $(0,2)$-bi-hyperideal of an ordered semihypergroup $(H, \circ, \leq)$ with a zero. If $(A^2] \neq \{0\}$; then, $A = (H \circ a^2]$ for every $a \in A \setminus \{0\}$.*

An ordered semihypergroup H with zero is called a 0-$(0,2)$-*bisimple* if $(H^2] \neq \{0\}$ and $\{0\}$ is the only proper $(0,2)$-bi-hyperideal of H.

Corollary 2.11. *An ordered semihypergroup H with zero scalar is 0-$(0,2)$-bisimple if and only if $(H \circ a^2] = H$ for every $a \in H \setminus \{0\}$.*

Proof. Assume that $(H \circ a^2] = H$ for all $a \in H \setminus \{0\}$. Let A be a $(0,2)$-bi-hyperideal of H such that $A \neq \{0\}$. Let $a \in A \setminus \{0\}$. Since, $H = (H \circ a^2] \subseteq (H \circ A^2] \subseteq (A] = A$; so, $A = H$. Since $H = (H \circ a^2] \subseteq (H \circ H] = (H^2]$. Then, $(H^2] \neq \{0\}$. Therefore, H is a 0-$(0,2)$-bisimple. The converse statement follows from Corollary 2.10. ∎

Theorem 2.12. *Let $(H, \circ, \leq)$ be an ordered semihypergroups with zero. Then, H is 0-$(0,2)$-bisimple if and only if H is a left 0-simple.*

Proof. Assume that H is 0-$(0,2)$-bisimple. If A is a left hyperideal of H, so A is a $(0,2)$-bi-hyperideal of H. Hence, $A = \{0\}$ or $A = H$. Conversely, assume that H is left 0-simple. Let $a \in H \setminus \{0\}$. Then, $(H \circ a] = H$, hence $H = (H \circ a] = ((H \circ a] \circ a] \subseteq ((H \circ a] \circ (a]] \subseteq ((H \circ a^2]] = (H \circ a^2]$. By corollary 2.11, H is 0-$(0,2)$-bisimple. ∎

Theorem 2.13. *Let $(H, \circ, \leq)$ be an ordered semihypergroups with zero. If A is a 0-minimal $(0,2)$-bi-hyperideal of H; then, either $(A^2] = \{0\}$ or A is left 0-simple.*

Proof. Assume that A is a 0-minimal $(0,2)$-bi-hyperideal of H such that $(A^2] \neq \{0\}$. By Corollary 2.10, $(H \circ a^2] = A$ for every $a \in A \setminus \{0\}$. Let $a \in A \setminus \{0\}$. Since

$$\begin{aligned}
(A \circ a^2] \circ H \circ (A \circ a^2] &\subseteq (A \circ a^2 \circ H \circ A \circ a^2] \\
&\subseteq (A \circ A^2 \circ H \circ A \circ a^2] \\
&= (A \circ A \circ (A \circ H \circ A) \circ a^2] \\
&\subseteq (A^3 \circ a^2] \\
&\subseteq (A \circ a^2] \quad \text{and} \\
H \circ (A \circ a^2]^2 &= H \circ (A \circ a^2] \circ (A \circ a^2] \\
&\subseteq (H \circ A \circ a^2 \circ A \circ a^2] \\
&\subseteq (H \circ A \circ A^2 \circ A \circ a^2] \\
&= ((H \circ A^2) \circ A \circ A \circ a^2] \\
&\subseteq (A^3 \circ a^2] \\
&\subseteq (A \circ a^2].
\end{aligned}$$

Thus, $(A \circ a^2]$ is a $(0,2)$-bi-hyperideal of H contained in A. Hence, $(A \circ a^2] = \{0\}$ or $(A \circ a^2] = A$. Since $(H \circ a^2] = A$ for every $a \in A \setminus \{0\}$. Then, $a^2 \neq \{0\}$. Therefore, there exist $0 \neq x \in a^2 \subseteq A$. Clearly, $x^2 \neq \{0\}$ and $x^2 \subseteq a^2 \circ a^2 \subseteq A \circ a^2 \subseteq (A \circ a^2]$. Hence, $(A \circ a^2] = A$ and conclude by Corollary 2.11 that A is 0-$(0,2)$-bisimple. By Theorem 2.12, applies A is left 0-simple. ∎

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