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On the Chebyshev Polynomial Bounds for λ-Convex and μ-Starlike Functions

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Abstract In this paper, we obtain initial coefficient estimates for a general subclass of univalent functions by using the Chebyshev polynomials and also we find Fekete-Szegö inequalities for this class.

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1. Introduction

Let $\mathcal{A}$ be the class of all analytic functions in the unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$, satisfying the conditions $f(0) = 0$ and $f'(0) = 1$, then each function $f \in \mathcal{A}$ has the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (z \in \Delta). \tag{1.1}$$

We shall denote by $\mathcal{S}$ the class of functions in $\mathcal{A}$ that are also univalent in Δ.

We say that an analytic function f is subordinate to an analytic function g, written $f \prec g$, provided there is an analytic function w defined on Δ with $w(0) = 0$ and $|w(z)| < 1$ satisfying $f(z) = g(w(z))$.

A number of vital and well explored subclasses of class $\mathcal{S}$ are the class $\mathcal{S}^*(\alpha)$ of starlike functions of order α in Δ and the class $\mathcal{K}(\alpha)$ of convex functions of order α in Δ. By definition, we have

$$\mathcal{S}^*(\alpha) = \left\{ f \in \mathcal{A} : \Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha\,;\ z \in \Delta,\ 0 \le \alpha < 1 \right\}$$

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and

$$\mathcal{K}(\alpha)=\left\{f\in\mathcal{A}:\Re\left(1+\frac{zf''(z)}{f'(z)}\right)>\alpha\,;\ z\in\Delta,\ 0\leq\alpha<1\right\},$$

respectively. In particular, we set $\mathcal{S}^*(0)=\mathcal{S}^*$ and $\mathcal{K}(0)=\mathcal{K}$.

The arithmetic means of some functions and expressions is very frequently used in geometric function theory. Making use of the arithmetic means Mocanu [1] introduced the class of $\lambda-$convex $(0\leq\lambda\leq1)$ functions as follows:

$$\mathcal{M}_\lambda:=\left\{f\in\mathcal{A}:\Re\left((1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right)>0\,;\ z\in\Delta\right\},$$

which, in some case, proclaims the class of starlike and in the other, convex functions. In general, the class of $\lambda-$convex functions determines the arithmetic bridge between starlikeness and convexity.

Using the geometric means, Lewandowski et al. [2] defined the class of $\mu-$starlike functions $(0\leq\mu\leq1)$ as follows:

$$\mathcal{L}_\mu:=\left\{f\in\mathcal{A}:\Re\left(\left(\frac{zf'(z)}{f(z)}\right)^\mu\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\mu}\right)>0\,;\ z\in\Delta\right\}.$$

We note that the class of $\mu-$starlike functions also, constitute the geometric bridge between starlikeness and convexity. The significance of Chebyshev polynomial in numerical analysis is increased in both theoretical and practical points of view. Out of four kinds of Chebyshev polynomials, many researchers dealing with orthogonal polynomials of Chebyshev. For a brief history of Chebyshev polynomials of first kind $T_n(t)$, the second kind $U_n(t)$ and their applications one can refer [3–5]. The Chebyshev polynomials of the first and second kinds are well known and they are defined by

$$T_n(t)=\cos n\theta\quad\text{and}\quad U_n(t)=\frac{\sin(n+1)\theta}{\sin\theta}\qquad(-1<t<1)$$

where n denotes the polynomial degree and $t=\cos\theta$.

Definition 1.1. A function $f\in\mathcal{A}$ given by (1.1) is said to be in the class $\mathcal{M}(\lambda,\delta,\mu,t)$, $\lambda\geq0$, $0\leq\mu\leq1$, $\delta\geq0$ and $t\in(1/2,1]$, if the following subordination holds for all $z\in\Delta$

$$(1-\lambda)\left(\frac{zf'(z)}{f(z)}\right)^\delta+\lambda\left(\frac{zf'(z)}{f(z)}\right)^\mu\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\mu}\prec H(z,t):=\frac{1}{1-2tz+z^2}. \tag{1.2}$$

We note that if $t=\cos\alpha$, where $\alpha\in(-\pi/3,\pi/3)$, then

$$H(z,t)=\frac{1}{1-2\cos\alpha z+z^2}=1+\sum_{n=1}^{\infty}\frac{\sin(n+1)\alpha}{\sin\alpha}z^n\quad(z\in\Delta).$$

Thus

$$H(z,t)=1+2\cos\alpha z+(3\cos^2\alpha-\sin^2\alpha)z^2+\ldots\quad(z\in\Delta).$$

From [7], we can write

$$H(z,t)=1+U_1(t)z+U_2(t)z^2+\ldots\quad(z\in\Delta,\quad t\in(-1,1))$$

where

$$U_{n-1} = \frac{\sin(n \arccos t)}{\sqrt{1-t^2}} \quad (n \in \mathbb{N})$$

are the Chebyshev polynomials of the second kind and we have

$$U_n(t) = 2tU_{n-1}(t) - U_{n-2}(t),$$

and

$$U_1(t) = 2t, \quad U_2(t) = 4t^2 - 1, \quad U_3(t) = 8t^3 - 4t, \quad U_4(t) = 16t^4 - 12t^2 + 1, \dots. \tag{1.3}$$

The generating function of the first kind of Chebyshev polnomial $T_n(t)$, $t \in [-1, 1]$, is given by

$$\sum_{n=0}^{\infty} T_n(t) z^n = \frac{1 - tz}{1 - 2tz + z^2} \qquad (z \in \Delta).$$

The first kind of Chebyshev polnomial $T_n(t)$ and second kind of Chebyshev polnomial $U_n(t)$ are connected by:

$$\frac{dT_n(t)}{dt} = nU_{n-1}(t); \quad T_n(t) = U_n(t) - tU_{n-1}(t); \quad 2T_n(t) = U_n(t) - U_{n-2}(t).$$

Remark 1.2. It is interesting to note that for restricted values of the parameters involved in the class gives the following special subclasses.
(**i**) A function $f(z) \in \mathcal{A}$ is said to be in the class $\mathcal{M}(0, 1, \mu, t) := \mathcal{N}(t)$, $t \in (1/2, 1]$, if the following subordination holds:

$$\frac{zf'(z)}{f(z)} \prec H(z,t) = \frac{1}{1 - 2tz + z^2} \qquad (z \in \Delta).$$

(**ii**) A function $f(z) \in \mathcal{A}$ is said to be in the class $\mathcal{M}(1, \delta, 0, t) = \mathcal{H}(t)$, $t \in (1/2, 1]$, if the following subordination holds:

$$1 + \frac{zf''(z)}{f'(z)} \prec H(z,t) = \frac{1}{1 - 2tz + z^2} \qquad (z \in \Delta).$$

This class was introduced and studied by Dziok et al. [4].
(**iii**) A function $f(z) \in \mathcal{A}$ is said to be in the class $\mathcal{M}(\lambda, 1, 0, t) = \mathcal{K}(\lambda, t)$, $0 \le \lambda \le 1$ and $t \in (1/2, 1]$, if the following subordination holds:

$$(1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1 + \frac{zf''(z)}{f'(z)}\right) \prec H(z,t) = \frac{1}{1 - 2tz + z^2} \qquad (z \in \Delta).$$

This class was introduced and studied by Altınkaya and Yalçın [6].
(**iv**) A function $f(z) \in \mathcal{A}$ is said to be in the class $\mathcal{M}(1, \delta, \mu, t) := \mathcal{L}(\mu, t)$, $0 \le \mu \le 1$ and $t \in (1/2, 1]$, if the following subordination holds:

$$\left(\frac{zf'(z)}{f(z)}\right)^{\mu}\left(1 + \frac{zf''(z)}{f'(z)}\right)^{1-\mu} \prec H(z,t) = \frac{1}{1 - 2tz + z^2} \qquad (z \in \Delta).$$

In the present paper, motivated by the earlier work of Dziok et al. [4] and Altınkaya and Yalçın [6], we use the Chebyshev polynomials expansions to provide estimates for the initial coefficients of univalent functions in $\mathcal{M}(\lambda, \delta, \mu, t)$. We also solve Fekete-Szegö problem for functions in this class.

2. Coefficient Bounds for the Function Class $\mathcal{M}(\lambda,\delta,\mu,t)$

Throughout this paper, we assume that

$$\lambda \geq 0,\ 0 \leq \mu \leq 1,\ \delta \geq 0 \text{ and } t \in (1/2, 1].$$

Theorem 2.1. *Let the function $f \in \mathcal{A}$ given by (1.1) be in the class $\mathcal{M}(\lambda,\delta,\mu,t)$. Then*

$$|a_2| \leq \frac{2t}{|(1-\lambda)\delta + \lambda(2-\mu)|}, \tag{2.1}$$

$$|a_3| \leq \frac{t}{|(1-\lambda)\delta + \lambda(3-2\mu)|} \max\left\{1, \left|\frac{4t^2-1}{2t} + \frac{(1-\lambda)\delta(3-\delta) - \lambda(\mu^2+5\mu-8)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2} t\right|\right\} \tag{2.2}$$

and

$$\left|a_3 - \eta a_2^2\right| \leq \begin{cases} \frac{t}{|(1-\lambda)\delta+\lambda(3-2\mu)|} & ,\eta \in [\eta_1,\eta_2] \\ \frac{t}{|(1-\lambda)\delta+\lambda(3-2\mu)|}\left|\frac{4t^2-1}{2t} + \frac{(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t - 4\eta\frac{(1-\lambda)\delta+\lambda(3-2\mu)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t\right| & ,\eta \notin [\eta_1,\eta_2] \end{cases}, \tag{2.3}$$

where

$$\eta_1 = \frac{2t^2\left\{2[(1-\lambda)\delta+\lambda(2-\mu)]^2+(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)\right\}-(1+2t)[(1-\lambda)\delta+\lambda(2-\mu)]^2}{8t^2[(1-\lambda)\delta+\lambda(3-2\mu)]} \tag{2.4}$$

and

$$\eta_2 = \frac{2t^2\left\{2[(1-\lambda)\delta+\lambda(2-\mu)]^2+(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)\right\}-(1-2t)[(1-\lambda)\delta+\lambda(2-\mu)]^2}{8t^2[(1-\lambda)\delta+\lambda(3-2\mu)]}. \tag{2.5}$$

All of the inequalities are sharp.

Proof. Let the function $f(z)$ given by (1.1) be in the class $\mathcal{M}(\lambda,\delta,\mu,t)$. From (1.2), we have

$$\begin{aligned} & (1-\lambda)\left(\frac{zf'(z)}{f(z)}\right)^{\delta} + \lambda\left(\frac{zf'(z)}{f(z)}\right)^{\mu}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\mu} \\ = \ & 1 + U_1(t)w(z) + U_2(t)w^2(z) + \cdots \end{aligned} \tag{2.6}$$

for some analytic function

$$w(z) = c_1 z + c_2 z^2 + c_3 z^3 + \cdots \qquad (z \in \Delta), \tag{2.7}$$

such that $w(0) = 0$ and $|w(z)| < 1$. It is well-known that if $|w(z)| < 1$, $z \in \Delta$, then

$$|c_j| \leq 1 \quad \text{for all} \quad j \in \mathbb{N} = \{1, 2, \ldots\} \tag{2.8}$$

and

$$\left|c_2 - \mu c_1^2\right| \leq \max\{1, |\mu|\}, \quad \text{for all} \quad \mu \in \mathbb{C}. \tag{2.9}$$

From (2.6) and (2.7), we have

$$\begin{aligned}&(1-\lambda)\left(\frac{zf'(z)}{f(z)}\right)^{\delta}+\lambda\left(\frac{zf'(z)}{f(z)}\right)^{\mu}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\mu}\\ =\;&1+U_1(t)c_1z+\left[U_1(t)c_2+U_2(t)c_1^2\right]z^2+\cdots.\end{aligned} \tag{2.10}$$

Equating the coefficients in (2.10), we get

$$[(1-\lambda)\delta+\lambda(2-\mu)]a_2=U_1(t)c_1 \tag{2.11}$$

and

$$2[(1-\lambda)\delta+\lambda(3-2\mu)]a_3-[(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)]\frac{a_2^2}{2}=U_1(t)c_2+U_2(t)c_1^2. \tag{2.12}$$

Then, by using (1.3), (2.8) and (2.11), we get

$$|a_2|\le\frac{2t}{|(1-\lambda)\delta+\lambda(2-\mu)|}.$$

By using (2.11) and (2.12) for some $\eta\in\mathbb{R}$, we get

$$\left|a_3-\eta a_2^2\right|\le\frac{U_1(t)}{2\,|(1-\lambda)\delta+\lambda(3-2\mu)|}\times\left|c_2-\tau c_1^2\right|,$$

where

$$\tau=-\frac{U_2(t)}{U_1(t)}-\frac{(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)}{2\,[(1-\lambda)\delta+\lambda(2-\mu)]^2}U_1(t)+2\eta\frac{[(1-\lambda)\delta+\lambda(3-2\mu)]}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}U_1(t).$$

From (2.9), it follows that

$$\begin{aligned}\left|a_3-\eta a_2^2\right|\;\le\;&\frac{U_1(t)}{2\,|(1-\lambda)\delta+\lambda(3-2\mu)|}\\ &\times\max\left\{1,\left|\frac{U_2(t)}{U_1(t)}+\frac{[(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)]}{2[(1-\lambda)\delta+\lambda(2-\mu)]^2}U_1(t)\right.\right.\\ &\left.\left.-2\eta\frac{[(1-\lambda)\delta+\lambda(3-2\mu)]}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}U_1(t)\right|\right\}.\end{aligned}$$

Next, using (1.3) in the above equation, we have

$$\begin{aligned}\left|a_3-\eta a_2^2\right|\;\le\;&\frac{t}{|(1-\lambda)\delta+\lambda(3-2\mu)|}\\ &\times\max\left\{1,\left|\frac{4t^2-1}{2t}+\frac{(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t\right.\right.\\ &\left.\left.-4\eta\frac{(1-\lambda)\delta+\lambda(3-2\mu)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t\right|\right\}.\end{aligned}$$

Since $t>0$, we get

$$\left|\frac{4t^2-1}{2t}+\frac{(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t-4\eta\frac{(1-\lambda)\delta+\lambda(3-2\mu)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t\right|\le 1,$$

if and only if $\eta_1\le\eta\le\eta_2$ where η_1 and η_2 are given in (2.4) and (2.5), respectively. So we obtain the inequality (2.3). If we take $\eta=0$, then we obtain the inequality (2.2).

The equality (2.6) with $w(z)=z$ generate the function $\hat{f}\in\mathcal{M}(\lambda,\delta,\mu,t)$ such that

$$\begin{aligned}\hat{f}(z) &= z+\frac{2t}{(1-\lambda)\delta+\lambda(2-\mu)}z^2\\ &+\frac{t}{(1-\lambda)\delta+\lambda(3-2\mu)}\left[\frac{4t^2-1}{2t}+\frac{(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t\right]z^3+\cdots,\end{aligned}$$

which shows that the inequalities (2.1), and (2.2) for $\left|\frac{4t^2-1}{2t}+\frac{(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t\right|\geq 1$ are sharp. Also, in this case

$$\begin{aligned}\left|a_3-\eta a_2^2\right| &= \frac{t}{\left|(1-\lambda)\delta+\lambda(3-2\mu)\right|}\times\\ &\left|\frac{4t^2-1}{2t}+\frac{(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t-4\eta\frac{(1-\lambda)\delta+\lambda(3-2\mu)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t\right|,\end{aligned}$$

which shows the sharpness of (2.3) for $\eta\notin[\eta_1,\eta_2]$. On the other hand, the equality (2.6) with $w(z)=z^2$ generate the function $\check{f}\in\mathcal{M}(\lambda,\delta,\mu,t)$ such that

$$\check{f}(z)=z+\frac{t}{(1-\lambda)\delta+\lambda(3-2\mu)}z^3+\cdots,$$

which shows the sharpness of (2.2) for $\left|\frac{4t^2-1}{2t}+\frac{(1-\lambda)\delta(3-\delta)-\lambda(\mu^2+5\mu-8)}{[(1-\lambda)\delta+\lambda(2-\mu)]^2}t\right|\leq 1$, and (2.3) for $\eta\in[\eta_1,\eta_2]$. This completes the proof of Theorem 2.1. ∎

3. Corollaries and Consequences

Taking different restricted values to parameter which involved in Theorem 2.1, we get the following corollaries.

Corollary 3.1. *Let $f\in\mathcal{N}(t)$. Then*

$$\begin{aligned}|a_2| &\leq 2t,\\ |a_3| &\leq 4t^2-\frac{1}{2},\\ \left|a_3-\eta a_2^2\right| &\leq \begin{cases} t &, \ \eta\in[\eta_1,\eta_2]\\ \left|4(1-\eta)t^2-\frac{1}{2}\right| &, \ \eta\notin[\eta_1,\eta_2]\end{cases},\end{aligned}$$

where

$$\eta_1=\frac{8t^2-2t-1}{8t^2}\quad \text{and}\quad \eta_2=\frac{8t^2+2t-1}{8t^2}.$$

All of the inequalities are sharp.

Corollary 3.2. [4] *Let $f\in\mathcal{H}(t)$. Then*

$$\begin{aligned}|a_2| &\leq t,\\ |a_3| &\leq \frac{4}{3}t^2-\frac{1}{6},\\ \left|a_3-\eta a_2^2\right| &\leq \begin{cases} \frac{t}{3} &, \ \eta\in[\eta_1,\eta_2]\\ \left|\frac{4-3\eta}{3}t^2-\frac{1}{6}\right| &, \ \eta\notin[\eta_1,\eta_2]\end{cases},\end{aligned}$$

where

$$\eta_1 = \frac{8t^2 - 2t - 1}{6t^2} \quad \text{and} \quad \eta_2 = \frac{8t^2 + 2t - 1}{6t^2}.$$

All of the inequalities are sharp.

Corollary 3.3. *Let $f \in \mathcal{K}(\lambda, t)$. Then*

$$|a_2| \le \frac{2t}{1+\lambda},$$

$$|a_3| \le \begin{cases} \frac{t}{1+2\lambda} & , \quad \frac{1}{2} < t < \frac{(1+\lambda)^2 + (1+\lambda)\sqrt{5\lambda^2+22\lambda+9}}{4(\lambda^2+5\lambda+2)} \\ \frac{2(\lambda^2+5\lambda+2)t^2}{(1+\lambda)^2(1+2\lambda)} - \frac{1}{2(1+2\lambda)} & , \quad \frac{(1+\lambda)^2 + (1+\lambda)\sqrt{5\lambda^2+22\lambda+9}}{4(\lambda^2+5\lambda+2)} \le t \le 1 \end{cases},$$

$$\left|a_3 - \eta a_2^2\right| \le \begin{cases} \frac{t}{1+2\lambda} & , \quad \eta \in [\eta_1, \eta_2] \\ \left| \frac{2\left(\lambda^2+5\lambda+2\right) - 4(1+2\lambda)\eta}{(1+\lambda)^2(1+2\lambda)} t^2 - \frac{1}{2(1+2\lambda)} \right| & , \quad \eta \notin [\eta_1, \eta_2] \end{cases},$$

where

$$\eta_1 = \frac{4(\lambda^2+5\lambda+2)t^2 - (1+\lambda)^2(1+2t)}{8(1+2\lambda)t^2}$$

and

$$\eta_2 = \frac{4(\lambda^2+5\lambda+2)t^2 - (1+\lambda)^2(1-2t)}{8(1+2\lambda)t^2}.$$

All of the inequalities are sharp.

Remark 3.4. We note that Corollary 3.3 is an improvement of the results obtained by Altınkaya and Yalçın [6].

Corollary 3.5. *Let $f \in \mathcal{L}(\mu, t)$. Then*

$$|a_2| \le \frac{2t}{2-\mu},$$

$$|a_3| \le \frac{2\left(\mu^2 - 13\mu + 16\right)t^2 - (2-\mu)^2}{2(2-\mu)^2(3-2\mu)},$$

$$\left|a_3 - \eta a_2^2\right| \le \begin{cases} \frac{t}{3-2\mu} & , \quad \eta \in [\eta_1, \eta_2] \\ \left| \frac{\left(\mu^2-13\mu+16\right) - 4(3-2\mu)\eta}{(2-\mu)^2(3-2\mu)} t^2 - \frac{1}{2(3-2\mu)} \right| & , \quad \eta \notin [\eta_1, \eta_2] \end{cases},$$

where

$$\eta_1 = \frac{2(\mu^2 - 13\mu + 16)t^2 - (2-\mu)^2(1+2t)}{8(3-2\mu)t^2}$$

and

$$\eta_2 = \frac{2(\mu^2 - 13\mu + 16)t^2 - (2-\mu)^2(1-2t)}{8(3-2\mu)t^2}.$$

All of the inequalities are sharp.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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