Singular Vector Ensemble Forecast of Tropical Cyclone

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Abstract : Singular vector is a method to generate several initial conditions for weather prediction models from the same observation data. In this paper, a singular vector associated with a barotropic model is applied to tropical cyclone track forecasts by creating 50 initial conditions. These initial conditions are used to create 50 track predictions with slightly different initial conditions, called ensemble forecast. The ensemble mean should have more forecast skill than a single forecast. In addition, probability forecast of storm track based on the ensemble is also possible.

Keywords : Singular Vector, Barotropic Model, Ensemble Prediction System.

1   Introduction

At the present time, forecasting of the atmosphere is always uncertain especially the movement of tropical cyclones. This uncertainty happens because the atmosphere is sensitive to initial condition. Although, there are now highly complex models available as weather forecast models, the results have not been satisfied. A part of this problem is because tropical cyclones interact with the environment. The concept of numerical weather prediction is emphasized on environment, which is large scale and does not require high resolution model. However, to use a high resolution weather prediction model for operational forecast, a very large, fast and expensive computer is required. Another option to improve accuracy of weather forecast is to reduce uncertainty in the output from numerical models by performing several forecasts of the same event from several initial conditions. This is the concept of ensemble forecast.

From the fact that the atmosphere is sensitive to initial condition, using only single initial state value means that the result will be only one value in many possible values. So if there are many initial conditions, there also are many possible solutions which represent the possibility of the future states of the atmosphere. One way to get more initial condition values is to generate initial conditions based on the observed initial conditions by using singular vector technique.

2   Barotropic Model (Holtan,1992)

The barotropic model is a simple model for the atmosphere. This model is based on the assumptions; (1) horizontal motion, (2) non-divergent, (3) density is constant and (4) no friction. So the horizontal equations of motion governing the non-divergent barotropic flow is

X-Momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -g\frac{\partial z}{\partial x} + fv \tag{2.1}$$

Y-Momentum:

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\frac{\partial z}{\partial y} - fu \tag{2.2}$$

The continuity equation for the system is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.3}$$

Where u and v are horizontal velocity components, g is gravity, $f = 2\Omega\sin\phi$ is the Coriolis parameter where ϕ is latitude, Ω is the earth angular velocity. Differentiating equation (2.2) by $\partial/\partial x$ and equation (2.1) by $\partial/\partial y$ and taking the difference between the two equations, the vorticity equation for the non-divergent barotropic flow is obtained as

$$\frac{d}{dt}\zeta_a = 0 \tag{2.4}$$

Where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the relative vorticity and $\zeta_a = \zeta + f$ is the absolute vorticity. This is, the absolute vorticity of a fluid is conserved following the motion. In Cartesian coordinates, the non-divergent part of the flow can be expressed in terms of a streamfunction, ψ, such that;

$$u_\psi = -\frac{\partial\psi}{\partial y} \qquad \text{and} \qquad v_\psi = \frac{\partial\psi}{\partial x} \tag{2.5}$$

In streamfunction form, the relative vorticity , ζ , may be expressed as follows;

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \nabla^2\psi \tag{2.6}$$

The barotropic model is based on one equation and one unknown, the streamfunction, ψ;

$$\frac{\partial}{\partial t}\nabla^2\psi = -J(\psi, \nabla^2\psi) - \beta\frac{\partial\psi}{\partial x} \tag{2.7}$$

where J is the Jacobian operator, $\beta = \frac{\partial f}{\partial y}$ is the beta parameter and f is the coriolis parameter. Equation(2.7) from the basic of the non-divergent barotropic model. The equation has one unknown, ψ (streamfunction). The integration procedure consists of replacing the derivatives of the prediction equation (2.7) with finite difference approximations at a discrete set of points in space and time.

3 Singular Vectors (SVs) Method (Kalnay.,2003)

The initial conditions for the ensemble prediction system (EPS) are designed to represent the uncertainties in the operational analysis. They are created by adding to the operation analysis perturbations which produce the fastest energy growth during the first stage of the forecast period, defined using the singular vector technique.

The aim is to find those perturbations to a given initial state which grow most rapidly. The problem can be formalized mathematically and solutions found if certain assumptions and choices are made. The main assumption is that the perturbations grow linearly in time. The choices include deciding how to measure the size of a perturbation (choice of norms), what regions to focus on (target areas) and how long to allow the perturbations to grow. The singular vectors are the perturbations with the greatest linear growth over the specified time interval for given norms and target areas. Different choices may lead to different sets of singular vectors.

The calculation of the singular vectors only becomes feasible if the evolution of the perturbations is linear (the rate of growth is constant over time). In the atmosphere this assumption is generally valid for small perturbations for one or two days ahead. To determine the growth of a perturbation over time a forecast model is needed, but this model must be specifically designed for the linear evolution of small perturbations. Technically this model is the tangent-linear version of the full (non-linear) operational forecast model. A second model, the adjoint of the tangent linear model is also needed in the singular vector calculation.

3.1 Tangent Linear Model

Consider a nonlinear Model

$$x(t) \quad = \quad M\left[x(t_0)\right] \tag{3.1}$$

where $x(t_0)$ is the state at initial time, $x(t)$ is the state at time t and M is the time integration of the numerical scheme from the initial condition to time t.
The Tangent Linear Model(TLM), is defined such that;

$$L \quad = \quad \frac{\partial M}{\partial x} \tag{3.2}$$

where L is a tangent linear model of M.
So the TLM evolves a perturbation in time;

$$y(t) \quad = \quad L(t_0, t)\, y(t_0) \tag{3.3}$$

where $y(t_0)$ is the perturbation state at initial time $t = 0$ and $y(t)$ is the perturbation state at time t.

3.2 Adjoint Model

The adjoint tangent linear model is the transpose of the tangent linear model. It is defined with respect to the inner product of two arbitrary vectors.

$$\langle L\, u,\, v \rangle \quad = \quad \langle u,\, L^T v \rangle \tag{3.4}$$

where u and v are arbitrary vectors and L^T is the adjoint TLM.

3.3 Calculation of Singular Vectors

For an interval (t_0, t_1), the tangent linear model is a matrix that when applied to a small initial perturbation $y(t_0)$ produces the final perturbation $y(t_1)$;

$$y(t_1) \quad = \quad L(t_0, t_1) \, y(t_0) \tag{3.5}$$

Singular value decomposition theory indicates that for any matrix L there exist two orthogonal matrices U, V such that

$$U^T L V \quad = \quad S \tag{3.6}$$

where

$$S \quad = \quad \begin{bmatrix} \sigma_1 & 0 & \ldots & 0 \\ 0 & \sigma_2 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & \sigma_n \end{bmatrix} \quad \text{and} \quad UU^T \quad = \quad I, \quad VV^T \quad = \quad I \tag{3.7}$$

S is a diagonal matrix whose elements are the singular values of L.
Left multiply equation (3.6) by U, to get

$$UU^T L V \quad = \quad US$$

$$LV \quad = \quad US \quad \text{i.e.,} \quad L\left(v_1, \ldots, v_n\right) \quad = \quad \left(\sigma_1 u_1, \ldots, \sigma_n u_n\right) \tag{3.8}$$

where v_i are the columns of V and u_i are the columns of U. This implies that

$$Lv_i \quad = \quad \sigma_i u_i \tag{3.9}$$

Equation (3.9) defines the v_is as the *right singular vectors of L*, hereafter referred to as *initial singular vectors*, since they are valid at the beginning of the optimization interval over which L is defined.
Right multiply equation (3.6) by V^T to obtain;

$$U^T L V V^T \quad = \quad SV^T$$

$$U^T L \quad = \quad SV^T \tag{3.10}$$

Transposing equation (3.10);

$$L^T U \quad = \quad VS \text{i.e.,} \quad L^T\left(u_1, \ldots, u_n\right) \quad = \quad \left(\sigma_1 v_1, \ldots, \sigma_n v_n\right) \tag{3.11}$$

so that

$$L^T u_i \quad = \quad \sigma_i v_i \tag{3.12}$$

The u_is are the *left singular vectors of L* and will be referred to as *final (or evolved) singular vectors*, since they correspond to the end of the interval of optimization. From equation (3.9);

$$Lv_i = \sigma_i u_i$$

multiply by L^T; to obtain

$$L^T L v_i = \sigma_i L^T u_i$$

From equation (3.12);

$$L^T u_i = \sigma_i v_i$$
$$L^T u_i = \sigma_i \sigma_i (\sigma_i v_i)$$
$$L^T L v_i = \sigma_i^2 v_i \qquad (3.13)$$

Therefore, the initial singular vectors can be obtained as the eigenvectors of $L^T L$, a normal matrix whose eigenvalues are the squares of the singular values.

4   Experiment and Results

Three cases (simple, complex and backtracking tracks) are used to perform the experiments as shown in Figures 1-3. All experiments were run with the barotropic model for 72 hours and the time step is half an hour. The output interval from the model forecast is 6 hours. So there were 12 outputs within the 72 hours forecast.
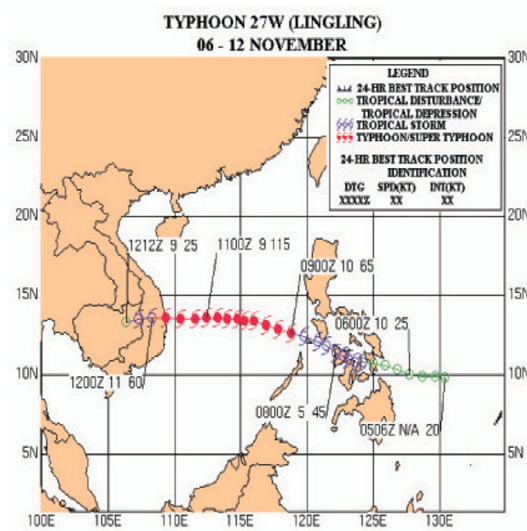

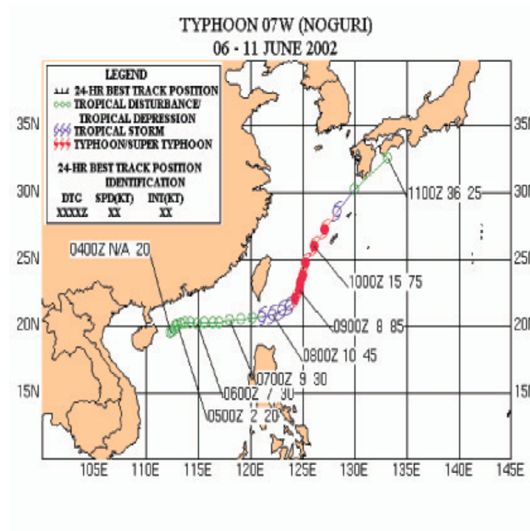Figure 1: Simple track of a tropical cyclone (Typhoon LINGLING)

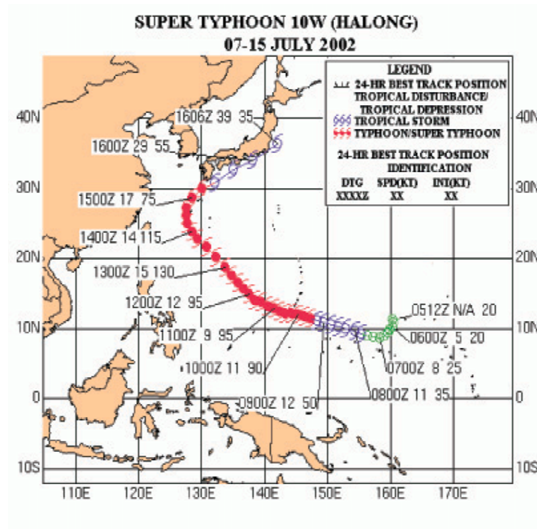Figure 2: Complex track of a tropical cyclone (Typhoon NOGURI)

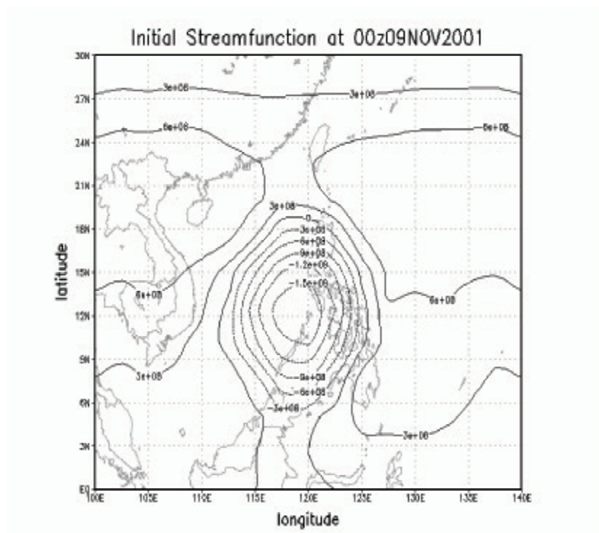Figure 3: Backtracking of a tropical cyclone (Super Typhoon HALOG)

The data for the model are prepared using data from the European Center for Medium Range Weather Forecasting (ECMWF). These data were downloaded from *http://data.ecnwf.int/data/era40_daily.* The downloaded data can be classified in Table 1. Since the downloaded data are low-resolution (2.5 degree), the center of tropical cyclone

	*Case*1	*Case*2	*Case*3
Type	*Pressure levels*	*Pressure levels*	*Pressure level*
Date	$09 - 11 - 2001$	$09 - 06 - 2002$	$10 - 07 - 2002$
Time	$00:00$	$00:00$	$00:00$
Parameter	*u, v velocity*	*u, v velocity*	*u, v velocity*
Level	$500\ mb$	$500\ mb$	$500\ mb$
Resolution	$2.5\ degree$	$2.5\ degree$	$2.5\ degree$

Table 1: Data from the ECMWF.

of initial streamfunction from running barotropic model of all three cases have different positions from analysis data as show in Figure 4, Figure 6 and Figure 8. So it is important to modify initial streamfunction of these 3 cases and make the storm location close to the positions in the analysis data by using Rankine Vortex technique (Teinh and Krishnamurti.,1992) as show in Figure 5, Figure 7 and Figure 9.

Figure 4: Initial Streamfunction of Case 1, 9 November 2001, 00 UTC (Universal Time Constant Zone)

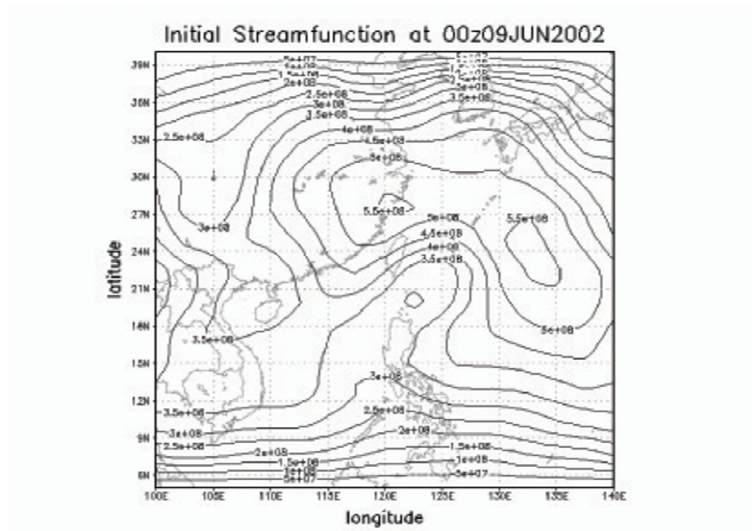Figure 5: Initial Streamfunction by using ranking vortex of Case 1, 9 November 2001, 00 UTC

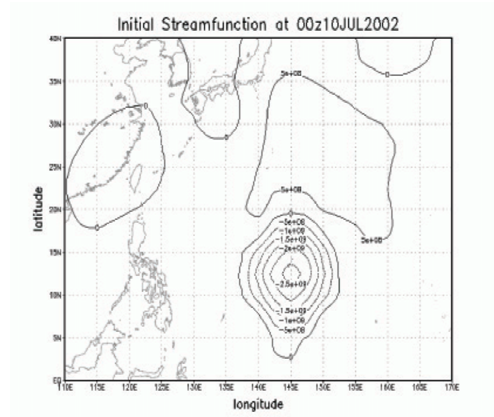Figure 6: Initial Streamfunction of Case 2, 9 June 2002, 00 UTC

Figure 7: Initial Streamfunction by using ranking vortex of Case 2, 9 June 2002, 00 UTC

Figure 8: Initial Streamfunction of Case 3, 10 July 2002, 00 UTC

Figure 9: Initial Streamfunction by using ranking vortex of Case 3, 10 July 2002, 00 UTC

4.1 The Results of Running the Barotropic Model for Case 1 (Simple Track)

The output of running the model for Case 1 (simple track) on November 9 th, 2001 are shown in this section. The domain for running the model is $100°E - 140°E$, $0°N - 30°N$.
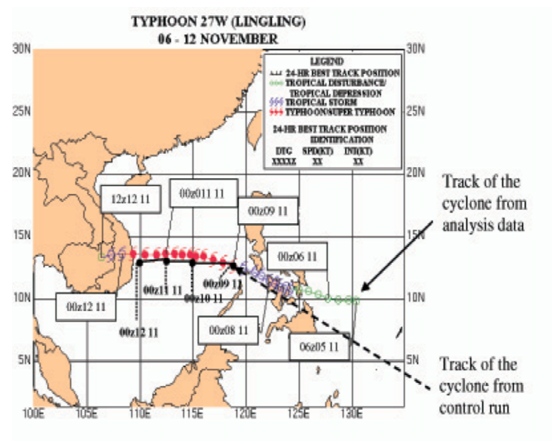
Figure 10: The comparison between the observed track and the control run of Case 1

Figure 10 show the comparison between the observed track and the control run. From the figure, the control run gives the forecast track which is in agreement whit the observed track.

4.2   The Results of Singular Vector for Case 1 (Simple Track).

The correct initial condition can be used for generating new initial condition by using singular vector method. The results are new initial data that could be used for next forecasting. In this paper, the experiments generated new initial data by using 50 singular vector members. Then, there members were used in running the forecasts many times from different initial conditions, called ensemble prediction system (EPS). Results from the experiments are shown below;
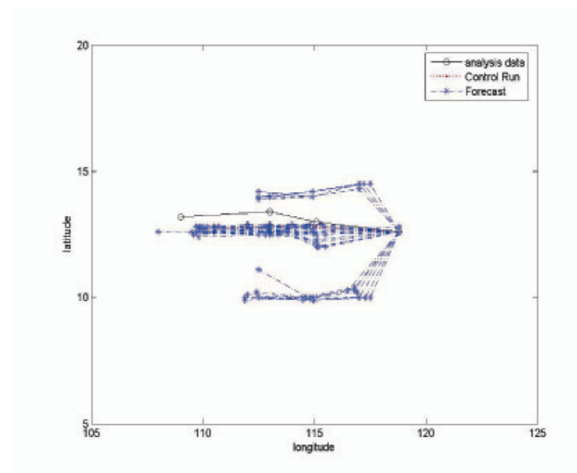
Figure 11: Ensemble 50 singular vector members of Case 1

Figure 11 shows that the results of ensemble can be divided into three groups; (1) the first group is close to the control run, (2) the second group is above the control run and (3) the third one is below the control run. In each group, the number of member is difference. The first one has more members than the others. Moreover, it is also in the same direction with the control run. In contrast, members in the second and third groups have different northward and southward directions, respectively. This is because of the rotation and expansion of singular vector calculation process. Thus, singular vector method generates initial conditions that can be used in Case 1 successfully.

4.3 The Results of Running the Barotropic model for Case 2 (Complex Track).

The output of running the model for Case 2 (complex track) on June 9 th, 2002 are shown in this section. The domain for running the model is $100°E - 140°E$, $5°N - 40°N$. Figure
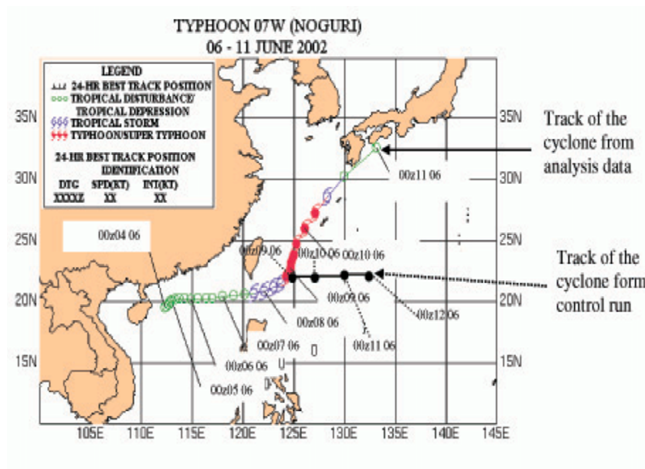

Figure 12: The comparison between the observed track and the control run of Case 2

12 shows that the comparison between the observed track and the control run. From the figure, track is different from the observed track. There are 2 reasons for this difference: (1) the barotropic model is a simple and uncomplicated model. So it is not able to forecast complex cyclone movement, (2) the resolution of the model is low.

4.4 The Results of Singular Vector for Case 2 (Complex Track).

Figure 13 shows that the results of ensemble can be divided into three groups; (1) the first group is close to the control run, (2) the second group is above the control run and (3) the third one is below the control run. In each group, the number of member is difference. The first one has more members than the others. However, all the results of ensemble are below to the observed track because of the limitation of barotropic model in forecasting any complex track of cyclone, and the resolution of model is low.
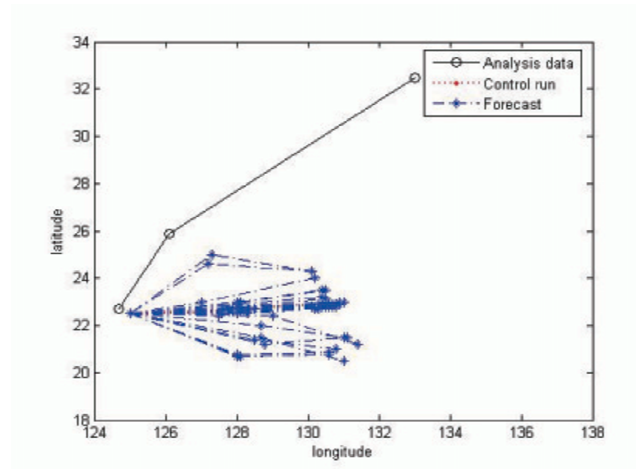
Figure 13: Ensemble 50 singular vector members of Case 2

4.5 The Results of Running the Barotropic Model for Case 3 (Backtracking).

The output of running the model for Case 3 (Backtracking) on July 10 th, 2002 are shown in this section. The domain for running the model is $110°E - 170°E$, $0°N - 40°N$.
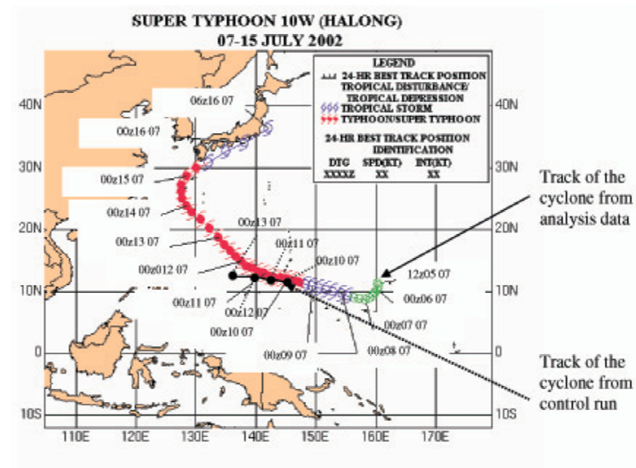

Figure 14: The comparison between the observed track and the control run of Case 3

Figure 14 shows that the comparison between the observed track and the control run. From the figure, the forecast of the control run track is different from the observed track. There are 2 reasons for this difference;(1) the barotropic model is a simple and uncomplicated model. So it is not able to forecast complex cyclone movement, (2) the resolution of the model is low.

4.6   The Results of Singular Vector for Case 3 (Backtracking).
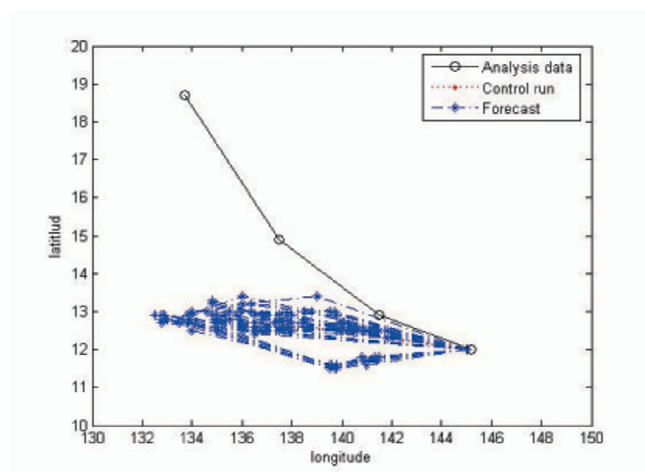

Figure 15: Ensemble 50 singular vector members of Case 3

Figure 15 shows that the results of ensemble can be divided into three groups; (1) the first group is close to the control run, (2) the second group is above the control run and (3) the third one is below the control run. In each group, the number of member is difference. The first one has more members than the others. However, all the results of ensemble are below to the observed track because of the limitation of barotropic model in forecasting any complex track of cyclone, and the resolution of model is low.

5   Conclusion

From the experiments, three cases has been used in generating initial conditions using singular vector method and the results are shown that singular vector method can be used to generate initial conditions. That result in different track forecast.

For Case 1 (simple track) the ensemble forecast give the result that agree with the observed data. As for 2nd (complex track) and 3rd (backtracking) cases, the ensemble results are different from the observed track. There are 2 reasons for this difference;

1. The barotropic model is a simple and uncomplicated model. So it is not able to forecast cyclone with complicated track.

2. The resolution of the model is low.

References

[1] European Centre for Medium-Range Weather Forecasts.Data.
http://data.ecmwf.int/data/

[2] Holton, James R., 1992: An introduction to Dynamic Meteorology (3th edition). Academic press,London,48,511.

[3] Institute of Global Environment and Society.GrADS Program.
http://grads.iges.org.

[4] Kalnay,Eugenia.,2003:Atmospheric Modeling,Data Assimilation and Predictability. Combridge,UK,35,205-260.

[5] Trinh Van Thu and T.N.Krishnamurti., 1992:Vortex Initialization for Typhoon Track Prediction. Meteorol.Atmos.Phys 47,117-126.

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