Thai Journal of **Math**ematics
Volume 19 Number 2 (2021)
Pages 479–491

http://thaijmath.in.cmu.ac.th

ISSN 1686-0209

Forecasting Art Prices with Bayesian Models

Vandana[1,*], **Deepmala**[2], **Krzysztof Drachal**[3] **and Lakshmi Narayan Mishra**[4,5]

[1] *School of Studies in Mathematics, Pt. Ravishankar Shukla University, Raipur-492010, (C.G.), India*
e-mail : vdrai1988@gmail.com

[2] *Mathematics Discipline, PDPM Indian Institute of Information Technology, Design and Manufacturing, Jabalpur 482 005, Madhya Pradesh, India*
e-mail : dmrai23@gmail.com

[3] *Faculty of Economic Sciences, University of Warsaw, Poland*
e-mail : kdrachal@wne.uw.edu.pl

[4] *Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology (VIT) University, Vellore 632 014, Tamil Nadu, India*
e-mail : lakshminarayanmishra04@gmail.com, lakshminarayan.mishra@vit.ac.in

[5] *L. 1627 Awadh Puri Colony Beniganj, Phase-III, Opposite-Industrial Training Institute (I.T.I.), Ayodhya Main Road Faizabad 224 001, Uttar Pradesh, India*

Abstract In this paper several potential art price determinants are considered. For example, stock market indices, other commodity prices, exchange rates, GDP, disposable income, consumption, interest rates, etc. The analysis is based on quarterly data starting in 1998 and ending in 2015. The methodology is based on BMA (Bayesian Model Averaging) and DMA (Dynamic Model Averaging), which is applicable in case of the uncertainty about the suitable predictors. Prices of various type of art goods are analysed. The results suggest that art market is quite a complex one and even in case of including many predictors it is hard to model. However, it is found that DMA outperforms BMA.

MSC: 62P20; 62M10
Keywords: art market; Bayesian forecasting; Bayesian model averaging; BMA; dynamic model averaging; DMA; forecasting art prices

Submission date: 13.07.2017 / Acceptance date: 11.01.2019

1. Introduction

Forecasting art prices is quite a hard problem. First of all, there is no commonly used model. Moreover, many researches focus just on the construction of the art price index itself. Some researchers even doubt if the price of an art good can be modelled similarly as other financial assets.

However, if one has a reasonable art price index, it is quite hard to tell which economic variables should be included in the potential model, as the independent variables. Actually, a Bayesian econometrics and especially Bayesian Model Averaging (BMA) seems

*Corresponding author.

Published by The Mathematical Association of Thailand.

to be an interesting tool is such a case. For it allows to initially consider many models with various (just potentially interesting) independent variables. Such a methodology has been successfully applied, for example, in medicine or macroeconomics.

In this paper the currently available literature is scanned first to collect quite a big set of potential art prices determinants. Next, the BMA methodology is shortly reminded. Finally, the BMA model is estimated and discussed.

2. Literature Review

The art market is quite a complex one. Moreover, investing in art goods is perceived as a very specific kind of an investment. This is because art goods are non-homogeneous and indivisible. The risk of buying forged good is very high and the valuation of a good is extremely complex [1]. It might be subject of personal emotions, cultural aspects, current trends, etc. [2].

For example, Baumol stated that no fundamental value can be given to paintings in the context of the supply and demand market forces [3]. Similarly, some researchers criticize the concept of creating any art price index [4]. Others point out that many transactions on the art market are outside any institution and/or without a broker, making the market more informal. This results in small market liquidity and poor availability of information [5].

Despite these facts, art markets received much attention also as tools in portfolio diversification [6–8]. Some researchers advocate treating an art good in a similar way as other market goods [9, 10]. As a result, various art price indices have been proposed. For example, Mei-Moses Fine Art Index, Sotheby's Art Index, Art Price Index, Hislop's Art Sales Index and Gabrius Index, etc. However, still much more researches have been devoted to construct hedonistic regression models, than to model the general behaviour of the art market [11–14].

But even if art prices are studied with certain econometric model there is still much uncertainty which determinants should be included in the model. Most often the following factors are concluded to play the important role: income per capita, oil prices, equity markets, interest rates, exchange rates, inflation and house prices [7, 8, 15–20].

Recently, there is also much awareness on the impact of Chinese market [21].

3. Data

The analyses covered the period beginning on the first quarter of 1998 and ending on the first quarter of 2015. Quarterly data were taken. Art Price Global Index (AGI) was taken as the art price index [22]. In particular, this index consists of 12 sub-indices.

First of all, it differentiates geographically for U.S. prices (U.S.) and worldwide art market prices (Global). Secondly, different types of art goods are considered: Paintings, Drawings, Photographies, Prints and Sculptures. Finally, goods from different time periods are considered: Old Masters, 19th Century, Modern Art, Post-War and Contemporary. The prices were taken in USD.

The following time series were taken as possible art price determinants:

- MSCI – index as a representation of developed stock markets,
- EM – the MSCI index but reduced to the emerging stock markets,
- SHC – Shanghai Composite Index (China stock market),
- VIX – implied volatility of S&P 500,

- P/E – price to earnings index for S&P 500,
- CRB – The Thomson Reuters/Core Commodity Index,
- GOLD – gold price,
- WTI – West Texas Intermediate crude oil price,
- TWEXB – trade-weighted U.S. dollar index,
- USSTHPI – all-transactions house price index for U.S.,
- GDP – U.S. real gross domestic product per capita,
- DPI – real disposable personal income per capita in U.S.,
- PCE – real personal consumption expenditure in U.S.,
- US10BY – 10-year U.S. bond yield,
- US30BY – 30-year U.S. bond yield,
- LR – bank prime loan rate in U.S.,
- M1 – M1 money aggregate in U.S.,
- M2 – M2 money aggregate in U.S.,
- UNEMP – unemployment rate in U.S.

These time series were obtained from freely available sources [23–25].

First, all time series, except VIX, P/E, US10BY, US30BY, LR and UNEMP were re-scaled in order to be based at 100 for the first observation (the first quarter of 1998).

In the next step, first differences were taken in order to guarantee stationary of the data (see Tab. 1). Standard tests were performed: ADF, KPSS and Phillips–Perron [26]. It can be seen, that assuming 5% significance level, all time series, except USSTHPI and UNEMP, are stationary. Therefore, for these two time series second differences were obtained, which can be treated as stationary (see Tab. 2). Similarly, for all AGI art price indices the first differences could be taken as stationary (see Tab. 3).

All computations were done in R software [27].

4. Methodology

When dealing with uncertainty about predictors which should be included in the model, various methods have been proposed. Especially, Bayesian econometrics is useful in such a case [28–30]. Indeed, for example, Bayesian Model Averaging (BMA) has been successfully applied in various sciences. In particular, it gained much interest in economics and finance [31–33].

For the Reader's convenience a short overview of the main idea of BMA is reminded herein. However, for the details the original work should be consulted [34].

Let us consider the linear model

$$y = X\beta + \epsilon,$$

where y is the dependent variable (herein: the art prices) and X is a matrix of independent variables. The model coefficients are β (constant is included) and it is assumed that $\epsilon \sim N(0, \sigma^2 I)$. Suppose that X consists of many variables, and it is not clear which ones should be used in the model.

In BMA all variations possible to obtain from X are considered. In other words, if there are K potential independent variables, then 2^K models can be constructed. Let us denote a given model by M_i. Then, $i = \{1, 2, \ldots, 2^K\}$.

First, a prior $p(M_i)$ is specified. Usually (as in this paper) all models are treated as equally "good", i.e., $p(M_i) = \frac{1}{2^K}$.

Then, posterior model probabilities should be computed, i.e.:

$$p(M_i|y,X) = \frac{p(y|M_i,X)p(M_i)}{\sum_{j=1}^{2^K} p(y|M_j,X)p(M_j)},$$

where $p(y|M_i,X)$ is the marginal likelihood of the i-th model. With these posteriori probabilities as weights, the final forecast is formulated as a weighted average over all 2^K models.

Additionally, the Occam's Window can be applied. Herein, the best models based on BIC (Bayesian Information Criterion) were taken, and then all the models that were 20 times less likely than the best model were excluded [35].

It is worth to notice that BMA is a special case of Dynamic Model Averaging (DMA) [36]. Indeed, DMA allows for both parameters and state space model to vary in time. The state space model is given by

$$y_t = x_t^i \beta_t^i + \epsilon_t^i, \tag{4.1}$$

$$\beta_t^i = \beta_{t-1}^i + \delta_t^i. \tag{4.2}$$

Errors are normally distributed: $\epsilon_t^i \sim N(0,V_t^i)$ and $\delta_t^i \sim N(0,W_t^i)$. V_0^i and W_0^i have to be specified. For example, $V_0^i := I$, where I is the unit matrix. (However, as this value should be specified in accordance with the variability of the variables used, herein it was set $V_0^i := 100I$.) The specification of W_0^i is proposed in [36].

Further, V_t^i and W_t^i are computed recursively using the Kalman filter updating [36]. In this procedure certain "forgetting factor" is used, which herein was assumed to be 0.99, following some previous researches with DMA [37, 38]. (This parameter corresponds to variability of W_t^i and setting it to 1 is one of the two conditions, when DMA reduces to BMA.)

Further, the recursive updating is done, in the following way:

$$\pi_{t|t-1,i} = \frac{(\pi_{t-1|t-1,i})^\alpha + c}{\sum_{j=1}^{2^K}(\pi_{t-1|t-1,j})^\alpha + c},$$

$$\pi_{t|t,i} = \frac{\pi_{t|t-1,i} f_i(y_t|y_0,y_1,\ldots,y_{t-1})}{\sum_{j=1}^{2^K} \pi_{t|t-1,j} f_j(y_t|y_0,y_1,\ldots,y_{t-1})}.$$

In order to avoid possible zeros due to numerical approximations a small constant c is added. For example, $c = \frac{0.001}{2^K}$. Similarly, as with the first "forgetting factor", herein it was set $\alpha = 0.99$. (The second requirement for DMA to reduce to BMA is setting $\alpha = 1$.)

The final DMA forecast is given by

$$\hat{y}_t = \sum_{j=1}^{2^K} \pi_{t|t-1,j} \hat{y}_t^j,$$

where $\hat{y}_t^j$ is the forecast produced by the j-th model.

5. Results

The estimations were done with BMA package for R [35].

Tab. 4 and Tab. 5 present coefficients for the best model for each type of art good. By the best model is understood the one with the highest posteriori probability. Posteriori probabilities, as well as, R^2 coefficients are also presented. It can be observed that the coefficients of a given independent variable have the same sign for every type of art goods, except SHC (which for Old Masters is negative). Moreover, for 3 types of art goods the R^2 coefficient for the best model is higher than 50%. On the other hand, for 3 types – it is less than 30%. Posteriori probability for Contemporary art goods is over 20%. But this is an exception. All other models have posteriori probabilities below 11%, which means that it is not a one "preferred" model.

From Tab. 6 and Tab. 7 it can be seen that the expected values of coefficients have quite often different signs for different types of an art good. However, coefficients for VIX, P/E and CRB have the same sign for every type of an art good.

Finally, in Tab. 8 and Tab. 9 are presented probabilities that the given coefficient is not equal to 0. It can be seen that certain variables are important in determining prices of particular types of art goods, whether there is no independent variable which would be highly important for every type of an art good. This can be interpreted that the art market is very differentiated and different predictors play important role for various types of art goods. This can serve as a suggestion that in modelling art prices it is better to focus on the selected type of an art good than to consider the overall index.

Setting a benchmark of 50% for posterior probabilities that each variable is non-zero, it was found that MSCI and EM did not exceed this value for any type of an art good. For Paintings this criterion was met by VIX, P/E, CRB, GDP, M1 and M2. For Prints – by VIX, P/E, WTI, USSTHPI and PCE. For Sculptures – by VIX, P/E, WTI and USSTHPI. For Photographies – by WTI only. For Drawings – by UNEMP only. For Old Masters – by SHC and CRB. For 19th Century – by VIX, SHC and WTI. For Modern Art – by P/E, TWEXB, US10BY, M1 and M2. For Post-War – by SHC and M1. For Contemporary – by SHC and WTI. For U.S. – by P/E and PCE. For Global – by VIX, GDP and PCE.

The highest number of determinants (i.e., 6) was found for Paintings. Initially, 19 variables were considered. Therefore, it can be seen that estimation of BMA and setting some reasonable benchmark for posterior probabilities resulted in narrowing the set of determinants.

As for Paintings BMA found 6 important determinants according to the described procedure, further comparisons were made. In particular, BMA unrestricted by the Occam's Window for these 6 variables was estimated and DMA for these 6 variables (with "forgetting factors" set to 0.99, 0.98, ... , 0.90, as mentioned previously). The results (root mean squared errors) are presented in Tab. 10. It can be seen that, if the "forgetting factor" in DMA is set to 0.99 than worse forecast is obtained. However, if smaller "forgetting factors" are considered forecasts are better than that of BMA. The best forecast (according to root mean squared error) is obtained when "forgetting factors" are 0.91.

Let us notice that the above conclusions above BMA and DMA are still valid, if the naive forecast is taken as the benchmark. However, in order to stay in the time varying background, the naive forecast was computed in DMA framework. In other words, Eq. 4.1 and Eq. 4.2 were modified in such a way that the only considered model is the one with constant solely (see Tab. 10).

Finally, from Fig. 1 it can be seen that posteriori probabilities for various independent variables (i.e., the sum of posteriori probabilities of models including a given independent variable) vary significantly in time. This corresponds to the situation that it is not a one model which is preferred during the whole time. As a result model averaging plays an important role. Indeed, certain changes can be noticed around the time of the recent global financial crisis. The assumption that the parameters are different before and after the recent global financial crisis [39] is another argument in favour of using time varying models.

6. Conclusions

In this paper Bayesian Model Averaging (BMA) was applied to art prices. Various models were estimated. It was found that according to BMA methodology it is quite hard to select macroeconomic determinants of art prices, which would be common for different types of art goods. However, BMA identified certain determinants for markets of particular art goods.

Setting a benchmark of 50% for posterior probabilities that each variable is non-zero, it was found that MSCI, EM and US30BY did not exceed this value for any type of an art good. VIX met this criterion for Paintings, Prints, Sculptures, 19th Century and Global. P/E – for Paintings, Prints, Sculptures, Modern Art, and U.S. WTI – for Prints, Sculptures, Photographies, 19th Century and Contemporary. SHC – for Old Masters, 19th Century, Post-War and Contemporary. PCE – for Prints, U.S. and Global. M1 – for Paintings, Modern Art and Post-War. M2 – for Paintings and Modern Art. CRB – for Paintings and Old Masters. GDP – for Paintings and Global. USSTHPI – for Prints and Sculptures. PCE – for Global only. UNEMP – for Drawings only. TWEXB – for Modern Art only. US10BY – for Modern Art only.

This is quite in an agreement with previous researches. There are certain suggestions that equity and commodity markets (especially oil price) have impact on art prices. However, it seems that in case of developed economies they are rather market stress indices than the level of the market index itself. In none models MSCI have posterior probability over 50%. Neither did EM. However the Chinese SHC was important in certain models.

Furthermore, it was found that Dynamic Model Averaging (DMA) can lead to better forecasts than BMA (Bayesian Model Averaging).

Despite this conclusions, further investigation is definitely necessary.

Table 1. Stationarity tests at the level of 1st differences

	ADF stat	ADF p-val	KPSS stat	KPSS p-val	PP stat	PP p-val
MSCI	-3.46	0.05	0.10	0.10	-57.50	0.01
EM	-4.41	0.01	0.06	0.10	-47.77	0.01
SHC	-4.13	0.01	0.04	0.10	-44.21	0.01
VIX	-4.27	0.01	0.03	0.10	-61.08	0.01
P/E	-4.64	0.01	0.03	0.10	-31.14	0.01
CRB	-4.47	0.01	0.10	0.10	-46.69	0.01
GOLD	-2.35	0.43	0.21	0.10	-81.54	0.01
WTI	-4.56	0.01	0.12	0.10	-40.15	0.01
TWEXB	-2.67	0.30	0.18	0.10	-41.79	0.01
USSTHPI	-1.36	0.83	0.74	0.01	-14.44	0.26
GDP	-3.13	0.12	0.34	0.10	-44.08	0.01
DPI	-3.88	0.02	0.23	0.10	-89.86	0.01
PCE	-2.54	0.36	0.69	0.01	-32.67	0.01
US10BY	-4.26	0.01	0.02	0.10	-62.08	0.01
US30BY	-4.51	0.01	0.02	0.10	-61.84	0.01
LR	-3.26	0.09	0.11	0.10	-20.74	0.04
M1	-2.81	0.25	2.01	0.01	-33.16	0.01
M2	-3.35	0.07	1.26	0.01	-43.35	0.01
UNEMP	-2.72	0.28	0.29	0.10	-19.17	0.07

Table 2. Stationarity tests at the level of 2nd differences

	ADF stat	ADF p-val	KPSS stat	KPSS p-val	PP stat	PP p-val
USSTHPI	-3.88	0.02	0.03	0.10	-50.19	0.01
UNEMP	-4.87	0.01	0.05	0.10	-82.18	0.01

Table 3. Stationarity tests at the level of 1st differences

	ADF stat	ADF p-val	KPSS stat	KPSS p-val	PP stat	PP p-val
Paintings	-3.34	0.07	0.37	0.09	-25.35	0.01
Prints	-3.23	0.09	0.16	0.10	-24.70	0.02
Sculptures	-3.59	0.04	0.21	0.10	-30.40	0.01
Photographies	-4.10	0.01	0.10	0.10	-32.95	0.01
Drawings	-3.21	0.09	0.04	0.10	-30.20	0.01
Old Masters	-3.34	0.07	0.18	0.10	-50.16	0.01
19th Century	-3.10	0.13	0.30	0.10	-28.06	0.01
Modern Art	-2.48	0.38	0.10	0.10	-20.07	0.05
Post-War	-2.82	0.24	0.11	0.10	-25.31	0.01
Contemporary	-4.01	0.01	0.14	0.10	-33.64	0.01
U.S.	-3.72	0.03	0.16	0.10	-30.57	0.01
Global	-4.71	0.01	0.16	0.10	-114.39	0.01

TABLE 4. Coefficients for the best models

	Paintings	Prints	Sculptures	Photographies	Drawings	U.S.
Intercept	-3.90	-1.28	-2.16	1.14	2.65	0.54
MSCI	.	.	.	.	.	.
EM	.	.	.	.	.	.
SHC	.	.	.	.	.	.
VIX	0.39	0.36	0.69	.	.	.
P/E	-0.15	-0.15	.	.	-0.29	-0.17
CRB	0.16	.	.	.	.	.
GOLD	.	.	.	.	.	.
WTI	.	0.06	0.06	0.09	.	.
TWEXB	.	.	.	.	.	.
USSTHPI	.	0.73	1.63	.	.	.
GDP	2.69	.	.	.	.	.
DPI	.	.	.	.	.	.
PCE	.	2.28	3.30	.	.	2.69
US10BY	.	.	.	.	.	4.52
US30BY	.	.	.	.	.	.
LR	.	.	.	.	.	.
M1	-1.43	.	.	.	.	-0.49
M2	2.63	.	.	.	.	.
UNEMP	.	.	.	.	.	.
R^2	0.58	0.54	0.46	0.22	0.07	0.43
post prob	0.08	0.03	0.04	0.04	0.03	0.02

TABLE 5. Coefficients for the best models – cont.

	Old Masters	19th Century	Modern Art	Post-War	Contemporary	Global
Intercept	-0.45	-2.12	-1.14	5.00	0.86	-2.07
MSCI	.	.	.	.	.	.
EM	.	.	.	.	.	.
SHC	-0.04	0.04	.	0.08	0.14	.
VIX	.	0.38	.	.	.	1.05
P/E	.	-0.12	-0.21	.	.	.
CRB	0.13	.	.	0.26	.	.
GOLD	.	.	.	.	.	.
WTI	.	0.05	.	.	0.10	.
TWEXB	.	.	-1.18	.	.	.
USSTHPI	.	.	.	.	.	.
GDP	.	.	.	.	.	11.07
DPI	.	.	.	.	.	.
PCE	.	2.06	.	.	.	.
US10BY	.	.	3.64	.	.	.
US30BY	.	.	.	.	.	.
LR	.	.	.	.	.	.
M1	.	.	-1.13	-1.14	.	.
M2	.	.	1.57	.	.	.
UNEMP	.	.	.	.	.	.
R^2	0.17	0.51	0.58	0.37	0.41	0.20
post prob	0.10	0.07	0.11	0.07	0.21	0.10

Table 6. Expected values of coefficients

	Paintings	Prints	Sculptures	Photographies	Drawings	U.S.
Intercept	-3.92	-1.97	-1.75	0.30	0.56	0.44
MSCI	0.00	0.02	0.00	-0.01	0.00	0.00
EM	0.00	-0.01	0.00	-0.04	-0.04	0.00
SHC	0.00	0.01	0.00	0.00	0.00	0.01
VIX	0.43	0.34	0.62	0.01	0.01	0.08
P/E	-0.14	-0.17	-0.08	-0.05	-0.14	-0.16
CRB	0.14	0.01	0.00	0.02	0.08	0.01
GOLD	0.00	0.00	0.00	0.00	0.01	0.00
WTI	0.01	0.05	0.05	0.09	0.00	0.01
TWEXB	-0.11	-0.13	0.00	-0.01	-0.27	-0.01
USSTHPI	0.33	0.44	1.44	-0.01	-0.01	0.01
GDP	2.29	0.29	1.33	0.16	0.01	0.88
DPI	0.01	0.09	0.02	0.86	-0.01	0.00
PCE	0.17	1.78	1.42	1.05	0.39	1.55
US10BY	1.02	0.02	0.32	0.21	0.03	3.16
US30BY	-0.79	0.00	0.07	0.57	0.11	0.88
LR	-0.07	-0.44	0.01	0.33	0.02	0.59
M1	-1.40	-0.14	-0.28	-0.17	0.13	-0.24
M2	2.60	0.46	0.49	0.08	0.54	-0.01
UNEMP	0.03	0.04	1.73	0.45	-8.31	0.62

Table 7. Expected values of coefficients – cont.

	Old Masters	19th Century	Modern Art	Post-War	Contemporary	Global
Intercept	-0.34	-1.76	-0.58	3.44	1.02	-2.00
MSCI	0.00	0.00	0.02	0.01	0.00	-0.06
EM	0.00	0.00	-0.02	0.01	0.00	0.00
SHC	-0.03	0.04	0.00	0.08	0.14	-0.02
VIX	0.02	0.36	0.01	0.00	0.01	0.95
P/E	0.00	-0.11	-0.19	-0.05	-0.02	0.00
CRB	0.14	0.03	0.02	0.16	0.02	0.00
GOLD	0.00	0.00	0.00	0.00	0.00	0.03
WTI	-0.01	0.04	0.00	0.03	0.10	0.00
TWEXB	0.01	-0.21	-1.10	-0.08	0.00	-0.02
USSTHPI	0.01	0.00	0.00	-0.01	0.01	0.00
GDP	-0.01	-0.16	0.03	0.12	0.00	11.54
DPI	-0.06	0.00	0.00	0.04	0.12	0.65
PCE	-0.04	1.71	0.09	0.32	0.03	-0.64
US10BY	0.35	0.05	2.57	0.02	-0.11	-0.02
US30BY	-0.15	0.02	0.53	0.03	0.42	0.01
LR	0.02	-0.11	-0.17	-0.15	-0.15	-0.19
M1	-0.01	0.00	-0.98	-0.92	-0.08	-0.01
M2	-0.01	0.00	1.20	0.22	0.00	-0.02
UNEMP	0.11	0.13	-0.02	0.11	0.21	1.35

Table 8. Posterior probabilities (in %) that each variable is non-zero

	Paintings	Prints	Sculptures	Photographies	Drawings	U.S.
Intercept	100.00	100.00	100.00	100.00	100.00	100.00
MSCI	4.00	8.80	4.00	7.10	5.00	2.70
EM	3.80	18.80	3.70	27.20	24.00	4.60
SHC	5.60	39.50	3.70	3.60	2.10	41.00
VIX	100.00	99.70	100.00	5.10	4.40	30.90
P/E	91.30	100.00	59.60	25.20	49.00	96.40
CRB	95.80	10.90	6.30	13.00	36.10	17.40
GOLD	9.30	1.70	5.00	3.40	8.40	1.80
WTI	41.30	100.00	99.80	97.60	4.90	27.50
TWEXB	19.80	25.50	3.70	2.80	19.30	3.80
USSTHPI	41.50	57.30	98.90	2.70	3.40	4.20
GDP	89.50	17.50	49.70	6.40	2.60	35.80
DPI	4.00	12.80	5.20	40.30	2.60	1.90
PCE	10.10	76.10	49.40	24.50	11.10	54.40
US10BY	20.60	3.20	13.60	6.40	3.10	76.00
US30BY	13.00	2.50	6.50	12.40	4.30	22.70
LR	5.30	20.70	4.00	8.20	2.50	19.20
M1	100.00	24.40	32.40	18.20	15.20	47.60
M2	100.00	40.00	27.60	5.80	24.70	5.70
UNEMP	2.80	3.00	32.70	6.80	56.20	14.20

Table 9. Posterior probabilities (in %) that each variable is non-zero – cont.

	Old Masters	19th Century	Modern Art	Post-War	Contemporary	Global
Intercept	100.00	100.00	100.00	100.00	100.00	100.00
MSCI	3.70	3.80	10.50	6.00	2.60	11.40
EM	3.90	9.20	24.40	8.00	2.90	3.60
SHC	71.50	95.20	4.20	92.90	100.00	20.20
VIX	14.10	98.40	7.00	3.10	3.20	89.40
P/E	3.40	84.40	100.00	28.30	13.70	3.50
CRB	94.10	32.20	23.40	63.90	12.20	3.10
GOLD	3.90	3.80	2.10	2.30	2.60	22.50
WTI	23.80	99.70	10.20	48.50	100.00	6.00
TWEXB	4.10	34.00	100.00	8.40	2.50	3.40
USSTHPI	3.70	3.30	2.10	2.40	2.60	2.90
GDP	2.90	11.60	4.50	6.20	2.50	100.00
DPI	8.00	3.20	2.00	4.30	9.40	23.60
PCE	4.30	73.30	8.90	11.40	2.80	10.10
US10BY	11.20	5.30	76.90	2.30	4.30	3.00
US30BY	5.60	3.90	19.60	2.30	8.30	3.00
LR	3.30	8.00	10.40	5.40	5.80	4.20
M1	3.80	3.70	100.00	79.20	13.00	3.10
M2	4.00	3.10	78.60	13.00	3.80	3.20
UNEMP	4.70	6.20	2.20	3.00	4.60	10.80

Table 10. Forecasts comparison (RMSEs)

		"naive"
BMA	6.84	6.88
DMA (0.99)	7.03	6.90
DMA (0.98)	6.59	6.91
DMA (0.97)	6.66	6.91
DMA (0.96)	6.63	6.91
DMA (0.95)	6.59	6.91
DMA (0.94)	6.58	6.91
DMA (0.93)	6.56	6.90
DMA (0.92)	6.63	6.89
DMA (0.91)	6.48	6.88
DMA (0.90)	6.66	6.88

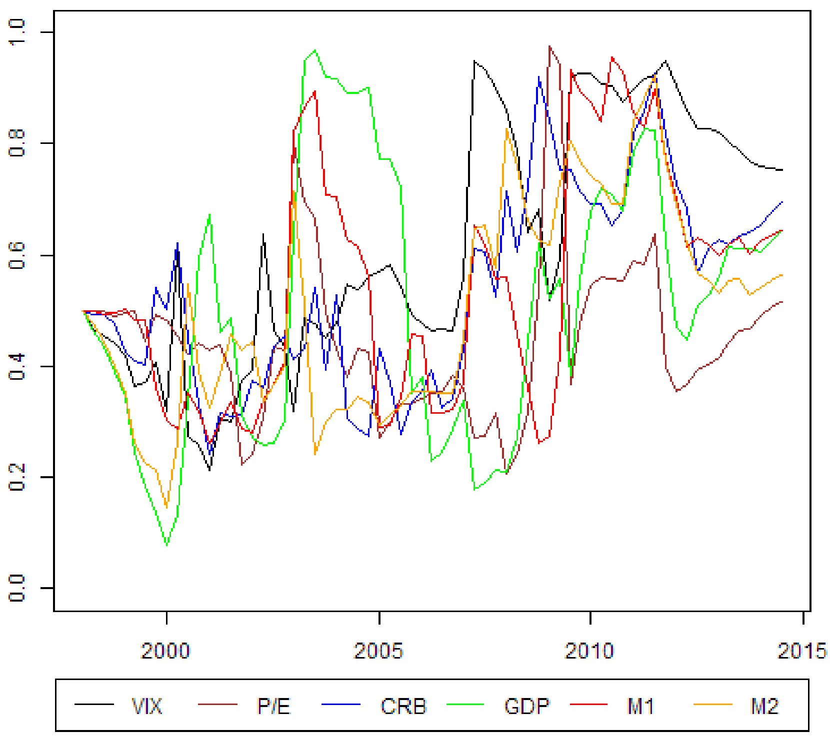

Figure 1. Posteriori probabilities for DMA with "forgetting factors" = 0.91

Acknowledgements

Research of K.D. was funded by the Polish National Science Centre grant under the contract number DEC-2015/19/N/HS4/00205.

References

[1] J. Orenstein, Show me the Monet: The suitability of product disparagement to art expert, George Mason Law Rev. 13 (2005) 905–934.

[2] J. Beckert, J. Rossel, The price of art, Eur. Soc. 15 (2013) 178–195.

[3] W. Baumol, Unnatural value: or art investment as a floating crap game, Am. Econ. Rev. 76 (1986) 10–14.

[4] H.W. Holub, M. Hutter, G. Tappeiner, Light and shadow in art price computation, J. Cult. Econ. 17 (1993) 49–69.

[5] L. Singer, Microeconomics of the art market, J. Cult. Econ. 2 (1978) 21–40.

[6] R. Campbell, Art as a financial investment, JAI 10 (2008) 64–81.

[7] A.C. Worthington, H. Higgs, Art as an investment: Short and long-term comovements in major painting markets, Empir. Econ 28 (2004) 649–668.

[8] A.C. Worthington, H. Higgs, Art as an investment: risk, return and portfolio diversification in major painting markets, Accounting and Finance 44 (2004) 257–271.

[9] B. Frey, W. Pommerehne, Muses and Markets: Explorations in the Economics of the Arts, Basil Blackwell, Oxford, 1989.

[10] L. Andre, G. Varet, On pricing the priceless: Comments on the economics of the visual art market, Eur. Econ. Rev. 39 (1995) 509–518.

[11] D. Sagot-Duvauroux, Art prices, In: (ed.) R. Towse, A Handbook of Cultural Economics, Edward Elgar, Cheltenham (2011), 43–48.

[12] V. Ginsburgh, D. Throsby, Handbook of the Economics of Art and Culture, Elsevier, Oxford, 2013.

[13] I. Robertson, Understanding Art Markets: Inside the World of Art and Business, Routledge, London, 2015.

[14] I. Robertson, D. Chong, The Art Business, Routledge, London, 2008.

[15] D. Hodgson, A. Seckin, Dynamic price dependence of Canadian and international art markets: an empirical analysis, Empir. Econ. 43 (2012) 867–890.

[16] V. Ginsburgh, P. Jeanfils, Long-term comovements in international markets for paintings, Eur. Econ. Rev. 38 (1995) 538–548.

[17] W. Goetzmann, L. Renneboog, C. Spaenjers, Art and money, Am. Econ. Rev. 101 (2011) 222–226.

[18] O. Chanel, Is art market behaviour predictable?, Eur. Econ. Rev. 39 (1995) 519–527.

[19] L. Figini, P. Onofri, Old Master paintings: price formation and public policy implications, In: (eds.) A. Marciano, J.-M. Josselin, Law and the State: A Political Economy Approach (2005), 381–398.

[20] G. Candela, A. Scorcu, A price index for art market auctions, J. Cult. Econ. 21 (1997) 175–196.

[21] O. Velthuis, S.B. Curioni, Cosmopolitan Canvases: The Globalization of Markets for Contemporary Art, Oxford University Press, Oxford, 2015.

[22] Artprice, Artprice Global Indices (2015) http://imgpublic.artprice.com/pdf/agi.xls.

[23] FRED, Federal Reserve Bank of St. Louis. Economic Research (2015) http://research.stlouisfed.org/fred2.

[24] Stooq, Stooq (2015) http://stooq.com.

[25] R. Shiller, Online data (2015) http://www.econ.yale.edu/ shiller/data.htm.

[26] P. Kennedy, A Guide to Econometrics, The MIT Press, Cambridge MA, 2003.

[27] R Core Team, R: A Language and Environment for Statistical Computing, R Foundation for Statistical Computing, Vienna, 2015.

[28] T.M. Fragoso, F.L. Neto, Bayesian model averaging: A systematic review and conceptual classification, Int. Stat. Rev. 86 (1) (2018) 1–28.

[29] J.A. Hoeting, D. Madigan, A.E. Raftery, C.T. Volinsky, Bayesian model averaging: A tutorial, Stat. Sci. 14 (1999) 382–417.

[30] T.A. Hakim, A.A. Karia, I. Bujang, Does goods and services tax stimulate economic growth? International evidence, J. Bus. Retail. Manag. Res. 10 (2016) 137–146.

[31] X. Sala-i-Martin, G. Doppelhofer, R.I. Miller, Determinants of long-term growth: A Bayesian averaging of classical estimates (BACE) approach, Am. Econ. Rev. 94 (2004) 813–835.

[32] X. Sala-i-Martin, I just ran two million regressions, Am. Econ. Rev. 87 (1997) 178–183.

[33] E. Moral-Benito, Model averaging in economics: an overview, J. Econ. Surv. 29 (2015) 46–75.

[34] A.E. Raftery, D. Madigan, J.A. Hoeting, Bayesian model averaging for linear regression models, JASA 92 (1997) 179–191.

[35] A.E. Raftery, J. Hoeting, C. Volinsky, I. Painter, K.Y. Yeung, BMA: Bayesian model averaging (2015) http://cran.r-project.org/package=BMA.

[36] A.E. Raftery, M. Karny, P. Ettler, Online prediction under model uncertainty via dynamic model averaging: Application to a cold rolling mill, Technometrics 52 (2010) 52–66.

[37] G. Aye, R. Gupta, S. Hammoudeh, W.J. Kim, Forecasting the price of gold using dynamic model averaging, IRFA 41 (2015) 257–266.

[38] K. Dedecius, I. Nagy, M. Karny, Parameter tracking with partial forgetting method, Internat. J. Adapt. Control Signal Process. 26 (2012) 1–12.

[39] T. Heryan, P.G. Tzeremes, The bank lending channel of monetary policy in EU countries during the global financial crisis, Econ. Model. 67 (2017) 10–22.