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Fixed Point Theorems via Absorbing Maps

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Abstract : The purpose of this paper is to obtain common fixed point theorems by using a new notion of absorbing maps in fuzzy metric space. In this paper we illustrate the properties of absorbing maps. Moreover we demonstrate the necessity of absorbing maps to find a common fixed point in fuzzy metric spaces and menger spaces. Our result generalizes many known results and explore the possibility of applying the notion of reciprocal continuity and absorbing maps to the problem of finding common fixed points of four mappings or sequence of mappings satisfying contractive type conditions in fuzzy metric spaces as well as probabilistics metric spaces without being continuous even at the fixed point.

Keywords : Absorbing Maps, Compatible maps, Reciprocal continuity, fuzzy metric space, common fixed point, continuous t-norm.
2000 Mathematics Subject Classification : 54H25, 47H10

1 Introduction

In 1965 Zadeh [33] introduced the notion of fuzzy sets. After this during the last few decades many authors have establish the existence of a lots of fixed point theorems in fuzzy setting: Badard [1], Bose and Sahani [2], Fang [7], Hadzic [13], Heilpern [14], Kaleva [17].

The aim of this paper is to introduce the new notion of absorbing maps which is neither a subclass of compatible maps nor a subclass of non-compatible maps. Also it is not necessary that absorbing maps commute at their coincidence points however if the mapping pair satisfy the contractive type condition then point wise absorbing maps not only commute at their coincidence points but it becomes a necessary condition for obtaining a common fixed point of mapping pair.

Let f and g are two self maps on a fuzzy metric space (X, M, *) then f is called g - absorbing if there exists a positive integer $R > 0$ such that

$$M(gx, gfx, t) \geq M(gx, fx, t/R) \ for \ all \ x \in X$$

Similarly g is called f - absorbing if there exists a positive integer $R > 0$ such that

$$M(fx, fgx, t) \geq M(fx, gx, t/R) \ for \ all \ x \in X$$

The map f is called point wise g - absorbing if for given $x \in X$, there exists a positive integer $R > 0$ such that

$$M(gx, gfx, t) \geq M(gx, fx, t/R) forall x \in X$$

similarly we can defined point wise f - absorbing maps.

2 Preliminaries and Notations

In this section we recall some definitions and known results in fuzzy metric space.

Definition 1. *[33] Let X be any non empty set. A fuzzy set A in X is a function with domain X and values in [0,1].*

Definition 2. *[25] A triangular norm * (shortly t-norm) is a binary operation on the unit interval [0, 1] such that for all $a, b, c, d \in [0, 1]$ the following conditions are satisfied:*
(1) $a * 1 = a$;
(2) $a * b = b * a$;
(3) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$;
(4) $a * (b * c) = (a * b) * c$.

Definition 3. *[9] The 3-tuple (X,M,*) is called a fuzzy metric space if X is an arbitrary non-empty set ,* is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$ and $s, t > 0$:*

(FM-1) $M(x, y, 0) = 0$,
(FM-2) $M(x, y, t) = 1$ for all $t > 0$ iff $x = y$,
(FM-3) $M(x, y, t) = M(y, x, t)$,
(FM-4) $M(x, y, t) * M(y, z, t) \geq M(x, z, t + s)$,
(FM-5) $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous
(FM-6) $\lim_{t\to\infty} M(x, y, t) = 1$.

Definition 4. *In the definition of George and Veeramani [9], M is a fuzzy set on $X^2 \times (0, \infty)$ and (FM-1), (FM-2), (FM-5) are replaced, respectively, with (GV-1), (GV-2), (GV-5) below (the axiom (GV-2) is reformulated as in [7, Remark 1]):*

(GV-1) $M(x, y, 0) > 0 \quad \forall\, t > 0$.
(GV-2) $M(x, x, t) = 1 \quad \forall\, t > 0$ and $x \neq y \Rightarrow M(x, y, t) < 1\ \forall\, t > 0$
(GV-5) $M(x, y, .) : (0, \infty) \to [0, 1]$ is continuous $\forall\, x, y \in X$.

Example 1. *[9] Let (X, d) be a metric space. Define $a * b = ab$ (or $a * b = \min\{a, b\}$) and for all $x, y \in X$ and $t > 0$, $M(x, y, t) = \frac{t}{t+d(x,y)}$. Then (X, M, *) is a fuzzy metric space. We call this fuzzy metric M induced by the metric d the standard fuzzy metric.*

Definition 5. *[32] A sequence $\{x_n\}$ in a fuzzy metric space (X, M,*) is called Cauchy if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ for each $t > 0$ as $n \to \infty$ uniformly on $p \in N$ (set of all natural numbers).*

*Fuzzy metric space (X, M, *) is said to complete if every Cauchy sequence in X converge to a point in X.*

A sequence $\{x_n\}$ in X is convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for each $t > 0$.

Definition 6. *[23] A pair (A, S) of self maps of a fuzzy metric space (X, M, *) is said to be reciprocal continuous if*

$\lim_{n\to\infty} ASx_n = Ax$ and $\lim_{n\to\infty} SAx_n = Sx$, whenever there exists a sequence $x \in X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$, for some $x \in X$.

If A and S are both continuous then they are obviously reciprocally continuous but the converse need not be true(see [23]).

We shall use the following lemmas to prove our next result without any further citation:

Lemma 1. *[21] If for all $x, y \in X$, $t > 0$ and $0 < k < 1$, $M(x, y, kt) \geq M(x, y, t)$, then $x = y$.*

Lemma 2. *[12] M(x, y, .) is non-decreasing for all x, y in X.*

Proof Suppose $M(x, y, t) > M(x, y, s)$ for some $0 < t < s$. Then $M(x, y, t) * M(y, y, s - t) \leq M(x, y, s) < M(x, y, t)$. By (FM-2), $M(y, y, s - t) = 1$, and thus $M(x, y, t) \leq M(x, y, s) < M(x, y, t)$ a contradiction.

3 Examples

In this section we have given some examples which are illustrate the properties of absorbing maps. Our first example shows that the class of absorbing maps is neither a sub class of compatible maps nor a sub class of non-compatible maps

Example 2. *[9] Let (X, d) be usual metric space where $X = [2, 20]$ and M be the usual fuzzy metric on (X, M, *) where $* = t_{min}$ be the induced fuzzy metric space with $M(x, y, t) = \frac{t}{t+d(x,y)}$ and $M(x, y, 0) = 0$ for $x, y \in X,\ t > 0$. We define mappings A, B, S and T by*

$fx = 6$ *if* $2 \leq x \leq 5$; $f6 = 6$; $fx = 10$ *if* $x > 6$; $fx = \frac{(x-1)}{2}$ *if* $x \in (5, 6)$

$gx = 2$ *if* $2 \leq x \leq 5$; $gx = \frac{(x+1)}{3}$ *if* $x > 5$

It is easy to see that both pairs (f, g) and (g, f) are not compatible but f is g - absorbing and g is f - absorbing. [Hint: Choose $x_n = 5 + \frac{1}{2n} : n \in N$]

Example 3. *let X = [0,1] be a metric space and d and M are same as in example 1. Define* $f, g : X \to X$ *by*

$$fx = \frac{x}{16} \text{ and } gx = 1 - \frac{x}{3}.$$

In this example we can see that

(1) f and g are compatible pair of maps and

(2) f is g - absorbing while g is f - absorbing

(Hint: Range of $f = [0, 1/16]$ *and range of* $g = [2/3, 1]$ *)*

Next we give an example to show that absorbing maps need not commute at their coincidence points, thus the notion of absorbing maps is different from other generalizations of commutativity which force the mapping to commute at coincidence points.

Example 4. *let X = [0,1] be a metric space and d and M are same as in example 1. Define* $f, g : X \to X$ *by*

$$fx = 1 \text{ for } x \neq 1; \; f1 = 0 \text{ and } gx = 1 for x \in X$$

Then the maps f is g - absorbing for any $R > 1$ *but the pair of maps (f, g) do not commute at their coincidence point x = 0.*

4 Main Results

Using the notion of point wise absorbing maps and reciprocal continuity of mappings we can widen the scope of many interesting results of fixed points in fuzzy metric spaces as well as menger spaces (eg. [3], [4], [5], [27], [28],[29],[30], [31],[18],[20],[21], [26],[6]).

Theorem 1. *Let P be point wise S - absorbing and Q be point wise T - absorbing self maps on a complete fuzzy metric space (X, M, *) with continuous t-norm defined by* $a * b = min\{a, b\}$ *where* $a, b \in [0, 1]$*, satisfying the conditions:*

(1.1) $P(X) \subseteq T(X)$, $Q(X) \subseteq S(X)$

(2.2) There exists $k \in (0, 1)$ *such that for every* $x, y \in X$ *and* $t > 0$,

$$M(Px, Qy, kt) \geq \min\{M(Sx, Ty, t), M(Px, Sx, t), M(Qy, Ty, t), M(Px, Ty, t)\}$$

(3.3) for all $x, y \in X$, $\lim_{t\to\infty} M(x, y, t) = 1$

If the pair of maps (P, S) is reciprocal continuous compatible maps then P, Q, S and T have a unique common fixed point in X.

Proof: let x_0 be any arbitrary point in X, construct a sequence $y_n \in X$ such that $y_{2n-1} = Tx_{2n-1} = Px_{2n-2}$ and $y_{2n} = Sx_{2n} = Qx_{2n+1}$, n = 1, 2, 3. This can be done by the virtue of (1.1). By using contractive condition we obtain,

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, kt) &= M(Px_{2n}, Qx_{2n+1}, kt) \\ &\geq \min\{M(Sx_{2n}, Tx_{2n+1}, t), M(Px_{2n}, Sx_{2n}, t), \\ &\quad M(Qx_{2n+1}, Tx_{2n+1}, t), M(Px_{2n}, Tx_{2n+1}, t)\} \\ &\geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), \\ &\quad M(y_{2n}, y_{2n+1}, t), 1\} \end{aligned}$$

which implies,

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)$$

in general,

$$M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t) \tag{1}$$

To prove $\{y_n\}$ is a Cauchy sequence, we have to show $M(y_n, y_{n+1}, t) \to 1$ (for $t > 0$ as $n \to \infty$ uniformly on $p \in N$), for this from (2.3) we have,

$$\begin{aligned} M(y_n, y_{n+1}, t) \geq M(y_{n-1}, y_n, t/k) \geq M(y_{n-2}, y_{n-1}, t/k^2) \geq .. \geq M(y_0, y_1, t/k^n) \\ \to 1 \ as \ n \to \infty \end{aligned}$$

for $p \in N$, by (1) we have

$$\begin{aligned} M(y_n, y_{n+p}, t) &\geq M(y_n, y_{n+1}, (1-k)t) * M(y_{n+1}, y_{n+p}, kt) \\ &\geq M(y_0, y_1, \frac{(1-k)t}{k^n}) * M(y_{n+1}, y_{n+2}, t) * \\ &\quad M(y_{n+2}, y_{n+p}, (k-1)t) \\ &\geq M(y_0, y_1, \frac{(1-k)t}{k^n}) * M(y_0, y_1, \frac{t}{k^n}) * M(y_{n+2}, y_{n+3}, t) * \\ &\quad M(y_{n+3}, y_{n+p}, (k-2)t) \\ &\geq M(y_0, y_1, \frac{(1-k)t}{k^n}) * M(y_0, y_1, \frac{t}{k^n}) * M(y_0, y_1, \frac{(1-k)t}{k^{n+2}}) \\ &\quad \cdots * M(y_0, y_1, \frac{(k-p)t}{k^{n+p+1}}) \end{aligned}$$

Thus $M(y_n, y_{n+p}, t) \to 1$ (for all $t > 0$ as $n \to \infty$ uniformly on $p \in N$). Therefore $\{y_n\}$ is a Cauchy sequence in X. But (X, M, *) is complete so there exists a point (say) z in X such that $\{y_n\} \to z$. Also, using (1.1) we have $\{Px_{2n-2}\}$, $\{Tx_{2n-1}\}$,

$\{Sx_{2n}\}$,$\{Qx_{2n+1}\} \to z$. Since the pair (P, S) is reciprocally continuous mappings, then we have,

$$\lim_{n\to\infty} PSx_{2n} = Pz \ and \ \lim_{n\to\infty} SPx_{2n} = Sz$$

and compatibility of P and S yields,

$$\lim_{n\to\infty} M(PSx_{2n}, SPx_{2n}, t) = 1$$

i.e. $M(Pz, Sz, t) = 1$. Hence Pz = Sz. Since $P(X) \subseteq T(X)$ then there exists a point u in X such that Pz = Tu. Now by contractive condition, we get,

$$\begin{aligned} M(Pz, Qu, kt) &\geq \min\{M(Sz, Tu, t), M(Pz, Sz, t), M(Qu, Tu, t), \\ &\qquad M(Pz, Tu, t)\} \\ &\geq \min\{M(Pz, Pz, t), M(Pz, Pz, t), M(Qu, Pz, t), \\ &\qquad M(Pz, Pz, t)\} \\ &> M(Pz, Qu, t) \end{aligned}$$

i.e. Pz = Qu. Thus Pz = Sz = Qu =Tu. Since P is S - absorbing then for $R > 0$ we have

$$M(Sz, SPz, t) \geq M(Sz, Pz, t/R) = 1$$

i.e. Pz = SPz = Sz. Now by contractive condition, we have,

$$\begin{aligned} M(Pz, PPz, t) = M(PPz, Qu, t) &\geq \min\{M(SPz, Tu, t), M(PPz, Su, t), \\ &\qquad M(Qu, Tu, t), M(PPz, Tu, t)\} \\ &= min\{M(Pz, Pz, t), M(PPz, Pz, t), M(Qu, Qu, t), \\ &\qquad M(PPz, Pz, t)\} \\ &= \text{M(PPz, Pz, t)}. \end{aligned}$$

i.e. PPz = Pz = SPz. Therefore Pz is a common fixed point of P and S. Similarly, T is Q - absorbing therefore we have,

$$M(Tu, TQu, t) \geq M(Tu, Qu, t/R) = 1$$

i.e. Tu = TQu = Qu. Now by contractive condition, we have

$$\begin{aligned} M(QQu, Qu, t) = M(Pz, QQu, t) &\geq \min\{M(Sz, TQu, t), M(Pz, Su, t), \\ &\qquad M(QQu, TQu, t), M(Pz, TQu, t)\} \end{aligned}$$

$$= min\{M(Sz, Qu, t), M(Pz, Pz, t), M(QQu, Qu, t), \\ M(Pz, Qu, t)\}$$

$$= \text{M(QQu, Qu, t)}.$$

i.e. QQu = Qu = TQu . Hence Qu = Pz is a common fixed point of P, Q, S and T. Uniqueness of Pz can easily follows from contractive condition. The proof is similar when Q and T are assumed compatible and reciprocally continuous. This completes the proof. □

Now we give an example to illustrate our theorem 1.

Example 5. *Let (X, d) be usual metric space where* $X = [2, 20]$ *and M be the usual fuzzy metric on (X, M, *) where* $* = t_{min}$ *be the induced fuzzy metric space with* $M(x, y, t) = \frac{t}{t+d(x,y)}$ *for* $x, y \in X,\ t > 0$. *We define mappings P, Q, S and* $T : X \to X$ *by*

P2 = 2, Px = 3 if $x > 2$,

S2 = 2, Sx = 6 if $x > 2$,

Qx = 2 if x = 2 or $x > 5$, *Qx = 6 if* $2 < x \leq 5$,

Tx = 2 if $2 \leq x \leq 5$, *Tx = x - 3 if* $x > 5$.

Then P, Q, S and T satisfy all the conditions of the above theorem with $k \in (3/4, 1)$ *and have a unique common fixed point x = 2. It may be noted that in this example* $P(X) = \{2, 3\} \subseteq T(X) = [2, 17]$ *and* $Q(X) = \{2, 6\} \subseteq S(X) = \{2, 6\}$ *and P and S are reciprocal continuous compatible mappings. But neither P nor S is continuous even at fixed point x = 2. The mappings Q and T are non-compatible but Q is point wise T - absorbing. To see Q and T are non-compatible let us consider the sequence* $\{x_n\}$ *in X defined by* $\{x_n = 5 + 1/n; n \geq 1\}$. *Then* $\{Tx_n\}, \{Qx_n\}, \{TQx_n\} \to 2$ *and* $\{QTx_n\} \to 6$.*Hence Q and T are non-compatible.*

Our theorem thus improves the results of Singh et al [27] in three ways. **Firstly**, by using the notion of both compatibility and point wise absorbing maps, it widens the scope of the study of common fixed point theorems from the class of compatible map to the wider class of point wise absorbing map. **Secondly**, our theorem does not force the maps to be continuous even at the common fixed point and **thirdly**, contractive condition of our theorem 2 is more general than the contractive condition of B. Singh et al [27](see remark).

Remark 1. $M(Ax, By, kt) \geq M(Sx, Ty, t)*M(Ax, Sx, t)*M(By, Ty, t)*M(By, Sx, 2t)*$
$M(Ax, Ty, t) \geq M(Sx, Ty, t)*M(Ax, Sx, t)*M(By, Ty, t)*M(Sx, Ty, t)*M(Ty, By, t)*$
$M(Ax, Ty, t) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$

Now we can prove our next theorem by assuming the range of one of the mappings P, Q, S or T to be a complete subspace of X.

Theorem 2. *Let P is point wise S-absorbing and Q be point wise T-absorbing pairs of self mappings of a fuzzy metric space (X, M, *) satisfying conditions:*

(2.1) $P(X) \subseteq T(X)$, $Q(X) \subseteq S(X)$

(2.2) There exists $k \in (0,1)$ *such that for every* $x, y \in X$ *and* $t > 0$,

$$M(Px, Qy, kt) \geq \min\{M(Sx, Ty, t), M(Px, Sx, t), M(Qy, Ty, t), M(Px, Ty, t)\}$$

(2.3) for all $x, y \in X$, $\lim_{t\to\infty} M(x, y, t) = 1$

If the range of one of the mappings P, Q, S or T be a complete subspace of X then P, Q, S and T have a unique common fixed point in X.

Proof: let x_0 be any arbitrary point in X, construct a sequence $y_n \in X$ such that

$$y_{2n-1} = Tx_{2n-1} = Px_{2n-2} \quad \text{and} \quad y_{2n} = Sx_{2n} = Qx_{2n+1}, n = 1, 2, 3, \ldots \tag{2.4}$$

This can be done by the virtue of (2.1) and by using the same techniques of above theorem we can show that $\{y_n\}$ is a Cauchy sequence. Let S(X) the range of X be a complete metric subspace than there exists a point Su such that $\lim_{n\to\infty} Sx_{2n} = Su$. By (2.4) we get $Qx_{2n+1} \to Su$,$Px_{2n-2} \to Su$, $Tx_{2n-1} \to Su$ and $\{y_n\} \to Su$ as $n \to \infty$. By using contractive condition we obtain,

$$M(Pu, Qx_{2n+1}, kt) \geq \min\{M(Su, Tx_{2n+1}, t), M(Pu, Su, t), M(Qx_{2n+1}, Tx_{2n+1}, t), M(Pu, Tx_{2n+1}, t)\}$$

Letting $n \to \infty$ we get

$$M(Pu, Su, kt) \geq \min\{M(Su, Su, t), M(Pu, Su, t), M(Su, Su, t), M(Pu, Su, t)\}$$

i.e. $Pu = Su$. Since $P(X) \subset T(X)$ then there exists $w \in X$ such that $Su = Tw$. Again by using contractive condition we get,

$$M(Pu, Qw, kt) \geq \min\{M(Su, Tw, t), M(Pu, Su, t), M(Qw, Tw, t), M(Pu, Tw, t)\}$$

i.e $Pu = Su = Qw = Tw$. Since P is pointwise S-absorbing then we have

$$M(Su, SPu, t) \geq M(Su, Qu, t/R)$$

i.e. $Su = SPu = SSu$, and similarly Q is pointwise T-absorbing then we have

$$M(Tw, TQw, t) \geq M(Tw, Qw, t/R)$$

i.e. $Tw = TQw = QQW$. Thus $Su(= Tw)$ is a common fixed point of P, Q, S and T.Uniquness of common fixed point follows from contractive condition.The proof is similar when $T(X)$, the range of T is assumed to be a complete subspace of X. Moreover, Since $P(X) \subset T(X)$ and $Q(x) \subset S(X)$. The proof follows on similar line when either the range of P or the range of Q is assumed complete. This completes the proof of the theorem.

Remark 2. We now demonstrate that point wise absorbing map is a necessary condition for the existence of common fixed point of contractive mapping pairs in fuzzy metric spaces. So let us assume that the self-mappings A and S of a fuzzy metric space (X, M, *) satisfy the contractive condition

$$M(Ax, Ay, kt) \geq \min\{M(Sx, Sy, t), M(Ax, Sx, t), M(Ay, Sy, t), M(Ax, Sy, t), M(Ay, Sx, t)\}$$

Which is one of the general contractive condition for a pair of mappings. Further suppose if possible suppose that A fails to be point wise R-S absorbing and yet have a common fixed point z (say). Then z = Az = Sz and there exists x X such that Ax = Sx but $Sx \neq SAx$. Clearly $z \neq x$ for if z = x then we get Sx = SAx. Moreover $Az \neq Ax$ for if Az = Ax then SAx = SAz = Sz = z and also Sx = Ax = Az = z. So again we get Sx = SAx. Hence we conclude that $Ax \neq Az$. But then we have,

$$M(Ax, Az, kt) \geq \min\{M(Sx, Sz, t), M(Ax, Sx, t), M(Az, Sz, t), M(Ax, Sz, t), M(Az, Sx, t)\}$$

$$= \min\{M(Ax, Az, t), 1, 1, M(Ax, Az, t), M(Az, Ax, t)\}$$

$$= M(Ax, Az, t) \qquad .$$

thus by lemma 1 we get a contradiction. Hence A must be point wise S - absorbing.There for point wise g - absorbing map is also a necessary condition for the existence of common fixed points for pair of mappings satisfying contractive type condition in fuzzy metric spaces.

Remark 3. The known common fixed point theorems in fuzzy metric spaces or Probabilistic metric spaces for four mappings satisfying compatibility (or semi-compatibility/ compatibility of type(A)/compatibility of type($\alpha/A1$ or $\beta/A2$) conditions or similar results involving a sequence of mappings require one of the mappings in compatible pair to be continuous. For example, Theorem 4.1 of Cho [3] et al, assumes one of A, B, S and T to be continues with compatibility of type (A) of (A, S) and (B, T). Theorem 4.1 of Cho et al [4], assumes the mappings S and T to be continuous with compatibility of type (β) of (P, S) and (Q, T). Theorem 4.1 of Sharma [26], assumes the mappings A, B, S and T to be continues with compatibility of type (A) of (P, AB) and (Q, ST). Likewise, the main theorem of Chung et al [6], assumes one of A, B, S and T to be continues with R-weak commutativity of (A, S) and (B, T) and Khan et al [18] in their Theorem

4.2 assumes one of S and T to be continuous with compatibility of type (A_1 or A_2) of (P, S) and (Q, T). Similarly, the main theorem of Cho-Hong [5] assume S and T to be continuous with the compatibility of (A, S) and (B, T). Theorem 3.1 of Kutukchu [20] assumes AB or L to be continuous with compatibility of (AB, L). One more theorem of Kutukchu [19] assumes the maps S to be continuous and the pair of maps (S, T_n) to be commuting maps. Similarly, B. Singh et al [27], [30] in their theorems 3.1 assumes one of A or S is continuous with semi-compatibility/compatibility of (A, S). We observe that it has been shown by the authors [31], [18], [32] that continuity of A or S with compatibility of type (A) or (A, S) implies compatibility of (A, S) . Similarly continuity of B or T with compatibility of type (α/A_1) or (β/A_2) implies compatibility of (B, T) and reciprocal continuity of (A, S) with semi compatibility implies compatibility of (A, S).

Remark 4. It is obvious that in most of the fixed point theorems in Menger Spaces as well as fuzzy metric spaces to prove the sequence of iterates of a point is a Cauchy sequence a particular class of t-norm is required. In our above theorem we have assumed the t-norm as min norm, however, adopting the approach of Liu et al [16] one can easily replace the condition of min norm by a larger class of t-norm called Hadzic [13]type t-norm (in short H-type t-norm).

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