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K_{2n+1} That Are $(2n+1)$-Color n Sequentially Hamiltonian

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Abstract It is known that K_{2n+1} is the sum of n spanning cycles. We assign colors from $2n+1$ colors to each line of K_{2n+1}. We find that, with some condition, it is possible to assign colors to K_{2n+1} such that each point is adjacent to $2n$ lines of different colors and each of n hamiltonian cycles has $2n+1$ lines of different colors.

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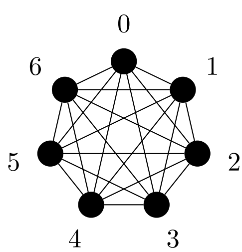
1. Introduction

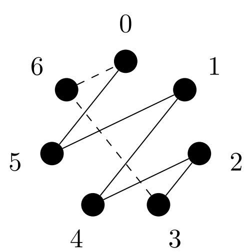
In this paper, we shall generally use definitions from [1]. We are discussing some properties of complete graph K_{2n+1}. From [1], we have the following theorem.

Theorem 1.1 (see [1]). *The graph K_{2n+1} is the sum of n spanning cycles.*

For example, K_7 is the sum of 3 spanning (hamiltonian) cycles, see [1].



K_7



(a)

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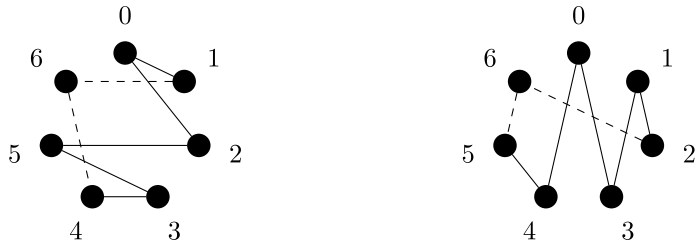


(b) (c)

Fig. 1.1

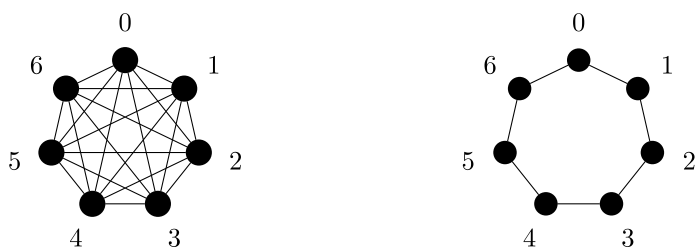
Some n spanning cycles of K_{2n+1}, with some conditions, can have some interesting properties.

Definition 1.2. K_{2n+1} is called n sequentially hamiltonian if K_{2n+1} can have n spanning cycles of which all lines of the first cycle, all lines of the second cycle, all lines of the third cycle, $\ldots$, all lines of the n-th cycle have distances $1, 2, 3, \ldots, n$ respectively.

Theorem 1.3 tell us which K_{2n+1} can be n sequentially hamiltonian.

Theorem 1.3 (see [2]). *For every* $n \geq 1$, K_{2n+1} *is* n *sequentially hamiltonian if and only if* $2n+1$ *is prime number.*

For example, from Theorem 1.1, K_7 is the sum of 3 spanning cycles, see Fig. 1.1. But, since 7 is prime number, from Theorem 1.3, K_7 can also be 3 sequentially hamiltonian, see Fig.1.2.



K_7 (a)

All lines have distances 1

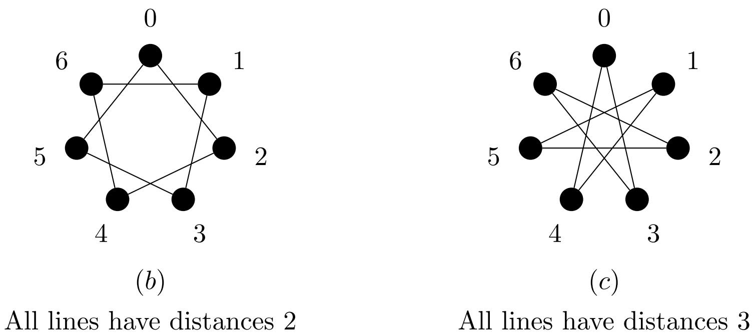


Fig. 1.2

If we assign colors from $2n+1$ colors to each line of K_{2n+1}, we call K_{2n+1} as $(2n+1)-$color. Given $(2n+1)-$color K_{2n+1}, we know that K_{2n+1} is the sum of n spanning cycles. Can we assign colors to lines of K_{2n+1} such that each line of the n hamiltonian cycles has $2n+1$ different colors, and also each point of K_{2n+1} is adjacent to $2n$ lines of different colors.

Definition 1.4. Let K_{2n+1} be n sequentially hamiltonian. We call K_{2n+1} as $(2n+1)-$color n sequentially hamiltonian if K_{2n+1} is $(2n+1)-$color and each hamiltonian cycle has $2n+1$ lines of different colors, and also each point is adjacent to $2n$ lines of different colors.

For example, consider 5-color K_5 in Fig. 1.3.

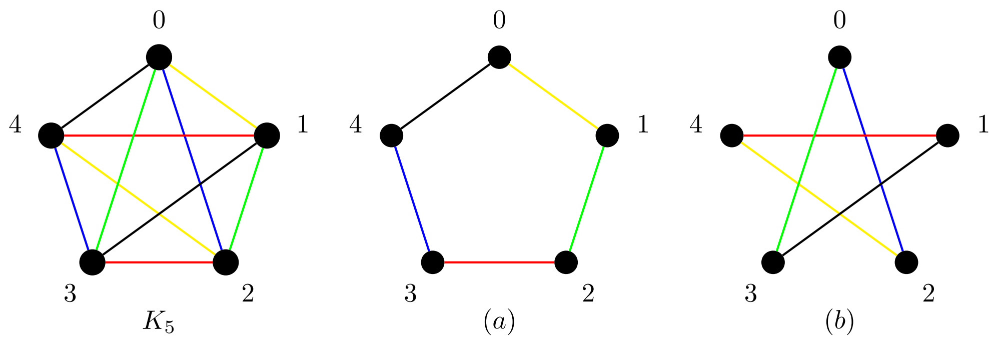


Fig. 1.3

There are 2 hamiltonian cycles

$$0\ 1\ 2\ 3\ 4\ 0$$

and

$$0\ 2\ 4\ 1\ 3\ 0.$$

We can see that each line of the first cycle, and each line of the second cycle has 5 different colors, and each point of K_5 is adjacent to 4 lines of different colors. So K_5 is 5-color 2 sequentially hamiltonian.

For another example, consider 7-color K_7 in Fig. 1.4.

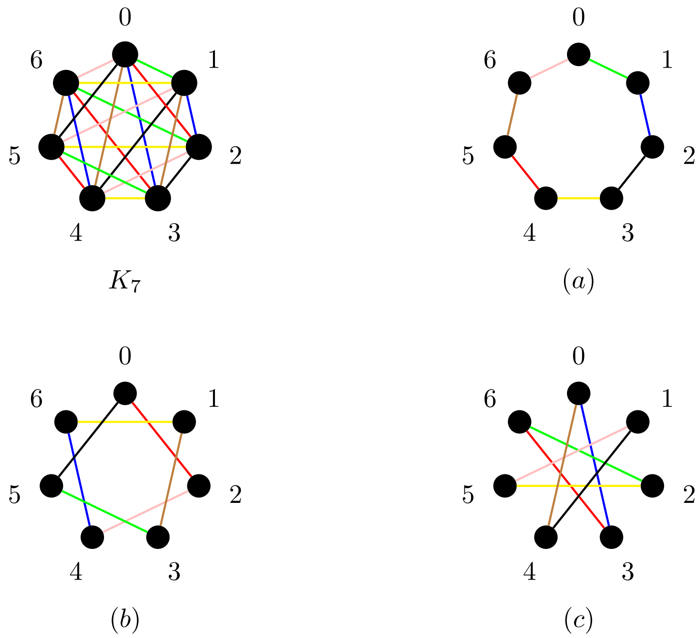


Fig. 1.4

There are 3 hamiltonian cycles

0 1 2 3 4 5 6 0
0 2 4 6 1 3 5 0
and 0 3 6 2 5 1 4 0.

We can see that each line of the first, second, and third hamiltonian cycles has 7 different colors. Also, each point of the K_7 is adjacent to 6 lines of different colors. Therefore, K_7 is 7-color 3 sequentially hamiltonian.

We show, see Fig. 2.1 in Section 2, how to assign colors to K_7 so that K_7 becomes 7-color 3 sequentially hamiltonian.

Now look at K_9, we find that K_9 has no hamiltonian cycle of which each line has distance three, see Fig. 1.5.

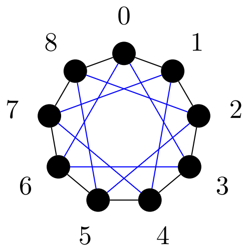


Fig. 1.5

So, K_9 is not 3 sequentlially hamiltonian, and therefore K_9 is not 9-color 3 sequentially hamiltonian.

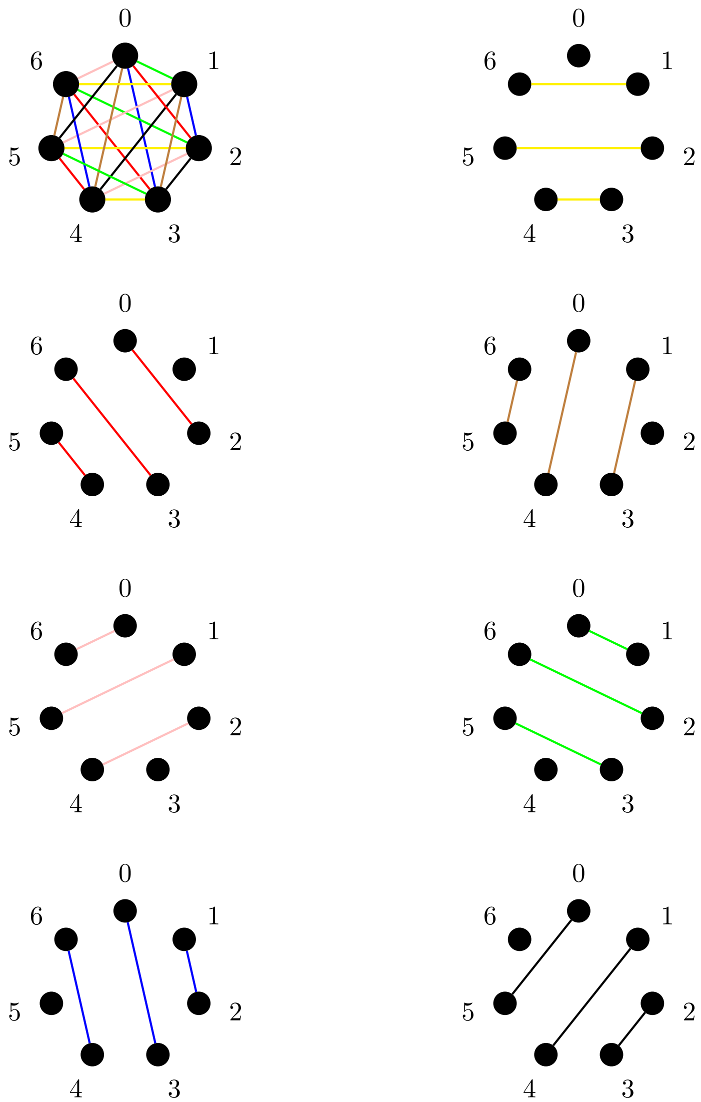
2. n Sequentially Hamiltonian and $(2n+1)-$Color n Sequentially Hamiltonian

In this section we show that it is possible to assign $2n+1$ colors to lines of K_{2n+1} that is n sequentially hamiltonian such that K_{2n+1} becomes $(2n+1)-$color n sequentially hamiltonian.

Theorem 2.1. *If K_{2n+1} is n sequentially hamiltonian, then K_{2n+1} can become $(2n+1)-$color n sequentially hamiltonian.*

Proof. Consider K_{2n+1} that is n sequentially hamiltonian. Let $0, 1, 2, 3, \ldots, 2n$ be points of K_{2n+1}, and let $c_0, c_1, c_2, c_3, \ldots, c_{2n}$ be $2n+1$ different colors. Consider point 0, for example, of K_{2n+1}, and all n lines that are joined between two equidistant points from 0, i.e. lines $\{1, 2n\}, \{2, 2n-1\}, \{3, 2n-2\}, \ldots, \{n, 2n-(n-1)\}$. We assign color c_0 to all these n lines. Note that these lines have distance 1, distance 2, ... , distance n. Next, consider point 1, and all n lines that are joined between two equidistant points from 1. We assign color c_1 to all these lines. We consider points $2, 3, 4, \ldots, 2n$ and repeat the same process that we have done to point 0, and, point 1. Now, we have assigned colors, from $2n+1$ colors, to all lines of K_{2n+1}. We note that all lines of distance 1 have different colors. In fact , all lines of distances k $(k = 1, 2, 3, \ldots, n)$ have different colors. Also, we note that every point of K_{2n+1} is adjacent to $2n$ lines of different colors. From Definition 1.2 , K_{2n+1} has n hamiltonian cycles, from which one cycle has distance 1 for all its lines, one cycle has distance 2 for all its lines, ... , and one cycle has distance n for all its lines. Therefore, each cycle of K_{2n+1} has $2n+1$ lines of different colors, so K_{2n+1} is $(2n+1)-$color n sequentially hamiltonian. ∎

The following Fig. 2.1 illustrates the proof of Theorem 2.1 when 2n+1=7.



Assigning 7 colors to certain lines, K_7 becomes 7-color 3 sequentially hamiltonian

Fig. 2.1

References

[1] F. Harary, Graph Theory, Addison-Wesley Publishing Company, Boston, 1969.

[2] H. Yingtaweesittikul, V. Longani, A property of K_{2n+1} as the sum of n spanning cycles, (to appear).