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Dedicated to Prof. Suthep Suantai on the occasion of his 60^{th} anniversary

Practical Exponential Stability of Discrete Time Impulsive System with Delay

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Abstract In this paper studies practical exponential stability of discrete-time impulsive system with delay. By using Lyapunov functions and Razumikhin type technique, some criteria for practical exponential stability of discrete-time impulsive system with delay are achieved. Moreover, some numerical example is given to show the effectiveness of our theoretical result.

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1. Introduction

A discrete-time system is a more natural way to represent systems such as numerical analysis and population models [1–4]. In most dynamics systems, the state always instantly changes at a certain moment. It is natural to assume that such dynamical systems with abrupt changes spontaneously occur, and this kind of system is called impulsive systems. Systems with impulses provide a natural framework for mathematical modeling of many real world phenomena in which the state undergo abrupt changes. For example, many processes and events studied in chemical, physics, population dynamics, biotechnology, economics, and technology do exhibit impulse effects [5, 6]. In [7], the authors studied an oscillation theorem for nonlinear hyperbolic systems with impulses. Moreover, the real processes in our world always involve time-delay systems, see [8–10]. Therefore, impulsive systems with time delay have been investigated extensively over the past decades [11–18].

The theory of stability have been published in many areas [19, 20]. In case of exponential stability, it is required that all solutions starting near an equilibrium point not only stay nearby, but tend to the equilibrium point very fast with exponential decay rate.

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Meanwhile, for one of the essential types of stability is practical stability, it only needs to stabilize a system into a region of phase space, namely the system may oscillate near the state in which the performance is still acceptable. Theory of practical stability has been widely considered for continuous-time systems [21–23]. In [24], the authors focused on the practical stability of nonlinear time-varying cascade systems. On the other hand, there are few results for discrete-time systems [25–28]. In [27], the authors studied the practical stability and controllability for a class of nonlinear discrete systems with time delay. The practical stability of impulsive discrete systems with time delays in some cases, was discussed in [25]. In [28], the authors used the Razumikhin-type technique to derive the exponentially practical stability of discrete time singular system with delay and disturbance. Therefore, more attention has been paid to the theory of practical stability of the impulsive discrete-time system with delay.

This work aims to establish and improve the criteria for practical exponential stability of the discrete-time impulsive system with delay by using Razumikhin type technique. The paper is organized as follows: In Section 2 some notations and definitions are introduced. In Section 3, we present some criteria for practical exponential stability of discrete-time impulsive system with delay. In Section 4, we give some example to show the effectiveness of our theoretical result. Section 5 concludes the paper.

2. Preliminaries

Let R^n denote the n dimensional Euclidean space, $\|x\|$ is the Euclidean norm of vector x. Given a positive integer τ, for any function $\phi : N_{-\tau} \longrightarrow R^n$, we define $\|\phi\| = \max_{\theta \in N_{-\tau}}\{\|\phi(\theta)\|\}$, $N = \{0, 1, 2, \ldots\}$ and $N_{-\tau} = \{-\tau, -\tau-1, \ldots, -1, 0\}$.

Consider the following discrete time impulsive system with delay.

$$\begin{cases} x(k+1) = f(k, x_k), & k \neq k_m \\ x(k_m) = J_m(x(k_m - 1)), & k = k_m \\ x(s) = \phi(s), & s \in N_{k_0-\tau} \end{cases} \tag{2.1}$$

where $x(k) \in R^n$, x_k is defined $x_k(s) = x(k+s)$ for any $s \in N_{-\tau}$. We assume $f : N \times N_{k_0-\tau} \to R^n$,$J_m : R^n \to R^n$ for $m \in N$, and the impulsive moment satisfy $0 \leq k_0 < k_1 < k_2 < \ldots, k_m \to \infty$ for $m \to \infty$. Furthermore, we assume $f(k, 0) = 0, J_m(0) = 0$, so the systems (2.1) admit the trivial solution. Let $x(k; k_0, \phi)$ denote the trajectory of system (2.1) with initial value ϕ.

Definition 2.1. The trivial solution of system (2.1) is globally practically exponentially stable in the $p^{th}-$ moment, if, for any $k \geq k_0$ there exist constant $0 < \lambda < 1, M \geq 0, r > 0$ such that

$$\|x(k; k_0, \phi)\|^p \leq M\|\phi\|^p e^{-\lambda(k-k_0)} + r.$$

3. Main Result

In this section, we consider practical exponential stability of discrete-time impulsive system with delay (2.1) as follow the result.

Theorem 3.1. *If there exist positive number* $a, c_1, c_2, p, \gamma, q, \beta, \eta$*;*

$$q > \gamma > 0, \ \frac{e-1}{e} < \beta \leq 1, \ \eta < \beta a$$

and Lyapunov function $V(k, x(k))$ such that following condition hold:

$$\begin{aligned}
(i) \quad & c_1\|x\|^p \leq V(k,x(k)) \leq c_2\|x\|^p + a, \quad \forall k \geq k_0 - \tau, \quad x \in R^n, \\
(ii) \quad & \text{If} \quad V(k+s, x(k+s)) < qV(k+1, x(k+1)) \quad \text{with} \quad s \in N_{-\tau} \\
& \text{then} \quad \Delta V(k,x(k)) = V(k+1,x(k+1)) - V(k,x(k)), \\
& \qquad \leq -\beta V(k,x(k)) + \eta \quad \text{hold}, \\
(iii) \quad & V(k_m, x(k_m)) \leq \gamma V(k_m - 1, x(k_m - 1)), \quad m \in N, \quad x \in R^n.
\end{aligned}$$

Then, the trivial solution of system (2.1) is globally practically exponentially stable in the $p^{th}-$ moment.

Proof. Case I. $\quad q > \gamma \geq 1$.
Since $q > \gamma \geq 1$, then there exist $0 < \lambda < 1, q^*$ such that

$$q > q^* e^{\lambda(\tau+1)} > q^* > \gamma e^{\lambda(\tau+1)} \geq e^{\lambda(\tau+1)}.$$

For $k \in [k_0 - \tau, k_0]$, from (i), we can see that

$$\begin{aligned}
V(k,x(k)) \leq c_2\|x\|^p + a & \leq c_2\|x\|^p e^{-\lambda(k-k_0)} + a, \\
& \leq c_2\|\phi\|^p e^{-\lambda(k-k_0)} + a. \qquad (3.1)
\end{aligned}$$

We claim that

$$V(k,x(k)) \leq c_2\|\phi\|^p e^{-\lambda(k-k_0)} + a, \quad k \in [k_{m-1}, k_m - 1]. \qquad (3.2)$$

Now, we will prove (3.2) by using mathematical induction.
First, we show that (3.2) holds for $m = 1$, namely

$$V(k,x(k)) \leq c_2\|\phi\|^p e^{-\lambda(k-k_0)} + a, \quad k \in [k_0, k_1 - 1]. \qquad (3.3)$$

We assume (3.3) were not true, then there exists $k \in [k_0, k_1 - 1]$ such that

$$V(k,x(k)) > c_2\|\phi\|^p e^{-\lambda(k-k_0)} + a,$$

and let

$$k^* = \min\Big\{k \in [k_0, k_1 - 1] / V(k,x(k)) > c_2\|\phi\|^p e^{-\lambda(k-k_0)} + a\Big\}.$$

From (3.1) and definition of k^*, we have

$$V(k,x(k)) \leq c_2\|\phi\|^p e^{-\lambda(k-k_0)} + a, \quad k \in [k_0 - \tau, k^* - 1].$$

Therefore, for any $s \in N_{-\tau}$, we have

$$\begin{aligned}
V(k^* - 1 + s, x(k^* - 1 + s)) & \leq c_2\|\phi\|^p e^{-\lambda(k^* - 1 + s - k_0)} + a, \\
& = c_2\|\phi\|^p e^{-\lambda(s-1)} e^{-\lambda(k^* - k_0)} + a, \\
& \leq c_2 e^{\lambda(\tau+1)}\|\phi\|^p e^{-\lambda(k^* - k_0)} + a e^{\lambda(\tau+1)}, \\
& = e^{\lambda(\tau+1)}\Big[c_2\|\phi\|^p e^{-\lambda(k^* - k_0)} + a\Big], \\
& < qV(k^*, x(k^*)).
\end{aligned}$$

Let $\overline{k} = k^* - 1$, then we get

$$V(\overline{k} + s, x(\overline{k} + s)) \leq qV(\overline{k} + 1, x(\overline{k} + 1)).$$

By condition (ii), we have

$$\begin{aligned} V(k^*, x(k^*)) &\leq (1-\beta)\; V(k^*-1, x(k^*-1)) + \eta, \\ &\leq (1-\beta)\Big[c_2\|\phi\|^p e^{-\lambda(k^*-1-k_0)} + a\Big] + \eta, \\ &\leq (1-\beta)e^{\lambda} c_2\|\phi\|^p e^{-\lambda(k^*-k_0)} + a - \beta a + \eta, \end{aligned}$$

since, maximal value of $\frac{e^{\lambda}-1}{e^{\lambda}}$, $0 < \lambda < 1$ is $\frac{e-1}{e}$.
So, we choose

$$\frac{e-1}{e} < \beta \leq 1, \quad \eta < \beta a,$$

which is contradiction to the definition of k^*.
Hence (3.3) holds.
Now, we assume (3.2) holds for $m \in N$, namely

$$V(k, x(k)) \leq c_2\|\phi\|^p e^{-\lambda(k-k_0)} + a, \quad k \in [k_{m-1}, k_m - 1].$$

Next, we will show that

$$V(k, x(k)) \leq c_2\|\phi\|^p e^{-\lambda(k-k_0)} + a, \quad k \in [k_m, k_{m+1} - 1]. \tag{3.4}$$

First, we will prove limiting condition holds, when impulsive instant, namely

$$V(k, x(k)) \leq \frac{c_2}{q^*}\|\phi\|^p e^{-\lambda(k-k_0)} + \frac{a}{q^*}, \quad k \in [k_{m-1}, k_m - 1]. \tag{3.5}$$

We assume (3.5) were not true, there exists a

$$k^* = \min\Big\{k \in [k_{m-1}, k_m - 1]/V(k, x(k)) > \frac{c_2}{q^*}\|\phi\|^p e^{-\lambda(k-k_0)} + \frac{a}{q^*}\Big\},$$

and we know that

$$V(k, x(k)) \leq \tfrac{c_2}{q^*}\|\phi\|^p e^{-\lambda(k-k_0)} + \tfrac{a}{q^*}, \quad k \in [k_{m-1}, k^* - 1],$$

then, we have

$$\begin{aligned} V(k^*-1+s, x(k^*-1+s)) &\leq c_2\|\phi\|^p e^{-\lambda(k^*-1+s-k_0)} + a, \\ &\leq e^{\lambda(\tau+1)}\Big[c_2\|\phi\|^p e^{-\lambda(k^*-k_0)} + a\Big], \\ &< q^* e^{\lambda(\tau+1)}\Big[c_2\|\phi\|^p e^{-\lambda(k^*-k_0)} + a\Big], \\ &< qV(k^*, x(k^*)). \end{aligned}$$

So, we get

$$V(\bar{k}+s, x(\bar{k}+s)) \leq qV(\bar{k}+1, x(\bar{k}+1)),$$

and from (ii), we have

$$V(k^*, x(k^*)) \leq (1-\beta)\; V(k^*-1, x(k^*-1)) + \eta,$$

which is contradiction, so (3.5) holds.
From (iii), we can see that

$$\begin{aligned} V(k_m, x(k_m)) &\leq \gamma V(k_m - 1, x(k_m - 1)), \\ &\leq \gamma \left[\frac{c_2}{q^*} \|\phi\|^p e^{-\lambda(k_m - 1 - k_0)} + \frac{a}{q^*} \right], \\ &= \frac{\gamma e^{\lambda} c_2}{q^*} \|\phi\|^p e^{-\lambda(k_m - k_0)} + \frac{\gamma a}{q^*}, \\ &< \frac{q^* c_2}{q^*} \|\phi\|^p e^{-\lambda(k_m - k_0)} + \frac{\gamma a}{q^*}, \\ &\leq c_2 \|\phi\|^p e^{-\lambda(k_m - k_0)} + a. \end{aligned} \tag{3.6}$$

Finally, we prove that (3.2) holds, assume (3.2) were not true, there exists a

$$k^* = \min\left\{ k \in [k_m, k_{m+1} - 1] / V(k, x(k)) > c_2 \|\phi\|^p e^{-\lambda(k - k_0)} + a \right\}.$$

For $k \in [k_m, k^* - 1]$, we have

$$V(k, x(k)) \leq c_2 \|\phi\|^p e^{-\lambda(k - k_0)} + a,$$

and from (3.6), we know that $k^* \in [k_m, k_{m+1} - 1]$.
So, we have

$$\begin{aligned} V(k^* - 1 + s, x(k^* - 1 + s)) &\leq c_2 \|\phi\|^p e^{-\lambda(k^* - 1 + s - k_0)} + a, \\ &\leq e^{\lambda(\tau+1)} \left[c_2 \|\phi\|^p e^{-\lambda(k^* - k_0)} + a \right], \\ &< q^* e^{\lambda(\tau+1)} \left[c_2 \|\phi\|^p e^{-\lambda(k^* - k_0)} + a \right], \\ &< q V(k^*, x(k^*)). \end{aligned}$$

Then

$$V(\bar{k} + s, x(\bar{k} + s)) \leq q V(\bar{k} + 1, x(\bar{k} + 1)),$$

and from (ii), we have

$$V(k^*, x(k^*)) \leq (1 - \beta)\, V(k^* - 1, x(k^* - 1)) + \eta,$$

which is contradiction, so (3.2) holds.
So, for all $k \geq k_0 - 1$, we have

$$V(k, x) \leq c_2 \|\phi\|^p e^{-\lambda(k - k_0)} + a,$$

and from (i), we know that

$$c_1 \|x\|^p \leq V(k, x(k)) \leq c_2 \|\phi\|^p e^{-\lambda(k - k_0)} + a.$$

Therefore, we have

$$\|x\|^p \leq \frac{c_2}{c_1} \|\phi\|^p e^{-\lambda(k - k_0)} + \frac{a}{c_1}.$$

Case II. $q > 1 > \gamma > 0$.
Since $q > 1 > \gamma > 0$, then there exists $0 < \lambda < 1, q^*$ such that

$$q > q^* e^{\lambda(\tau+1)} > q^* > e^{\lambda(\tau+1)} \geq \gamma e^{\lambda(\tau+1)}.$$

By a similar argument as case I, we have

$$\|x\|^p \leq \frac{c_2}{c_1}\|\phi\|^p e^{-\lambda(k-k_0)} + \frac{a}{c_1}.$$

Therefore, from case I and case II the trivial solution of system (2.1) is globally practically exponentially stable in the $p^{th}-$ moment. ■

Remark 3.2. From the methods of proof of Theorem 3.1, it is clear that these methods can be applied to discrete-time impulsive system with time-varying delay $\tau(k)$ with $0 \leq \tau(k) \leq \tau, \quad \tau > 0$.

4. Numerical Example

To illustrate the effectiveness of the result obtained in previous sections, we consider the discrete-time impulsive system with delay.

Example 4.1. Consider the following system:

$$\begin{cases} x(k+1) = -bx(k) + \frac{d}{1+x^2(k)}x(k-\tau) + \mu, & k \neq k_m \\ x(k_m) = \gamma x(k_m - 1), & k = k_m \\ x(s) = \phi(s), & s \in N_{-\tau} \end{cases} \tag{4.1}$$

where b, d are arbitrary constants and γ, μ are positive constants. If there are exist positive numbers $a, c_1, c_2, p, \gamma, q, \beta, \eta$ such that

$$\eta = \mu + (1 - |b| - |d|)a,$$
$$\beta \leq 1 - \frac{|b|}{1 - |d|q},$$

and

$$a > \frac{\mu}{\beta - (1 - |b| - |d|)},$$

then the system (4.1) is globally practically exponentially stable in the $p^{th}-$ moment.

Proof. we choose the Lyapunov function

$$V(k, x(k)) = |x(k)| + a,$$

then we have

(i) $\quad c_1\|x(k)\| \leq V(k, x(k)) = |x(k)| + a \leq c_2\|x(k)\| + a, \quad \forall k \geq k_0 - \tau.$

(ii) $\quad$ Assume $V(k+s, x(k+s)) < qV(k+1, x(k+1)),$

with $s \in N_{-\tau}$, then we have

$$\begin{aligned}
V(k+1,x(k+1)) &= |x(k+1)|+a,\\
&= |-bx(k)+\frac{d}{1+x^2(k)}x(k-\tau)+\mu|+a,\\
&\leq |b||x(k)|+|d||x(k-\tau)|+\mu+a,\\
&= |b||x(k)|+|b|a-|b|a+|d||x(k-\tau)|+|d|a-|d|a+\mu+a,\\
&= |b|V(k,x(k))+|d|V(k-\tau,x(k-\tau))+\mu+(1-|b|-|d|)a,\\
&\leq |b|V(k,x(k))+|d|qV(k+1,x(k+1))+\mu+(1-|b|-|d|)a,\\
&\leq \frac{|b|}{1-|d|q}V(k,x(k))+\mu+(1-|b|-|d|)a.
\end{aligned}$$

Thus, we have

$$\begin{aligned}
\Delta V(k,x(k)) &= V(k+1,x(k+1))-V(k,x(k)),\\
&\leq \frac{|b|}{1-|d|q}V(k,x(k))+\mu+(1-|b|-|d|)a-V(k,x(k)),\\
&= (\frac{|b|}{1-|d|q}-1)V(k,x(k))+\mu+(1-|b|-|d|)a,\\
&= -(1-\frac{|b|}{1-|d|q})V(k,x(k))+\mu+(1-|b|-|d|)a,\\
&= -(1-\frac{|b|}{1-|d|q})V(k,x(k))+\eta,\\
\eta &= \mu+(1-|b|-|d|)a.
\end{aligned}$$

From assumptions, we get

$$\Delta V(k,\overline{x}(k)) \leq -\beta V(k,\overline{x}(k))+\eta.$$

(iii) We have

$$\begin{aligned}
V(k_m,x(k_m)) &= |x(k_m)|+a,\\
&= |\gamma x(k_m-1)|+a,\\
&= \gamma\Big[|x(k_m-1)|+\frac{a}{\gamma}\Big],\\
&\leq \gamma\Big[|x(k_m-1)|+a\Big],\\
&= \gamma V(k_m-1,x(k_m-1)).
\end{aligned}$$

Therefore, from Theorem 3.1, we conclude that the system (4.1) is globally practically exponentially stable in the $p^{th}-$ moment. For simulation propose, we let $|b|=0.04,\ |d|=0.43,\ \tau=1,\ k_0=0,\ \mu=0.45,\ \lambda=0.05,\ q=1.4>\gamma=1.1>0.$ We can choose Lyapunov function

$$V(k,x(k)) = |x(k)|+1.25,$$

and there exist $c_1 = 1 = c_2$, $a = 1.25$, $p = 1$, $\beta = 0.9$ which satisfies Theorem 3.1 as follows:

$$
\begin{aligned}
&(i) \quad \|x(k)\| \leq V(k,x) \leq \|x(k)\| + 1.25,\\
&(ii) \quad \text{If } V(k-s, x(k-s)) < (1.4)V(k+1, x(k+1)), \quad s \in N_{-1},\\
&\qquad \text{then } \Delta V(k, x(k)) = V(k+1, x(k+1)) - V(k, x(k)),\\
&\qquad\qquad \leq (-0.9)V(k, x(k)) + 1.76,\\
&(iii) \quad V(k_m, x(k_m)) \leq (1.1)V(k_m - 1, x(k_m - 1)),
\end{aligned}
$$

and the trajectory of solution (4.1) with impulsive moments $k_m = 4k + 3$, $m \in N$ and initial values $x(-1) = 1.8$, $x(0) = 1.6$ is shown in FIGURE 1.

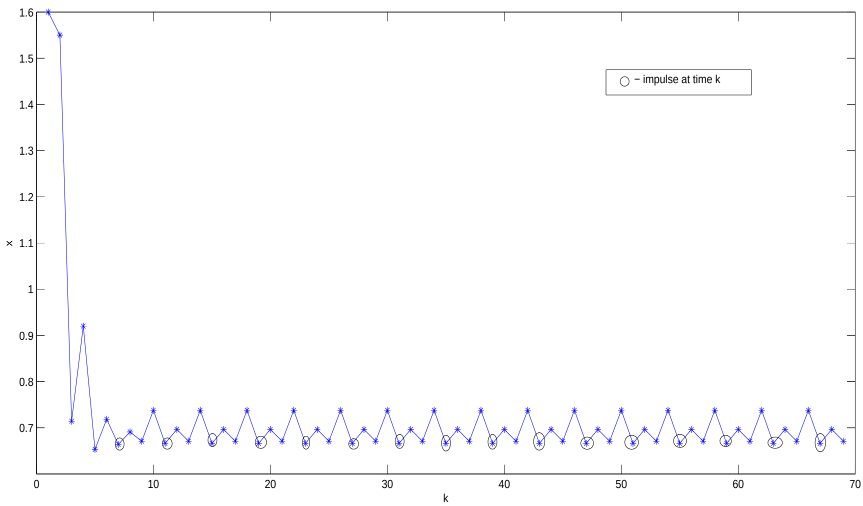

FIGURE 1. Numerical simulation of example with $k_m = 4k + 3$.

■

5. Conclusion

This paper establishes practical exponential stability results of discrete-time impulsive system with delay. For systems with delay, by using the Lyapunov stability theory and the Razumikhin type technique. Finally, a numerical example is presented to illustrate the effectiveness of the proposed results.

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References

[1] B. Liu, H. Marquez, Razumikhin-type stability theorems for discrete delay systems, Automatica (2007) 1219–1225.

[2] C. Li, S. Duan, S. Wu, X. Liao, Exponential stability of impulsive discrete systems with time delay and applications in stochastic neural networks: A Razumikhin approach, Neurocomputing 82 (2012) 29–36.

[3] G. Sun, Y. Zhang, Exponential stability of impulsive discrete-time stochastic BAM neural networks with time-varying delay, Neurocomputing (2014) 323–330.

[4] K. Wu, X. Ding, Impulsive stabilization of delay difference equation and its application in Nicholson's blowflies model, Adv. Diff. Equ. (2012) 1–11.

[5] V. Lakshmikantham, D. Bainov, P. Simeonov, Theory of Impulsive Differential Equations, World Scientific Pub. Co. Inc., 1989.

[6] M. Frigon, D. O'Regan, Impulsive differential equations with variable times, Nonlinear Analysis: Theory, Methods & Applications 26, 12 (1996) 1913–1922.

[7] B. Cui, M. Han, Oscillation theorems for nonlinear hyperbolic systems with impulses, Nonlinear Analysis: Real World Applications 9, 1 (2008) 94–102.

[8] W. Chartbupapan, O. Bagdasar, K. Mukdasai, A novel delay-dependent asymptotic stability conditions for differential and Riemann-Liouville fractional differential neutral systems with constant delays and nonlinear perturbation, Mathematics, 8 (1) 82 (2020) 1–10.

[9] W. Chartbupapan, T. Botmart, K. Mukdasai, N. Kaewbanjak, Non-differentiable delay-interval-dependent exponentially passive conditions for certain neutral integro-differential equation with time-varying delays, Thai J. Math. 18 (1) (2020) 233–251.

[10] P. Singkibud, L.T. Hiep, P. Niamsup, T. Botmart, K. Mukdasai, Delay-Dependent Robust Delay-dependent robust H_∞ performance for uncertain neutral systems with mixed time-varying delays and nonlinear perturbations, Math. Probl. Eng., 2018 (2018) 1–16.

[11] K. Zhang, X. Liu, Global exponential stability of nonlinear impulsive discrete systems with time delay, Chinese Control and Decision Conference (2013) 148–153.

[12] W. Chen, X. Lu, W. Zheng, Impulsive stabilization and impulsive synchronization of discrete-time delayed neural networks, IEEE Trans. Neural Netw. Learn. Syst. (2015) 734–748.

[13] X. Li, Z. Xiang, Observer design of discrete-time impulsive switched nonlinear systems with time-varying delays, Appl. Math. Comput. (2014) 327–339.

[14] S. Hinpang, X. Xiang, P. Sattayatham, Impulsive Periodic Control System with Parameter Perturbation, Thai J. Math. 4 (2006) 107–125.

[15] X. Li, Z. Xiang, H.R. Karimi, Asynchronously switched control of discrete impulsive switched systems with time delays, Inform. Sci. (2013) 132–142.

[16] X. Liu, Z. Zhang, Uniform asymptotic stability of impulsive discrete systems with time delay, Nonlinear Anal. 72 (2011) 4941–4950.

[17] Z. Li, J. Fang, Q. Miao, G. He, Exponential synchronization of impulsive discrete-time complex networks with time-varying delay, Neurocomputing (2017) 335–343.

[18] Z. Li, J. Fang, T. Huang, Q. Miao, H. Wang, Impulsive synchronization of discrete-time networked oscillators with partial input saturation, Inf. Sci. (2018) 531–541.

[19] B. Ghanmi, M.A. Hammami, N. Hadj Taied, Growth conditions for exponential stability of time-varying perturbed systems, Int. J. Control 89 (6) (2013).

[20] Y. Zhang, Exponential stability of impulsive discrete systems with time delay, App. Math. Lett. 25 (2012) 2290–2297.

[21] M. Dlala, M.A. Hammami, Uniform exponential practical stability of impulsive perturbed systems, J. Dym. & Control Syst. (2007) 373–386.

[22] B.B. Hamed, I. Ellouze, M.A. Hammami, Practical uniform stability of nonlinear differential delay equations, Mediterr. J. Math. 8 (2011) 603–616.

[23] R. Villafuerte, B. Saldivar, S. Mondi, Practical stability and stabilization of a class of nonlinear neutral type time delay systems with multiple delays: BMIs approaches, Internat. J. Control, Autom.Syst. 11 (5) (2013) 859–867.

[24] A. Benabdallah, I. Ellouze, M.A. Hammami, Practical stability of nonlinear time-varying cascade systems, Dym. & Control Syst. (2009) 45–62.

[25] L. Sun, C. Liu, X. Li, Practical stability of impulsive discrete systems with time delays, Abstr. Appl. Anal. (2014) 1–10.

[26] T. Caraballo, M.A. Hammami, L. Mchiri, On the practical global uniform asymptotic stability of stochastic differential equations, Stochastics (2015) 1–12.

[27] Q. Zhang, W. Liu, Z. Su, Practical stability and controllability for a class of nonlinear discrete systems with time delay, Nonlinear Dyn. Syst. Theory 10 (2) (2010) 161–174.

[28] S. Wangrat, P. Niamsup, Exponentially practical stability of discrete time singular system with delay and disturbance, Advances in Difference Equations. 130, 2018 (2018) 1–23.