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Dedicated to Prof. Suthep Suantai on the occasion of his 60^{th} anniversary

Neutrosophic Cubic Set Theory Applied to UP-Algebras

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Abstract Neutrosophic cubic sets which are the generalized form of fuzzy sets, cubic sets and neutrosophic sets and introduced by Jun et al. [Y.B. Jun, F. Smarandache, C.S. Kim, Neutrosophic cubic sets, New Math. Nat. Comput. 13 (1) (2017) 41–54]. In this paper, we applied the concept of neutrosophic cubic sets to UP-algebras and we introduce the concepts of neutrosophic cubic UP-subalgebras, neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, and neutrosophic cubic strong UP-ideals of UP-algebras. Moreover, we discuss the relations between neutrosophic cubic UP-subalgebras (resp., neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals and neutrosophic cubic strong UP-ideals) and their level subsets by means of interval-valued neutrosophic sets and neutrosophic sets.

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Keywords: UP-algebra; neutrosophic cubic UP-subalgebra; neutrosophic cubic near UP-filter; neutrosophic cubic UP-filter; neutrosophic cubic UP-ideal; neutrosophic cubic strong UP-ideal

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1. Introduction

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [1], BCI-algebras [2], BCH-algebras [3], B-algebras [4], KU-algebras [5], SU-algebras [6], UP-algebras [7] and others. They are strongly connected with logic. For example, BCI-algebras introduced by Iséki [2] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki [1, 2] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. The above-mentioned section has been derived from [8].

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The type of the logical algebra, a UP-algebra was introduced by Iampan [7], and it is known that the class of KU-algebras is a proper subclass of the class of UP-algebras. Later Somjanta et al. [9] studied fuzzy UP-subalgebras, fuzzy UP-ideals and fuzzy UP-filters of UP-algebras. Guntasow et al. [10] studied fuzzy translations of a fuzzy set in UP-algebras. Kesorn et al. [11] studied intuitionistic fuzzy sets in UP-algebras. Kaijae et al. [12] studied anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras of UP-algebras. Tanamoon et al. [13] studied Q-fuzzy sets in UP-algebras. Sripaeng et al. [14] studied anti Q-fuzzy UP-ideals and anti Q-fuzzy UP-subalgebras of UP-algebras. Dokkhamdang et al. [15] studied generalized fuzzy sets in UP-algebras. Songsaeng and Iampan [16, 17] studied $\mathcal{N}$-fuzzy UP-algebras and fuzzy proper UP-filters of UP-algebras. Senapati et al. [18, 19] studies cubic sets and interval-valued intuitionistic fuzzy structures in UP-algebras.

A fuzzy set f in a nonempty set S is a function from S to the closed interval $[0, 1]$. The concept of a fuzzy set in a nonempty set was first considered by Zadeh [20]. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere. Zadeh [21] introduced interval-value fuzzy sets. An interval-valued fuzzy set is defined by an interval-valued membership function. The concept of neutrosophic sets was introduced by Smarandache [22] in 1999. Wang et al. [23] introduced the concept of interval-valued neutrosophic sets in 2005. The interval-valued neutrosophic set is an instance of neutrosophic set which can be used in real scientific and engineering applications. Jun et al. [24] introduced the notion of interval-valued neutrosophic sets with applications in BCK/BCI-algebra, they also introduced the notion of interval-valued neutrosophic length of an interval-valued neutrosophic set, and investigate their properties and relations. In 2018-2019, Muhiuddin et al. [25–30] applied the notion of neutrosophic sets to semigroups, BCK/BCI-algebras. The concept of neutrosophic $\mathcal{N}$-structures and their applications in semigroups was introduced by Khan et al. [31] in 2017. Jun et al. [32] applied the concept of neutrosophic $\mathcal{N}$-structures to BCK/BCI-algebras in 2017. Songsaeng and Iampan [33] applied the concept of neutrosophic set to UP-algebras in 2019.

A cubic set in a nonempty set is a structure using an interval-value fuzzy set and a fuzzy set was introduced by Jun et al. [34] in 2012. People find that cubic sets have board applications in computer science and soft engineering. Jun et al. [35] applied the concept of cubic sets to a subgroup in 2011. Senapati [36] introduced the concept of cubic subalgebras and cubic closed ideals of B-algebras in 2015. Senapati et al. [18] introduced the concept of cubic set structure applied in UP-algebras in 2018.

A neutrosophic cubic set which is the generalized form of fuzzy sets, cubic sets and neutrosophic sets and introduced by Jun et al. [37] in 2017. The concept of truth-internals (indeterminacy-internals, falsity-internals) and truth-externals (indeterminacy-externals, falsity-externals) were introduced and related properties were investigated. Iqbal et al. [38] introduced the concept of neutrosophic cubic subalgebras and neutrosophic cubic closed ideals of B-algebras in 2016. Relation among neutrosophic cubic algebra with neutrosophic cubic ideals and neutrosophic closed ideals of B-algebras were studied and some related properties were investigated.

From literature review, we applied the concept of neutrosophic cubic sets to UP-algebras and we introduce the concepts of neutrosophic cubic UP-subalgebras, neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, and neutrosophic cubic strong UP-ideals of UP-algebras. Moreover, we discuss the relations between neutrosophic cubic UP-subalgebras (resp., neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals and neutrosophic cubic strong UP-ideals) and their level subsets by means of interval-valued neutrosophic sets and neutrosophic sets.

2. Basic Concepts and Preliminary Notes on UP-Algebras

Before we begin our study, we will give the definition and useful properties of UP-algebras.

Definition 2.1 ([7])**.** An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra*, where X is a nonempty set, $\cdot$ is a binary operation on X, and 0 is a fixed element of X (i.e., a nullary operation) if it satisfies the following axioms:

(UP-1): $(\forall x, y, z \in X)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,
(UP-2): $(\forall x \in X)(0 \cdot x = x)$,
(UP-3): $(\forall x \in X)(x \cdot 0 = 0)$, and
(UP-4): $(\forall x, y \in X)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y)$.

From [7], we know that the concept of UP-algebras is a generalization of KU-algebras (see [5]).

Example 2.2 ([39])**.** Let X be a universal set and let $\Omega \in \mathcal{P}(X)$, where $\mathcal{P}(X)$ means the power set of X. Let $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. Define a binary operation $\cdot$ on $\mathcal{P}_\Omega(X)$ by putting $A \cdot B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(X)$, where A^C means the complement of a subset A. Then $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 1 with respect to* Ω. Let $\mathcal{P}^\Omega(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$. Define a binary operation $*$ on $\mathcal{P}^\Omega(X)$ by putting $A * B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(X)$. Then $(\mathcal{P}^\Omega(X), *, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 2 with respect to* Ω. In particular, $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra and we shall call it the *power UP-algebra of type 1*, and $(\mathcal{P}(X), *, X)$ is a UP-algebra and we shall call it the *power UP-algebra of type 2*.

Example 2.3 ([15])**.** Let $\mathbb{N}_0$ be the set of all natural numbers with zero. Define two binary operations $\circ$ and $\bullet$ on $\mathbb{N}_0$ by

$$(\forall x, y \in \mathbb{N}_0)\left(x \circ y = \begin{cases} y & \text{if } x < y, \\ 0 & \text{otherwise} \end{cases}\right)$$

and

$$(\forall x, y \in \mathbb{N}_0)\left(x \bullet y = \begin{cases} y & \text{if } x > y \text{ or } x = 0, \\ 0 & \text{otherwise} \end{cases}\right).$$

Then $(\mathbb{N}_0, \circ, 0)$ and $(\mathbb{N}_0, \bullet, 0)$ are UP-algebras.

For more examples of UP-algebras, see [18, 19, 39–44].

In a UP-algebra $X = (X, \cdot, 0)$, the following assertions are valid (see [7, 42]).

$$(\forall x \in X)(x \cdot x = 0), \tag{2.1}$$

$$(\forall x, y, z \in X)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0), \tag{2.2}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0), \tag{2.3}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0), \tag{2.4}$$

$$(\forall x, y \in X)(x \cdot (y \cdot x) = 0), \tag{2.5}$$

$$(\forall x, y \in X)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x), \tag{2.6}$$

$$(\forall x, y \in X)(x \cdot (y \cdot y) = 0), \tag{2.7}$$

$$(\forall a, x, y, z \in X)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0), \tag{2.8}$$

$$(\forall a, x, y, z \in X)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0), \tag{2.9}$$

$$(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0), \tag{2.10}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0), \tag{2.11}$$

$$(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0), \text{ and} \tag{2.12}$$

$$(\forall a, x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0). \tag{2.13}$$

From [7], the binary relation $\leq$ on a UP-algebra $X = (X, \cdot, 0)$ defined as follows:

$$(\forall x, y \in X)(x \leq y \Leftrightarrow x \cdot y = 0).$$

In UP-algebras, 5 types of special subsets are defined as follows.

Definition 2.4 ([7, 9, 10, 45])**.** A nonempty subset S of a UP-algebra $X = (X, \cdot, 0)$ is called

(1) a *UP-subalgebra* of X if $(\forall x, y \in S)(x \cdot y \in S)$.
(2) a *near UP-filter* of X if
  (i) the constant 0 of X is in S, and
  (ii) $(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S)$.
(3) a *UP-filter* of X if
  (i) the constant 0 of X is in S, and
  (ii) $(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S)$.
(4) a *UP-ideal* of X if
  (i) the constant 0 of X is in S, and
  (ii) $(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$.
(5) a *strong UP-ideal* (renamed from a strongly UP-ideal) of X if
  (i) the constant 0 of X is in S, and
  (ii) $(\forall x, y, z \in X)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S)$.

Guntasow et al. [10] and Iampan [45] proved that the concept of UP-subalgebras is a generalization of near UP-filters, near UP-filters is a generalization of UP-filters, UP-filters is a generalization of UP-ideals, and UP-ideals is a generalization of strong UP-ideals. Furthermore, they proved that the only strong UP-ideal of a UP-algebra X is X.

In 1965, the concept of a fuzzy set in a nonempty set was first considered by Zadeh [20] as the following definition.

Definition 2.5. A *fuzzy set* (briefly, FS) in a nonempty set X (or a fuzzy subset of X) is defined to be a function $\lambda : X \to [0, 1]$, where $[0, 1]$ is the unit segment of the real line. Denote by $[0,1]^X$ the collection of all fuzzy sets in X. Define a binary relation $\leq$ on $[0,1]^X$ as follows:

$$(\forall \lambda, \mu \in [0,1]^X)(\lambda \leq \mu \Leftrightarrow (\forall x \in X)(\lambda(x) \leq \mu(x))). \quad (2.14)$$

Definition 2.6 ([9]). Let λ be a fuzzy set in a nonempty set X. The *complement of* λ, denoted by λ^C, is defined by

$$(\forall x \in X)(\lambda^C(x) = 1 - \lambda(x)). \quad (2.15)$$

Definition 2.7 ([46]). Let $\{\lambda_i \mid i \in J\}$ be a family of fuzzy sets in a nonempty set X. We define the *join* and the *meet* of $\{\lambda_i \mid i \in J\}$, denoted by $\vee_{i\in J}\lambda_i$ and $\wedge_{i\in J}\lambda_i$, respectively, as follows:

$$(\forall x \in X)((\vee_{i\in J}\lambda_i)(x) = \sup_{i\in J}\{\lambda_i(x)\}), \quad (2.16)$$

$$(\forall x \in X)((\wedge_{i\in J}\lambda_i)(x) = \inf_{i\in J}\{\lambda_i(x)\}). \quad (2.17)$$

In particular, if λ and μ be fuzzy sets in X, we have the join and meet of λ and μ as follows:

$$(\forall x \in X)((\lambda \vee \mu)(x) = \max\{\lambda(x), \mu(x)\}), \quad (2.18)$$

$$(\forall x \in X)((\lambda \wedge \mu)(x) = \min\{\lambda(x), \mu(x)\}), \quad (2.19)$$

respectively.

An *interval number* we mean a close subinterval $\tilde{a} = [a^-, a^+]$ of $[0,1]$, where $0 \leq a^- \leq a^+ \leq 1$. The interval number $\tilde{a} = [a^-, a^+]$ with $a^- = a^+$ is denoted by $\mathbf{a}$. Denote by $[[0,1]]$ the set of all interval numbers.

Definition 2.8 ([37]). Let $\{\tilde{a}_i \mid i \in J\}$ be a family of interval numbers. We define the *refined infimum* and the *refined supremum* of $\{\tilde{a}_i \mid i \in J\}$, denoted by $\mathrm{rinf}_{i\in J}\tilde{a}_i$ and $\mathrm{rsup}_{i\in J}\tilde{a}_i$, respectively, as follows:

$$\mathrm{rinf}_{i\in J}\{\tilde{a}_i\} = [\inf_{i\in J}\{a_i^-\}, \inf_{i\in J}\{a_i^+\}], \quad (2.20)$$

$$\mathrm{rsup}_{i\in J}\{\tilde{a}_i\} = [\sup_{i\in J}\{a_i^-\}, \sup_{i\in J}\{a_i^+\}]. \quad (2.21)$$

In particular, if $\tilde{a}_1$ and $\tilde{a}_2$ are interval numbers, we define the *refined minimum* and the *refined maximum* of $\tilde{a}_1$ and $\tilde{a}_2$, denoted by $\mathrm{rmin}\{\tilde{a}_1, \tilde{a}_2\}$ and $\mathrm{rmax}\{\tilde{a}_1, \tilde{a}_2\}$, respectively, as follows:

$$\mathrm{rmin}\{\tilde{a}_1, \tilde{a}_2\} = [\min\{a_1^-, a_2^-\}, \min\{a_1^+, a_2^+\}], \quad (2.22)$$

$$\mathrm{rmax}\{\tilde{a}_1, \tilde{a}_2\} = [\max\{a_1^-, a_2^-\}, \max\{a_1^+, a_2^+\}]. \quad (2.23)$$

Definition 2.9 ([37]). Let $\tilde{a}_1$ and $\tilde{a}_2$ be interval numbers. We define the symbols "$\succeq$", "$\preceq$", "$=$" in case of $\tilde{a}_1$ and $\tilde{a}_2$ as follows:

$$\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow a_1^- \geq a_2^- \text{ and } a_1^+ \geq a_2^+, \quad (2.24)$$

and similarly we may have $\tilde{a}_1 \preceq \tilde{a}_2$ and $\tilde{a}_1 = \tilde{a}_2$. To say $\tilde{a}_1 \succ \tilde{a}_2$ (resp., $\tilde{a}_1 \prec \tilde{a}_2$) we mean $\tilde{a}_1 \succeq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2$ (resp., $\tilde{a}_1 \preceq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2$).

Definition 2.10 ([21]). Let $\tilde{a}$ be an interval number. The *complement of* $\tilde{a}$, denoted by $\tilde{a}^C$, is defined by the interval number

$$\tilde{a}^C = [1 - a^+, 1 - a^-]. \quad (2.25)$$

In the [[0, 1]], the following assertions are valid (see [47]).

$$(\forall \tilde{a} \in [[0,1]])((\tilde{a}^C)^C = \tilde{a}), \tag{2.26}$$

$$(\forall \tilde{a} \in [[0,1]])(\text{rmax}\{\tilde{a},\tilde{a}\} = \tilde{a} \text{ and } \text{rmin}\{\tilde{a},\tilde{a}\} = \tilde{a}), \tag{2.27}$$

$$(\forall \tilde{a}_1,\tilde{a}_2 \in [[0,1]])(\text{rmax}\{\tilde{a}_1,\tilde{a}_2\} = \text{rmax}\{\tilde{a}_2,\tilde{a}_1\} \text{ and } \text{rmin}\{\tilde{a}_1,\tilde{a}_2\} = \text{rmin}\{\tilde{a}_2,\tilde{a}_1\}), \tag{2.28}$$

$$(\forall \tilde{a}_1,\tilde{a}_2 \in [[0,1]])(\text{rmax}\{\tilde{a}_1,\tilde{a}_2\} \succeq \tilde{a}_1 \text{ and } \tilde{a}_2 \succeq \text{rmin}\{\tilde{a}_1,\tilde{a}_2\}), \tag{2.29}$$

$$(\forall \tilde{a}_1,\tilde{a}_2 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow \tilde{a}_1^C \preceq \tilde{a}_2^C), \tag{2.30}$$

$$(\forall \tilde{a}_1,\tilde{a}_2,\tilde{a}_3,\tilde{a}_4 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2, \tilde{a}_3 \succeq \tilde{a}_4 \Rightarrow \text{rmin}\{\tilde{a}_1,\tilde{a}_3\} \succeq \text{rmin}\{\tilde{a}_2,\tilde{a}_4\}), \tag{2.31}$$

$$(\forall \tilde{a}_1,\tilde{a}_2,\tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2, \tilde{a}_3 \succeq \tilde{a}_2 \Leftrightarrow \text{rmin}\{\tilde{a}_1,\tilde{a}_3\} \succeq \tilde{a}_2), \tag{2.32}$$

$$(\forall \tilde{a}_1,\tilde{a}_2,\tilde{a}_3,\tilde{a}_4 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2, \tilde{a}_3 \succeq \tilde{a}_4 \Rightarrow \text{rmax}\{\tilde{a}_1,\tilde{a}_3\} \succeq \text{rmax}\{\tilde{a}_2,\tilde{a}_4\}), \tag{2.33}$$

$$(\forall \tilde{a}_1,\tilde{a}_2,\tilde{a}_3 \in [[0,1]])(\tilde{a}_2 \succeq \tilde{a}_1, \tilde{a}_2 \succeq \tilde{a}_3 \Leftrightarrow \tilde{a}_2 \succeq \text{rmax}\{\tilde{a}_1,\tilde{a}_3\}), \tag{2.34}$$

$$(\forall \tilde{a}_1,\tilde{a}_2 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow \text{rmin}\{\tilde{a}_1,\tilde{a}_2\} = \tilde{a}_2), \tag{2.35}$$

$$(\forall \tilde{a}_1,\tilde{a}_2 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow \text{rmax}\{\tilde{a}_1,\tilde{a}_2\} = \tilde{a}_1), \tag{2.36}$$

$$(\forall \tilde{a}_1,\tilde{a}_2 \in [[0,1]])(\text{rmin}\{\tilde{a}_1^C,\tilde{a}_2^C\} = \text{rmax}\{\tilde{a}_1,\tilde{a}_2\}^C), \tag{2.37}$$

$$(\forall \tilde{a}_1,\tilde{a}_2 \in [[0,1]])(\text{rmax}\{\tilde{a}_1^C,\tilde{a}_2^C\} = \text{rmin}\{\tilde{a}_1,\tilde{a}_2\}^C), \tag{2.38}$$

$$(\forall \tilde{a}_1,\tilde{a}_2,\tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \preceq \text{rmax}\{\tilde{a}_2,\tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \succeq \text{rmin}\{\tilde{a}_2^C,\tilde{a}_3^C\}), \tag{2.39}$$

$$(\forall \tilde{a}_1,\tilde{a}_2,\tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \succeq \text{rmax}\{\tilde{a}_2,\tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \preceq \text{rmin}\{\tilde{a}_2^C,\tilde{a}_3^C\}), \tag{2.40}$$

$$(\forall \tilde{a}_1,\tilde{a}_2,\tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \preceq \text{rmin}\{\tilde{a}_2,\tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \succeq \text{rmax}\{\tilde{a}_2^C,\tilde{a}_3^C\}), \text{ and} \tag{2.41}$$

$$(\forall \tilde{a}_1,\tilde{a}_2,\tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \succeq \text{rmin}\{\tilde{a}_2,\tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \preceq \text{rmax}\{\tilde{a}_2^C,\tilde{a}_3^C\}). \tag{2.42}$$

In 1975, the concept of an interval-valued fuzzy set in a nonempty set was first introduced by Zadeh [20] as the following definition.

Definition 2.11. An *interval-valued fuzzy set* (briefly, IVFS) in a nonempty set X is an arbitrary function $A : X \to [[0,1]]$. Let $IVFS(X)$ stands for the set of all IVFS in X. For every $A \in IVFS(X)$ and $x \in X, A(x) = [A^-(x), A^+(x)]$ is called the *degree of membership* of an element x to A, where A^-, A^+ are fuzzy sets in X which are called a *lower fuzzy set* and an *upper fuzzy set* in X, respectively. For simplicity, we denote $A = [A^-, A^+]$.

Definition 2.12 ([37])**.** Let A and B be interval-valued fuzzy sets in a nonempty set X. We define the symbols "$\subseteq$", "$\supseteq$", "$=$" in case of A and B as follows:

$$A \subseteq B \Leftrightarrow (\forall x \in X)(A(x) \preceq B(x)), \tag{2.43}$$

and similarly we may have $A \supseteq B$ and $A = B$.

Definition 2.13 ([21])**.** Let A be an interval-valued fuzzy set in a nonempty set X. The *complement of* A, denoted by A^C, is defined as follows: $A^C(x) = A(x)^C$ for all $x \in X$, that is,

$$(\forall x \in X)(A^C(x) = [1 - A^+(x), 1 - A^-(x)]). \tag{2.44}$$

We note that $A^{C^-}(x) = 1 - A^+(x)$ and $A^{C^+}(x) = 1 - A^-(x)$ for all $x \in X$.

Definition 2.14 ([21])**.** Let $\{A_i \mid i \in J\}$ be a family of interval-valued fuzzy sets in a nonempty set X. We define the *intersection* and the *union* of $\{A_i \mid i \in J\}$, denoted by $\cap_{i\in J}A_i$ and $\cup_{i\in J}A_i$, respectively, as follows:

$$(\forall x \in X)((\cap_{i\in J}A_i)(x) = \text{rinf}_{i\in J}\{A_i(x)\}), \tag{2.45}$$

$$(\forall x \in X)((\cup_{i\in J}A_i)(x) = \text{rsup}_{i\in J}\{A_i(x)\}). \tag{2.46}$$

We note that

$$(\forall x \in X)((\cap_{i\in J}A_i)^-(x) = (\wedge_{i\in J}A_i^-)(x) = \inf_{i\in J}\{A_i^-(x)\})$$

and

$$(\forall x \in X)((\cap_{i\in J}A_i)^+(x) = (\wedge_{i\in J}A_i^+)(x) = \inf_{i\in J}\{A_i^+(x)\}).$$

Similarly,

$$(\forall x \in X)((\cup_{i\in J}A_i)^-(x) = (\vee_{i\in J}A_i^-)(x) = \sup_{i\in J}\{A_i^-(x)\})$$

and

$$(\forall x \in X)((\cup_{i\in J}A_i)^+(x) = (\vee_{i\in J}A_i^+)(x) = \sup_{i\in J}\{A_i^+(x)\}).$$

In particular, if A_1 and A_2 are interval-valued fuzzy sets in X, we have the intersection and the union of A_1 and A_2 as follows:

$$(\forall x \in X)((A_1 \cap A_2)(x) = \text{rmin}\{A_1(x), A_2(x)\}), \tag{2.47}$$

$$(\forall x \in X)((A_1 \cup A_2)(x) = \text{rmax}\{A_1(x), A_2(x)\}). \tag{2.48}$$

In 1999, the concept of a neutrosophic set in a nonempty set was first considered by Smarandache [22] as the following definition.

Definition 2.15. A *neutrosophic set* (briefly, NS) in a nonempty set X is a structure of the form:

$$\Lambda = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}, \tag{2.49}$$

where $\lambda_T : X \to [0,1]$ is a *truth membership function*, $\lambda_I : X \to [0,1]$ is an *indeterminate membership function*, and $\lambda_F : X \to [0,1]$ is a *false membership function*. For our convenience, we will denote a NS as $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F) = (X, \lambda_{T,I,F}) = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}$.

Definition 2.16 ([22])**.** Let Λ be a NS in a nonempty set X. The NS $\Lambda^C = (X, \lambda_T^C, \lambda_I^C, \lambda_F^C)$ in X is called the *complement* of Λ in X.

In 2019, the concepts of a special neutrosophic UP-subalgebra, a special neutrosophic near UP-filter, a special neutrosophic UP-filter, a special neutrosophic UP-ideal, and a special neutrosophic strong UP-ideal of a UP-algebra were first considered by Songsaeng and Iampan [48] as the following definition.

Definition 2.17. A NS $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F)$ in a UP-algebra $X = (X, \cdot, 0)$ is called

(1) a *special neutrosophic UP-subalgebra* of X if

$$(\forall x, y \in X)(\lambda_T(x \cdot y) \le \max\{\lambda_T(x), \lambda_T(y)\}), \tag{2.50}$$

$$(\forall x, y \in X)(\lambda_I(x \cdot y) \ge \min\{\lambda_I(x), \lambda_I(y)\}), \tag{2.51}$$

$$(\forall x, y \in X)(\lambda_F(x \cdot y) \le \max\{\lambda_F(x), \lambda_F(y)\}). \tag{2.52}$$

(2) a *special neutrosophic near UP-filter* of X if

$$(\forall x \in X)(\lambda_T(0) \leq \lambda_T(x)), \tag{2.53}$$

$$(\forall x \in X)(\lambda_I(0) \geq \lambda_I(x)), \tag{2.54}$$

$$(\forall x \in X)(\lambda_F(0) \leq \lambda_F(x)), \tag{2.55}$$

$$(\forall x, y \in X)(\lambda_T(x \cdot y) \leq \lambda_T(y)), \tag{2.56}$$

$$(\forall x, y \in X)(\lambda_I(x \cdot y) \geq \lambda_I(y)), \tag{2.57}$$

$$(\forall x, y \in X)(\lambda_F(x \cdot y) \leq \lambda_F(y)). \tag{2.58}$$

(3) a *special neutrosophic UP-filter* of X if it satisfies the following conditions: (2.53), (2.54), (2.55), and

$$(\forall x, y \in X)(\lambda_T(y) \leq \max\{\lambda_T(x \cdot y), \lambda_T(x)\}), \tag{2.59}$$

$$(\forall x, y \in X)(\lambda_I(y) \geq \min\{\lambda_I(x \cdot y), \lambda_I(x)\}), \tag{2.60}$$

$$(\forall x, y \in X)(\lambda_F(y) \leq \max\{\lambda_F(x \cdot y), \lambda_F(x)\}). \tag{2.61}$$

(4) a *special neutrosophic UP-ideal* of X if it satisfies the following conditions: (2.53), (2.54), (2.55), and

$$(\forall x, y, z \in X)(\lambda_T(x \cdot z) \leq \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}), \tag{2.62}$$

$$(\forall x, y, z \in X)(\lambda_I(x \cdot z) \geq \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}), \tag{2.63}$$

$$(\forall x, y, z \in X)(\lambda_F(x \cdot z) \leq \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}). \tag{2.64}$$

(5) a *special neutrosophic strong UP-ideal* of X if it satisfies the following conditions: (2.53), (2.54), (2.55), and

$$(\forall x, y, z \in X)(\lambda_T(x) \leq \max\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\}), \tag{2.65}$$

$$(\forall x, y, z \in X)(\lambda_I(x) \geq \min\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\}), \tag{2.66}$$

$$(\forall x, y, z \in X)(\lambda_F(x) \leq \max\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\}). \tag{2.67}$$

In 2005, the concept of an interval neutrosophic set in a nonempty set was first considered by Wang et al. [23] as the following definition.

Definition 2.18. An *interval-valued neutrosophic set* (briefly, IVNS) in a nonempty set X is a structure of the form:

$$\mathbf{A} := \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}, \tag{2.68}$$

where A_T, A_I and A_F are interval-valued fuzzy sets in X, which are called an *interval truth membership function*, an *interval indeterminacy membership function* and an *interval falsity membership function*, respectively. For our convenience, we will denote a IVNS as $\mathbf{A} = (X, A_T, A_I, A_F) = (X, A_{T,I,F}) = \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}$.

Definition 2.19 ([23])**.** Let $\mathbf{A} = (X, A_T, A_I, A_F)$ be an IVNS in a nonempty set X. The IVNS $\mathbf{A}^C = (X, A_T^C, A_I^C, A_F^C)$ in X is called the *complement* of $\mathbf{A}$ in X.

In 2019, the concepts of an interval-valued neutrosophic UP-subalgebra, an interval-valued neutrosophic near UP-filter, an interval-valued neutrosophic UP-filter, an interval-valued neutrosophic UP-ideal, and an interval-valued neutrosophic strong UP-ideal of a UP-algebra were first considered by Songsaeng and Iampan [49] as the following definition.

Definition 2.20. An IVNS $\mathbf{A} = (X, A_T, A_I, A_F)$ in a UP-algebra $X = (X, \cdot, 0)$ is called

(1) an *interval-valued neutrosophic UP-subalgebra* of X if

$$(\forall x, y \in X)(A_T(x \cdot y) \succeq \text{rmin}\{A_T(x), A_T(y)\}), \tag{2.69}$$
$$(\forall x, y \in X)(A_I(x \cdot y) \preceq \text{rmax}\{A_I(x), A_I(y)\}), \tag{2.70}$$
$$(\forall x, y \in X)(A_F(x \cdot y) \succeq \text{rmin}\{A_F(x), A_F(y)\}). \tag{2.71}$$

(2) an *interval-valued neutrosophic near UP-filter* of X if

$$(\forall x \in X)(A_T(0) \succeq A_T(x)), \tag{2.72}$$
$$(\forall x \in X)(A_I(0) \preceq A_I(x)), \tag{2.73}$$
$$(\forall x \in X)(A_F(0) \succeq A_F(x)). \tag{2.74}$$
$$(\forall x, y \in X)(A_T(x \cdot y) \succeq A_T(y)), \tag{2.75}$$
$$(\forall x, y \in X)(A_I(x \cdot y) \preceq A_I(y)), \tag{2.76}$$
$$(\forall x, y \in X)(A_F(x \cdot y) \succeq A_F(y)). \tag{2.77}$$

(3) an *interval-valued neutrosophic UP-filter* of X if it holds the following conditions: (2.72), (2.73), (2.74), and

$$(\forall x, y \in X)(A_T(y) \succeq \text{rmin}\{A_T(x \cdot y), A_T(x)\}), \tag{2.78}$$
$$(\forall x, y \in X)(A_I(y) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\}), \tag{2.79}$$
$$(\forall x, y \in X)(A_F(y) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\}). \tag{2.80}$$

(4) an *interval-valued neutrosophic UP-ideal* of X if it holds the following conditions: (2.72), (2.73), (2.74), and

$$(\forall x, y, z \in X)(A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}), \tag{2.81}$$
$$(\forall x, y, z \in X)(A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}), \tag{2.82}$$
$$(\forall x, y, z \in X)(A_F(x \cdot z) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}). \tag{2.83}$$

(5) an *interval-valued neutrosophic strong UP-ideal* of X if it holds the following conditions: (2.72), (2.73), (2.74), and

$$(\forall x, y, z \in X)(A_T(x) \succeq \text{rmin}\{A_T((z \cdot y) \cdot (z \cdot x)), A_T(y)\}), \tag{2.84}$$
$$(\forall x, y, z \in X)(A_I(x) \preceq \text{rmax}\{A_I((z \cdot y) \cdot (z \cdot x)), A_I(y)\}), \tag{2.85}$$
$$(\forall x, y, z \in X)(A_F(x) \succeq \text{rmin}\{A_F((z \cdot y) \cdot (z \cdot x)), A_F(y)\}). \tag{2.86}$$

In 2012, the concept of a cubic set in a nonempty set was first considered by Jun et al. [34] as the following definition.

Definition 2.21. A *cubic set* (briefly, CS) in a nonempty set X is a structure of the form:

$$\mathbf{C} = \{(x, A(x), \lambda(x)) \mid x \in X\}, \tag{2.87}$$

where A is an interval-valued fuzzy set in X and λ is a fuzzy set in X. For our convenience, we will denote a CS as $\mathbf{C} = (X, A, \lambda) = \{(x, A(x), \lambda(x)) \mid x \in X\}$.

3. Neutrosophic Cubic Sets in UP-Algebras

In 2017, Jun et al. [37] introduced the concept of a neutrosophic cubic set in a nonempty set which extend the concept of a cubic sets to a neutrosophic set as the following definition.

Definition 3.1. A neutrosophic cubic set (briefly, NCS) in a nonempty set X is a pair $\mathscr{A} = (\mathbf{A}, \Lambda)$, where $\mathbf{A} = (X, A_T, A_I, A_F)$ is an interval-valued neutrosophic set in X and $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F)$ is a neutrosophic set in X. For simplicity, we denote $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in a nonempty set X is said to be *constant* if $A_T, A_I, A_F, \lambda_T, \lambda_I$, and λ_F are constant functions. The complement of a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ is defined to be the NCS $\mathscr{A}^C = (\mathbf{A}^C, \Lambda^C)$.

Now, we introduce the concepts of neutrosophic cubic UP-subalgebras, neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, and neutrosophic cubic strong UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

In what follows, let X denote a UP-algebra $(X, \cdot, 0)$ unless otherwise specified.

Definition 3.2. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is called a *neutrosophic cubic UP-subalgebra* of X if it holds the following conditions:

$$(\forall x, y \in X)\begin{pmatrix} A_T(x \cdot y) \succeq \text{rmin}\{A_T(x), A_T(y)\} \\ A_I(x \cdot y) \preceq \text{rmax}\{A_I(x), A_I(y)\} \\ A_F(x \cdot y) \succeq \text{rmin}\{A_F(x), A_F(y)\} \end{pmatrix} \tag{S1}$$

and

$$(\forall x, y \in X)\begin{pmatrix} \lambda_T(x \cdot y) \leq \max\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(x \cdot y) \geq \min\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(x \cdot y) \leq \max\{\lambda_F(x), \lambda_F(y)\} \end{pmatrix}. \tag{S2}$$

Proposition 3.3. *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X, then*

$$(\forall x \in X)\begin{pmatrix} A_T(0) \succeq A_T(x) \\ A_I(0) \preceq A_I(x) \\ A_F(0) \succeq A_F(x) \end{pmatrix} \tag{P1}$$

and

$$(\forall x \in X)\begin{pmatrix} \lambda_T(0) \leq \lambda_T(x) \\ \lambda_I(0) \geq \lambda_I(x) \\ \lambda_F(0) \leq \lambda_F(x) \end{pmatrix}. \tag{P2}$$

Proof. Let $\mathscr{A} = (\mathbf{A}, \Lambda)$ be a neutrosophic cubic UP-subalgebra of X. By (2.1) and (2.27), we have

$$(\forall x \in X)\begin{pmatrix} A_T(0) = A_T(x \cdot x) \succeq \text{rmin}\{A_T(x), A_T(x)\} = A_T(x) \\ A_I(0) = A_I(x \cdot x) \preceq \text{rmax}\{A_I(x), A_I(x)\} = A_I(x) \\ A_F(0) = A_F(x \cdot x) \succeq \text{rmin}\{A_F(x), A_F(x)\} = A_F(x) \\ \lambda_T(0) = \lambda_T(x \cdot x) \leq \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x) \\ \lambda_I(0) = \lambda_I(x \cdot x) \geq \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x) \\ \lambda_F(0) = \lambda_F(x \cdot x) \leq \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x) \end{pmatrix}.$$

■

Example 3.4. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	3	4
2	0	0	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

We define a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0	$([1,1],[0,0.3],[0.7,1])$	$(0,1,0)$
1	$([0.6,0.7],[0.4,0.5],[0.4,0.5])$	$(0.3,0.2,0.4)$
2	$([0.4,0.8],[0.1,0.4],[0.5,0.7])$	$(0.5,0.6,0.2)$
3	$([0.3,0.4],[0.8,0.9],[0.2,0.3])$	$(0.7,0.8,0.7)$
4	$([0.7,0.8],[0.2,0.4],[0.6,0.7])$	$(0.5,0.4,0.8)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X.

Definition 3.5. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is called a *neutrosophic cubic near UP-filter* of X if it holds the following conditions: (P1), (P2), and

$$(\forall x, y \in X)\begin{pmatrix} A_T(x \cdot y) \succeq A_T(y) \\ A_I(x \cdot y) \preceq A_I(y) \\ A_F(x \cdot y) \succeq A_F(y) \end{pmatrix} \tag{N1}$$

and

$$(\forall x, y \in X)\begin{pmatrix} \lambda_T(x \cdot y) \leq \lambda_T(y) \\ \lambda_I(x \cdot y) \geq \lambda_I(y) \\ \lambda_F(x \cdot y) \leq \lambda_F(y) \end{pmatrix}. \tag{N2}$$

Example 3.6. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	2	4
2	0	0	0	1	4
3	0	0	0	0	4
4	0	1	2	3	0

We define a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0	$([0.9,1],[0,0.1],[1,1])$	$(0,0.9,0.1)$
1	$([0.6,0.8],[0.1,0.3],[0.6,0.8])$	$(0.3,0.8,0.2)$
2	$([0.5,0.6],[0.3,0.4],[0.5,0.7])$	$(0.5,0.7,0.6)$
3	$([0.4,0.6],[0.5,0.6],[0.4,0.6])$	$(0.6,0.3,0.7)$
4	$([0.1,0.7],[0.8,0.9],[0.1,0.3])$	$(0.2,0.4,0.5)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X.

Definition 3.7. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is called a *neutrosophic cubic UP-filter* of X if it holds the following conditions: (P1), (P2), and

$$(\forall x, y \in X)\begin{pmatrix} A_T(y) \succeq \text{rmin}\{A_T(x \cdot y), A_T(x)\} \\ A_I(y) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\} \\ A_F(y) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\} \end{pmatrix} \tag{F1}$$

and

$$(\forall x, y \in X)\begin{pmatrix} \lambda_T(y) \leq \max\{\lambda_T(x \cdot y), \lambda_T(x)\} \\ \lambda_I(y) \geq \min\{\lambda_I(x \cdot y), \lambda_I(x)\} \\ \lambda_F(y) \leq \max\{\lambda_F(x \cdot y), \lambda_F(x)\} \end{pmatrix}. \tag{F2}$$

Example 3.8. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	3	3
3	0	1	2	0	3
4	0	1	2	0	0

We define a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0	$([0.9, 1], [0, 0.1], [0.8, 0.9])$	$(0, 1, 0.1)$
1	$([0.5, 0.8], [0.2, 0.3], [0.6, 0.7])$	$(0.2, 0.7, 0.2)$
2	$([0.3, 0.7], [0.4, 0.5], [0.5, 0.6])$	$(0.5, 0.5, 0.9)$
3	$([0.1, 0.4], [0.7, 0.9], [0.2, 0.4])$	$(0.7, 0.4, 0.3)$
4	$([0.1, 0.4], [0.7, 0.9], [0.2, 0.4])$	$(0.7, 0.4, 0.3)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X.

Definition 3.9. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is called a *neutrosophic cubic UP-ideal* of X if it holds the following conditions: (P1), (P2), and

$$(\forall x, y, z \in X)\begin{pmatrix} A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\} \\ A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\} \\ A_F(x \cdot z) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\} \end{pmatrix} \tag{I1}$$

and

$$(\forall x, y, z \in X)\begin{pmatrix} \lambda_T(x \cdot z) \leq \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\} \\ \lambda_I(x \cdot z) \geq \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\} \\ \lambda_F(x \cdot z) \leq \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\} \end{pmatrix}. \tag{I2}$$

Example 3.10. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	0	4
3	0	0	2	0	4
4	0	0	0	0	0

We define a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0	$([0.9, 1], [0.1, 0.3], [0.8, 0.9])$	$(0, 1, 0)$
1	$([0.7, 0.9], [0.3, 0.5], [0.5, 0.9])$	$(0.3, 0.6, 0.2)$
2	$([0.6, 0.8], [0.4, 0.7], [0.4, 0.6])$	$(0.5, 0.5, 0.7)$
3	$([0.6, 0.9], [0.3, 0.6], [0.5, 0.8])$	$(0.4, 0.6, 0.4)$
4	$([0.3, 0.5], [0.5, 0.9], [0.4, 0.5])$	$(0.6, 0.2, 0.9)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X.

Definition 3.11. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is called a *neutrosophic cubic strong UP-ideal* of X if it holds the following conditions: (P1), (P2), and

$$(\forall x, y, z \in X)\begin{pmatrix} A_T(x) \succeq \text{rmin}\{A_T((z \cdot y) \cdot (z \cdot x)), A_T(y)\} \\ A_I(x) \preceq \text{rmax}\{A_I((z \cdot y) \cdot (z \cdot x)), A_I(y)\} \\ A_F(x) \succeq \text{rmin}\{A_F((z \cdot y) \cdot (z \cdot x)), A_F(y)\} \end{pmatrix} \tag{B1}$$

and

$$(\forall x, y, z \in X)\begin{pmatrix} \lambda_T(x) \leq \max\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\} \\ \lambda_I(x) \geq \min\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\} \\ \lambda_F(x) \leq \max\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\} \end{pmatrix}. \tag{B2}$$

Example 3.12. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	4
3	0	1	0	0	4
4	0	1	0	3	0

We define a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0	$([0.5, 0.7], [0.3, 0.9], [0.4, 0.5])$	$(0.5, 0.4, 0.7)$
1	$([0.5, 0.7], [0.3, 0.9], [0.4, 0.5])$	$(0.5, 0.4, 0.7)$
2	$([0.5, 0.7], [0.3, 0.9], [0.4, 0.5])$	$(0.5, 0.4, 0.7)$
3	$([0.5, 0.7], [0.3, 0.9], [0.4, 0.5])$	$(0.5, 0.4, 0.7)$
4	$([0.5, 0.7], [0.3, 0.9], [0.4, 0.5])$	$(0.5, 0.4, 0.7)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X.

Theorem 3.13. *A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of X if and only if the IVNS $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra (resp., interval-valued neutrosophic near UP-filter, interval-valued neutrosophic UP-filter, interval-valued neutrosophic UP-ideal, interval-valued neutrosophic strong UP-ideal) of X and the NS Λ is a special neutrosophic UP-subalgebra (resp., special neutrosophic near UP-filter, special neutrosophic UP-filter, special neutrosophic UP-ideal, special neutrosophic strong UP-ideal) of X.*

Proof. It is straightforward by Definitions 2.17 and 2.20. ∎

Theorem 3.14. *A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is constant if and only if it is a neutrosophic cubic strong UP-ideal of X.*

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a constant neutrosophic cubic set in X. Then $A_T(x) = A_T(0), A_I(x) = A_I(0), A_F(x) = A_F(0), \lambda_T(x) = \lambda_T(0), \lambda_I(x) = \lambda_I(0)$, and $\lambda_F(x) = \lambda_F(0)$ for all $x \in X$. Then for all $x \in X, A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x), A_F(0) \succeq A_F(x), \lambda_T(0) \leq \lambda_T(x), \lambda_I(0) \geq \lambda_I(x)$, and $\lambda_F(0) \leq \lambda_F(x)$, and for all $x, y, z \in X$,

$$\begin{aligned}
\operatorname{rmin}\{A_T((z\cdot y)\cdot(z\cdot x)), A_T(y)\} &= \operatorname{rmin}\{A_T(0), A_T(0)\} \\
&= A_T(0) && ((2.27)) \\
&= A_T(x), \\
\operatorname{rmax}\{A_I((z\cdot y)\cdot(z\cdot x)), A_I(y)\} &= \operatorname{rmax}\{A_I(0), A_I(0)\} \\
&= A_I(0) && ((2.27)) \\
&= A_I(x), \\
\operatorname{rmin}\{A_F((z\cdot y)\cdot(z\cdot x)), A_F(y)\} &= \operatorname{rmin}\{A_F(0), A_F(0)\} \\
&= A_F(0) && ((2.27)) \\
&= A_F(x), \\
\max\{\lambda_T((z\cdot y)\cdot(z\cdot x)), \lambda_T(y)\} &= \max\{\lambda_T(0), \lambda_T(0)\} \\
&= \lambda_T(0) \\
&= \lambda_T(x), \\
\min\{\lambda_I((z\cdot y)\cdot(z\cdot x)), \lambda_I(y)\} &= \min\{\lambda_I(0), \lambda_I(0)\} \\
&= \lambda_I(0) \\
&= \lambda_I(x), \\
\max\{\lambda_F((z\cdot y)\cdot(z\cdot x)), \lambda_F(y)\} &= \max\{\lambda_F(0), \lambda_F(0)\} \\
&= \lambda_F(0) \\
&= \lambda_F(x).
\end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X.

Conversely, assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X. Then for all $x \in X$,

$$\begin{aligned}
A_T(x) &\succeq \operatorname{rmin}\{A_T((x\cdot 0)\cdot(x\cdot x)), A_T(0)\} \\
&= \operatorname{rmin}\{A_T(0\cdot(x\cdot x)), A_T(0)\} && ((\text{UP-3})) \\
&= \operatorname{rmin}\{A_T(x\cdot x), A_T(0)\} && ((\text{UP-2})) \\
&= \operatorname{rmin}\{A_T(0), A_T(0)\} && ((2.1)) \\
&= A_T(0) && ((2.27)) \\
&\succeq A_T(x),
\end{aligned}$$

$$\begin{aligned}
A_I(x) &\preceq \text{rmax}\{A_I((x\cdot 0)\cdot(x\cdot x)), A_I(0)\} \\
&= \text{rmax}\{A_I(0\cdot(x\cdot x)), A_I(0)\} && ((\text{UP-3})) \\
&= \text{rmax}\{A_I(x\cdot x), A_I(0)\} && ((\text{UP-2})) \\
&= \text{rmax}\{A_I(0), A_I(0)\} && ((2.1)) \\
&= A_I(0) && ((2.27)) \\
&\preceq A_I(x), \\
A_F(x) &\succeq \text{rmin}\{A_F((x\cdot 0)\cdot(x\cdot x)), A_F(0)\} \\
&= \text{rmin}\{A_F(0\cdot(x\cdot x)), A_F(0)\} && ((\text{UP-3})) \\
&= \text{rmin}\{A_F(x\cdot x), A_F(0)\} && ((\text{UP-2})) \\
&= \text{rmin}\{A_F(0), A_F(0)\} && ((2.1)) \\
&= A_F(0) && ((2.27)) \\
&\succeq A_F(x), \\
\lambda_T(x) &\leq \max\{\lambda_T((x\cdot 0)\cdot(x\cdot x)), \lambda_T(0)\} \\
&= \max\{\lambda_T(0\cdot(x\cdot x)), \lambda_T(0)\} && ((\text{UP-3})) \\
&= \max\{\lambda_T(x\cdot x), \lambda_T(0)\} && ((\text{UP-2})) \\
&= \max\{\lambda_T(0), \lambda_T(0)\} && ((2.1)) \\
&= \lambda_T(0) \\
&\leq \lambda_T(x), \\
\lambda_I(x) &\geq \min\{\lambda_I((x\cdot 0)\cdot(x\cdot x)), \lambda_I(0)\} \\
&= \min\{\lambda_I(0\cdot(x\cdot x)), \lambda_I(0)\} && ((\text{UP-3})) \\
&= \min\{\lambda_I(x\cdot x), \lambda_I(0)\} && ((\text{UP-2})) \\
&= \min\{\lambda_I(0), \lambda_I(0)\} && ((2.1)) \\
&= \lambda_I(0) \\
&\geq \lambda_I(x), \\
\lambda_F(x) &\leq \max\{\lambda_F((x\cdot 0)\cdot(x\cdot x)), \lambda_F(0)\} \\
&= \max\{\lambda_F(0\cdot(x\cdot x)), \lambda_F(0)\} && ((\text{UP-3})) \\
&= \max\{\lambda_F(x\cdot x), \lambda_F(0)\} && ((\text{UP-2})) \\
&= \max\{\lambda_F(0), \lambda_F(0)\} && ((2.1)) \\
&= \lambda_F(0) \\
&\leq \lambda_F(x).
\end{aligned}$$

Thus $A_T(0) = A_T(x), A_I(0) = A_I(x), A_F(0) = A_F(x), \lambda_T(0) = \lambda_T(x), \lambda_I(0) = \lambda_I(x)$, and $\lambda_F(0) = \lambda_F(x)$ for all $x \in X$. Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is constant. ■

Theorem 3.15. *Every neutrosophic cubic strong UP-ideal of X is a neutrosophic cubic UP-ideal.*

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x), A_F(0) \succeq A_F(x), \lambda_T(0) \leq \lambda_T(x), \lambda_I(0) \geq \lambda_I(x)$, and $\lambda_F(0) \leq \lambda_F(x)$. Let $x, y, z \in X$. Then

$$A_T(x \cdot z) = A_T(y) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\},$$
$$A_I(x \cdot z) = A_I(y) \preceq \text{rmax}\{A_T(x \cdot (y \cdot z)), A_T(y)\},$$
$$A_F(x \cdot z) = A_F(y) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\},$$
$$\lambda_T(x \cdot z) = A_T(y) \leq \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\},$$
$$\lambda_I(x \cdot z) = A_I(y) \geq \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\},$$
$$\lambda_F(x \cdot z) = A_F(y) \leq \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}.$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X. ■

The following example show that the converse of Theorem 3.15 is not true.

Example 3.16. From Example 3.10, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X. Since $\lambda_F(3) = 0.6 > 0.3 = \max\{\lambda_F((2 \cdot 0) \cdot (2 \cdot 3)), \lambda_F(0)\}$, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is not a neutrosophic cubic strong UP-ideal of X.

Theorem 3.17. *Every neutrosophic cubic UP-ideal of X is a neutrosophic cubic UP-filter.*

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x), A_F(0) \succeq A_F(x), \lambda_T(0) \leq \lambda_T(x), \lambda_I(0) \geq \lambda_I(x)$, and $\lambda_F(0) \leq \lambda_F(x)$. Let $x, y \in X$. Then

$$\begin{aligned}
A_T(y) &= A_T(0 \cdot y) && \text{((UP-2))}\\
&\succeq \text{rmin}\{A_T(0 \cdot (x \cdot y)), A_T(x)\}\\
&= \text{rmin}\{A_T(x \cdot y), A_T(x)\}, && \text{((UP-2))}\\
A_I(y) &= A_I(0 \cdot y) && \text{((UP-2))}\\
&\preceq \text{rmax}\{A_I(0 \cdot (x \cdot y)), A_I(x)\}\\
&= \text{rmax}\{A_I(x \cdot y), A_I(x)\}, && \text{((UP-2))}\\
A_F(y) &= A_F(0 \cdot y) && \text{((UP-2))}\\
&\succeq \text{rmin}\{A_F(0 \cdot (x \cdot y)), A_F(x)\}\\
&= \text{rmin}\{A_F(x \cdot y), A_F(x)\}, && \text{((UP-2))}\\
\lambda_T(y) &= \lambda_T(0 \cdot y) && \text{((UP-2))}\\
&\leq \max\{\lambda_T(0 \cdot (x \cdot y)), \lambda_T(x)\}\\
&= \max\{\lambda_T(x \cdot y), \lambda_T(x)\}, && \text{((UP-2))}\\
\lambda_I(y) &= \lambda_I(0 \cdot y) && \text{((UP-2))}\\
&\geq \min\{\lambda_I(0 \cdot (x \cdot y)), \lambda_I(x)\}\\
&= \max\{\lambda_I(x \cdot y), \lambda_I(x)\}, && \text{((UP-2))}\\
\lambda_F(y) &= \lambda_F(0 \cdot y) && \text{((UP-2))}\\
&\leq \max\{\lambda_F(0 \cdot (x \cdot y)), \lambda_F(x)\}\\
&= \max\{\lambda_F(x \cdot y), \lambda_F(x)\}. && \text{((UP-2))}
\end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X. ■

The following example show that the converse of Theorem 3.17 is not true.

Example 3.18. From Example 3.8, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X. Since $A_F(3 \cdot 4) = [0.2, 0.4] \not\succeq [0.5, 0.6] = \text{rmin}\{A_F(3 \cdot (2 \cdot 4)), A_F(2)\}$, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is not a neutrosophic cubic UP-ideal of X.

Theorem 3.19. *Every neutrosophic cubic UP-filter of X is a neutrosophic cubic near UP-filter.*

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x), A_F(0) \succeq A_F(x), \lambda_T(0) \leq \lambda_T(x), \lambda_I(0) \geq \lambda_I(x)$, and $\lambda_F(0) \leq \lambda_F(x)$. Let $x, y \in X$. Then

$$\begin{aligned}
A_T(x \cdot y) &\succeq \text{rmin}\{A_T(y \cdot (x \cdot y)), A_T(y)\} \\
&= \text{rmin}\{A_T(0), A_T(y)\} && \text{((2.5))} \\
&= A_T(y), \\
A_I(x \cdot y) &\preceq \text{rmax}\{A_I(y \cdot (x \cdot y)), A_I(y)\} \\
&= \text{rmax}\{A_I(0), A_I(y)\} && \text{((2.5))} \\
&= A_I(y), \\
A_F(x \cdot y) &\succeq \text{rmin}\{A_F(y \cdot (x \cdot y)), A_F(y)\} \\
&= \text{rmin}\{A_F(0), A_F(y)\} && \text{((2.5))} \\
&= A_F(y), \\
\lambda_T(x \cdot y) &\leq \max\{\lambda_T(y \cdot (x \cdot y)), \lambda_T(y)\} \\
&= \max\{\lambda_T(0), \lambda_T(y)\} && \text{((2.5))} \\
&= \lambda_T(y), \\
\lambda_I(x \cdot y) &\geq \min\{\lambda_I(y \cdot (x \cdot y)), \lambda_I(y)\} \\
&= \min\{\lambda_I(0), \lambda_I(y)\} && \text{((2.5))} \\
&= \lambda_I(y), \\
\lambda_F(x \cdot y) &\leq \max\{\lambda_F(y \cdot (x \cdot y)), \lambda_F(y)\} \\
&= \max\{\lambda_F(0), \lambda_F(y)\} && \text{((2.5))} \\
&= \lambda_F(y).
\end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X. ■

The following example show that the converse of Theorem 3.19 is not true.

Example 3.20. From Example 3.6, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X. Since $A_T(2) = [0.5, 0.6] \not\succeq [0.6, 0.8] = \text{rmin}\{A_T(1 \cdot 2), A_T(1)\}$, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is not a neutrosophic cubic UP-filter of X.

Theorem 3.21. *Every neutrosophic cubic near UP-filter of X is a neutrosophic cubic UP-subalgebra.*

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x), A_F(0) \succeq A_F(x), \lambda_T(0) \leq \lambda_T(x), \lambda_I(0) \geq \lambda_I(x)$, and $\lambda_F(0) \leq \lambda_F(x)$. Let $x, y \in X$. By (2.27), we have

$$\begin{aligned} A_T(x \cdot y) &\succeq A_T(y) \succeq \text{rmin}\{A_T(x), A_T(y)\}, \\ A_I(x \cdot y) &\preceq A_I(y) \preceq \text{rmax}\{A_I(x), A_I(y)\}, \\ A_F(x \cdot y) &\succeq A_F(y) \succeq \text{rmin}\{A_F(x), A_F(y)\}, \\ \lambda_T(x \cdot y) &\leq \lambda_T(y) \leq \max\{\lambda_T(x), \lambda_T(y)\}, \\ \lambda_I(x \cdot y) &\geq \lambda_I(y) \geq \min\{\lambda_I(x), \lambda_I(y)\}, \\ \lambda_F(x \cdot y) &\leq \lambda_F(y) \leq \max\{\lambda_F(x), \lambda_F(y)\}. \end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X. ■

The following example show that the converse of Theorem 3.21 is not true.

Example 3.22. From Example 3.4, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X. Since $\lambda_I(1 \cdot 2) = 0.2 < 0.6 = \lambda_I(2)$, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is not a neutrosophic cubic near UP-filter of X.

By Theorems 3.15, 3.17, 3.19, and 3.21 and Examples 3.16, 3.18, 3.20, and 3.22, we have that the concept of neutrosophic cubic UP-subalgebras is a generalization of neutrosophic cubic near UP-filters, neutrosophic cubic near UP-filters is a generalization of neutrosophic cubic UP-filters, neutrosophic cubic UP-filters is a generalization of neutrosophic cubic UP-ideals, and neutrosophic cubic UP-ideals is a generalization of neutrosophic cubic strong UP-ideals. Moreover, by Theorem 3.14, we obtain that neutrosophic cubic strong UP-ideals and constant neutrosophic cubic sets coincide.

Theorem 3.23. *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X satisfying the following condition:*

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} A_T(x) \succeq A_T(y) \\ A_I(x) \preceq A_I(y) \\ A_F(x) \succeq A_F(y) \\ \lambda_T(x) \leq \lambda_T(y) \\ \lambda_I(x) \geq \lambda_I(y) \\ \lambda_F(x) \leq \lambda_F(y) \end{cases} \right), \tag{3.1}$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X satisfying the condition (3.1). By Proposition 3.3, we have $\mathscr{A}$ satisfies the conditions (P1) and (P2). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

$$A_T(x \cdot y) = A_T(0) \succeq A_T(y), A_I(x \cdot y) = A_I(0) \preceq A_I(y), A_F(x \cdot y) = A_F(0) \succeq A_F(y),$$
$$\lambda_T(x \cdot y) = \lambda_T(0) \leq \lambda_T(y), \lambda_I(x \cdot y) = \lambda_I(0) \geq \lambda_I(y), \lambda_F(x \cdot y) = \lambda_F(0) \leq \lambda_F(y).$$

Case 2: $x \cdot y \neq 0$. Then

$$\begin{aligned} A_T(x \cdot y) &\succeq \operatorname{rmin}\{A_T(x), A_T(y)\} = A_T(y), \\ A_I(x \cdot y) &\preceq \operatorname{rmax}\{A_I(x), A_I(y)\} = A_I(y), \\ A_F(x \cdot y) &\succeq \operatorname{rmin}\{A_F(x), A_F(y)\} = A_F(y), \\ \lambda_T(x \cdot y) &\leq \max\{\lambda_T(x), \lambda_T(y)\} = \lambda_T(y), \\ \lambda_I(x \cdot y) &\geq \min\{\lambda_I(x), \lambda_I(y)\} = \lambda_I(y), \\ \lambda_F(x \cdot y) &\leq \max\{\lambda_F(x), \lambda_F(y)\} = \lambda_F(y). \end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X. ■

Theorem 3.24. *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X satisfying the following condition:*

$$A_T = A_I = A_F, \lambda_T = \lambda_I = \lambda_F, \tag{3.2}$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic near UP-filter of X satisfying the condition (3.2). Then $\mathscr{A}$ satisfies the conditions (P1) and (P2). Let $x \in X$. Then

$$\begin{aligned} A_T(0) &\succeq A_T(x) = A_I(x) \succeq A_I(0) = A_T(0) \\ A_I(0) &\preceq A_I(x) = A_T(x) \preceq A_T(0) = A_I(0) \\ A_F(0) &\succeq A_F(x) = A_I(x) \succeq A_I(0) = A_F(0) \\ \lambda_T(0) &\leq \lambda_T(x) = \lambda_I(x) \leq \lambda_I(0) = \lambda_T(0) \\ \lambda_I(x) &\geq \lambda_I(x) = \lambda_T(x) \geq \lambda_T(x) = \lambda_I(x) \\ \lambda_F(x) &\leq \lambda_F(x) = \lambda_I(x) \leq \lambda_I(x) = \lambda_F(x) \end{aligned}$$

Thus $A_T(0) = A_T(x), A_I(0) = A_I(x), A_F(0) = A_F(x), \lambda_T(0) = \lambda_T(x), \lambda_I(x) = \lambda_I(x)$, and $\lambda_F(x) = \lambda_F(x)$, that is, $\mathscr{A}$ is constant. By Theorem 3.14, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X. ■

Theorem 3.25. *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X satisfying the following condition:*

$$(\forall x, y, z \in X) \begin{pmatrix} A_T(y \cdot (x \cdot z)) = A_T(x \cdot (y \cdot z)) \\ A_I(y \cdot (x \cdot z)) = A_I(x \cdot (y \cdot z)) \\ A_F(y \cdot (x \cdot z)) = A_F(x \cdot (y \cdot z)) \\ \lambda_T(y \cdot (x \cdot z)) = \lambda_T(x \cdot (y \cdot z)) \\ \lambda_I(y \cdot (x \cdot z)) = \lambda_I(x \cdot (y \cdot z)) \\ \lambda_F(y \cdot (x \cdot z)) = \lambda_F(x \cdot (y \cdot z)) \end{pmatrix}, \tag{3.3}$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X satisfying the condition (3.3). Then $\mathscr{A}$ satisfies the conditions (P1) and (P2). Next, let $x, y, z \in X$.

Then

$$\begin{aligned}
A_T(x\cdot z) &\succeq \operatorname{rmin}\{A_T(y\cdot(x\cdot z)), A_T(y)\}\\
&= \operatorname{rmin}\{A_T(x\cdot(y\cdot z)), A_T(y)\},\\
A_I(x\cdot z) &\preceq \operatorname{rmax}\{A_I(y\cdot(x\cdot z)), A_I(y)\}\\
&= \operatorname{rmax}\{A_I(x\cdot(y\cdot z)), A_I(y)\},\\
A_F(x\cdot z) &\succeq \operatorname{rmin}\{A_F(y\cdot(x\cdot z)), A_F(y)\}\\
&= \operatorname{rmin}\{A_F(x\cdot(y\cdot z)), A_F(y)\},\\
\lambda_T(x\cdot z) &\leq \max\{\lambda_T(y\cdot(x\cdot z)), \lambda_T(y)\}\\
&= \max\{\lambda_T(x\cdot(y\cdot z)), \lambda_T(y)\},\\
\lambda_I(x\cdot z) &\geq \min\{\lambda_I(y\cdot(x\cdot z)), \lambda_I(y)\}\\
&= \min\{\lambda_I(x\cdot(y\cdot z)), \lambda_I(y)\},\\
\lambda_F(x\cdot z) &\leq \max\{\lambda_F(y\cdot(x\cdot z)), \lambda_F(y)\}\\
&= \max\{\lambda_F(x\cdot(y\cdot z)), \lambda_F(y)\}.
\end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X. ■

Theorem 3.26. *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x\cdot y \Rightarrow \begin{cases} A_T(z) \succeq \operatorname{rmin}\{A_T(x), A_T(y)\}\\ A_I(z) \preceq \operatorname{rmax}\{A_I(x), A_I(y)\}\\ A_F(z) \succeq \operatorname{rmin}\{A_F(x), A_F(y)\}\\ \lambda_T(z) \leq \max\{\lambda_T(x), \lambda_T(y)\}\\ \lambda_I(z) \geq \min\{\lambda_I(x), \lambda_I(y)\}\\ \lambda_F(z) \leq \max\{\lambda_F(x), \lambda_F(y)\} \end{cases}\right), \tag{3.4}$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the condition (3.4). Let $x, y \in X$. By (2.1), we have $(x\cdot y)\cdot(x\cdot y) = 0$, that is, $x\cdot y \geq x\cdot y$. It follows from (3.4) that

$$\begin{aligned}
A_T(x\cdot y) \succeq \operatorname{rmin}\{A_T(x), A_T(y)\}, A_I(x\cdot y) \preceq \operatorname{rmax}\{A_I(x), A_I(y)\},\\
A_F(x\cdot y) \succeq \operatorname{rmin}\{A_F(x), A_F(y)\}, \lambda_T(x\cdot y) \leq \max\{\lambda_T(x), \lambda_T(y)\},\\
\lambda_I(x\cdot y) \geq \min\{\lambda_I(x), \lambda_I(y)\}, \lambda_F(x\cdot y) \leq \max\{\lambda_F(x), \lambda_F(y)\}.
\end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-subalgebra of X. ■

Theorem 3.27. *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x\cdot y \Rightarrow \begin{cases} A_T(y) \succeq \operatorname{rmin}\{A_T(z), A_T(x)\}\\ A_I(y) \preceq \operatorname{rmax}\{A_I(z), A_I(x)\}\\ A_F(y) \succeq \operatorname{rmin}\{A_F(z), A_F(x)\}\\ \lambda_T(y) \leq \max\{\lambda_T(z), \lambda_T(x)\}\\ \lambda_I(y) \geq \min\{\lambda_I(z), \lambda_I(x)\}\\ \lambda_F(y) \leq \max\{\lambda_F(z), \lambda_F(x)\} \end{cases}\right), \tag{3.5}$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the condition (3.5). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leq x \cdot 0$. It follows from (3.5) that

$$\begin{aligned}
A_T(0) &\succeq \text{rmin}\{A_T(x), A_T(x)\} = A_T(x),\\
A_I(0) &\preceq \text{rmax}\{A_I(x), A_I(x)\} = A_I(x),\\
A_F(0) &\succeq \text{rmin}\{A_F(x), A_F(x)\} = A_F(x),\\
\lambda_T(0) &\leq \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x),\\
\lambda_I(0) &\geq \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x),\\
\lambda_F(0) &\leq \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x).
\end{aligned}$$

Next, let $x, y \in X$. By (2.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \geq x \cdot y$. It follows from (3.5) that

$$\begin{aligned}
A_T(y) \succeq \text{rmin}\{A_T(x \cdot y), A_T(x)\}, A_I(y) \preceq \text{rmax}\{A_I(x \cdot y), A_I(x)\},\\
A_F(y) \succeq \text{rmin}\{A_F(x \cdot y), A_F(x)\}, \lambda_T(y) \leq \max\{\lambda_T(x \cdot y), \lambda_T(x)\},\\
\lambda_I(y) \geq \min\{\lambda_I(x \cdot y), \lambda_I(x)\}, \lambda_F(y) \leq \max\{\lambda_F(x \cdot y), \lambda_F(x)\}.
\end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-filter of X. ■

Theorem 3.28. *If $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the following condition:*

$$(\forall a, x, y, z \in X)\left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} A_T(x \cdot z) \succeq \text{rmin}\{A_T(a), A_T(y)\}\\ A_I(x \cdot z) \preceq \text{rmax}\{A_I(a), A_I(y)\}\\ A_F(x \cdot z) \succeq \text{rmin}\{A_F(a), A_F(y)\}\\ \lambda_T(x \cdot z) \leq \max\{\lambda_T(a), \lambda_T(y)\}\\ \lambda_I(x \cdot z) \geq \min\{\lambda_I(a), \lambda_I(y)\}\\ \lambda_F(x \cdot z) \leq \max\{\lambda_F(a), \lambda_F(y)\} \end{cases} \right), \tag{3.6}$$

then $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the condition (3.6). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0)) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (3.6) that

$$\begin{aligned}
A_T(0) = A_T(0 \cdot 0) &\succeq \text{rmin}\{A_T(x), A_T(x)\} = A_T(x), && \text{((UP-2))}\\
A_I(0) = A_I(0 \cdot 0) &\preceq \text{rmax}\{A_I(x), A_I(x)\} = A_I(x), && \text{((UP-2))}\\
A_F(0) = A_F(0 \cdot 0) &\succeq \text{rmin}\{A_F(x), A_F(x)\} = A_F(x), && \text{((UP-2))}\\
\lambda_T(0) = \lambda_T(0 \cdot 0) &\leq \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), && \text{((UP-2))}\\
\lambda_I(0) = \lambda_I(0 \cdot 0) &\geq \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), && \text{((UP-2))}\\
\lambda_F(0) = \lambda_F(0 \cdot 0) &\leq \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x). && \text{((UP-2))}
\end{aligned}$$

Next, let $x, y, z \in X$. By (2.1), we have $(x\cdot(y\cdot z))\cdot(x\cdot(y\cdot z)) = 0$, that is, $x\cdot(y\cdot z) \geq x\cdot(y\cdot z)$. It follows from (3.6) that

$$\begin{aligned}
A_T(x \cdot z) &\succeq \mathrm{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\},\\
A_I(x \cdot z) &\preceq \mathrm{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\},\\
A_F(x \cdot z) &\succeq \mathrm{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\},\\
\lambda_T(x \cdot z) &\leq \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\},\\
\lambda_I(x \cdot z) &\geq \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\},\\
\lambda_F(x \cdot z) &\leq \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}.
\end{aligned}$$

Hence, $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic UP-ideal of X. ∎

Theorem 3.29. *A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X satisfies the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} A_T(z) \succeq A_T(y) \\ A_I(z) \preceq A_I(y) \\ A_F(z) \succeq A_F(y) \\ \lambda_T(z) \leq \lambda_T(y) \\ \lambda_I(z) \geq \lambda_I(y) \\ \lambda_F(z) \leq \lambda_F(y) \end{cases} \right) \tag{3.7}$$

if and only if $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X.

Proof. Assume that $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a NCS in X satisfying the condition (3.7). Let $x, y \in X$. By (UP-3) and (2.1), we have $x \cdot 0 = 0$, that is, $x \leq 0 = y \cdot y$. It follows from (3.7) that

$$\begin{aligned}
&A_T(x) \succeq A_T(y), A_I(x) \preceq A_I(y), A_F(x) \succeq A_F(y),\\
&\lambda_T(x) \leq \lambda_T(y), \lambda_I(x) \geq \lambda_I(y), \lambda_F(x) \leq \lambda_F(y).
\end{aligned}$$

Similarly,

$$\begin{aligned}
&A_T(y) \succeq A_T(x), A_I(y) \preceq A_I(x), A_F(y) \succeq A_F(x),\\
&\lambda_T(y) \leq \lambda_T(x), \lambda_I(y) \geq \lambda_I(x), \lambda_F(y) \leq \lambda_F(x).
\end{aligned}$$

Then

$$\begin{aligned}
&A_T(x) = A_T(y), A_I(x) = A_I(y), A_F(x) = A_F(y),\\
&\lambda_T(x) = \lambda_T(y), \lambda_I(x) = \lambda_I(y), \lambda_F(x) = \lambda_F(y).
\end{aligned}$$

Thus $\mathscr{A}$ is constant. By Theorem 3.14, we have $\mathscr{A} = (\mathbf{A}, \Lambda)$ is a neutrosophic cubic strong UP-ideal of X. ∎

For any fixed numbers $\alpha^+, \alpha^-, \beta^+, \beta^-, \gamma^+, \gamma^- \in [0, 1]$ such that $\alpha^+ > \alpha^-, \beta^+ > \beta^-, \gamma^+ > \gamma^-$ and a nonempty subset G of X, the NS ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}] = (X, {}^G\lambda_T[^{\alpha^-}_{\alpha^+}], {}^G\lambda_I[^{\beta^+}_{\beta^-}], {}^G\lambda_F[^{\gamma^-}_{\gamma^+}])$ in X, where ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}]$, ${}^G\lambda_I[^{\beta^+}_{\beta^-}]$, and ${}^G\lambda_F[^{\gamma^-}_{\gamma^+}]$ are fuzzy sets in X which are given as follows:

$${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \begin{cases} \alpha^- & \text{if } x \in G, \\ \alpha^+ & \text{otherwise,} \end{cases}$$

$$^{G}\lambda_I[^{\beta^+}_{\beta^-}](x) = \begin{cases} \beta^+ & \text{if } x \in G, \\ \beta^- & \text{otherwise}, \end{cases}$$

$$^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \begin{cases} \gamma^- & \text{if } x \in G, \\ \gamma^+ & \text{otherwise}. \end{cases}$$

For any fixed interval numbers $\tilde{a}^+, \tilde{a}^-, \tilde{b}^+, \tilde{b}^-, \tilde{c}^+, \tilde{c}^- \in [[0,1]]$ such that $\tilde{a}^+ \succ \tilde{a}^-, \tilde{b}^+ \succ \tilde{b}^-, \tilde{c}^+ \succ \tilde{c}^-$ and a nonempty subset G of X, the IVNS $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}] = (X, A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}], A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}], A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}])$ in X, where $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}]$, $A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}]$, and $A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}]$ are interval-valued fuzzy sets in X which are given as follows:

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x) = \begin{cases} \tilde{a}^+ & \text{if } x \in G, \\ \tilde{a}^- & \text{otherwise}, \end{cases}$$

$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x) = \begin{cases} \tilde{b}^- & \text{if } x \in G, \\ \tilde{b}^+ & \text{otherwise}, \end{cases}$$

$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x) = \begin{cases} \tilde{c}^+ & \text{if } x \in G, \\ \tilde{c}^- & \text{otherwise}. \end{cases}$$

We define the NCS $\mathscr{A}^G[[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}], [^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]] = (\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}], {}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}])$ in X.

Theorem 3.30 ([48]). *A NS* ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ *in* X *is a special neutrosophic UP-subalgebra (resp., special neutrosophic near UP-filter, special neutrosophic UP-filter, special neutrosophic UP-ideal, special neutrosophic strong UP-ideal) of* X *if and only if a nonempty subset* G *of* X *is a UP-subalgebra (resp., near UP-filter, UP-filter, UP-ideal, strong UP-ideal) of* X*.*

Theorem 3.31 ([49]). *An IVNS* $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ *in* X *is an interval-valued neutrosophic UP-subalgebra (resp., interval-valued neutrosophic near UP-filter, interval-valued neutrosophic UP-filter, interval-valued neutrosophic UP-ideal, interval-val- ued neutrosophic strong UP-ideal) of* X *if and only if a nonempty subset* G *of* X *is a UP-subalgebra (resp., near UP-filter, UP-filter, UP-ideal, strong UP-ideal) of* X*.*

Combining Theorems 3.13, 3.30, and 3.31, we have the following corollary.

Corollary 3.32. *A NCS* $\mathscr{A}^G[[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}], [^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]]$ *in* X *is a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of* X *if and only if a nonempty subset* G *of* X *is a UP-subalgebra (resp., near UP-filter, UP-filter, UP-ideal, strong UP-ideal) of* X*.*

4. Level Subsets of a Neutrosophic Cubic Set

In this section, we discuss the relationships among neutrosophic cubic UP-subalgebras (resp., neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, neutrosophic cubic strong UP-ideals) of UP-algebras and their level subsets.

Definition 4.1 ([9, 49])**.** Let f be a FS and A be an IVFS in a nonempty set X and let $t \in [0,1]$ and $\tilde{a} \in [[0,1]]$. The sets

$$U(f;t) = \{x \in X \mid f(x) \geq t\},$$
$$L(f;t) = \{x \in X \mid f(x) \leq t\},$$
$$E(f;t) = \{x \in X \mid f(x) = t\}$$

are called an *upper t-level subset*, a *lower t-level subset*, and an *equal t-level subset* of f, respectively, and the sets

$$U(A;\tilde{a}) = \{x \in X \mid A(x) \succeq \tilde{a}\}, \tag{4.1}$$
$$L(A;\tilde{a}) = \{x \in X \mid A(x) \preceq \tilde{a}\}, \tag{4.2}$$
$$E(A;\tilde{a}) = \{x \in X \mid A(x) = \tilde{a}\} \tag{4.3}$$

are called an upper $\tilde{a}$-level subset, a lower $\tilde{a}$-level subset, and an equal $\tilde{a}$-level subset of A, respectively.

Theorem 4.2 ([48])**.** *A NS Λ in X is a special neutrosophic UP-subalgebra (resp., special neutrosophic near UP-filter, special neutrosophic UP-filter, special neutrosophic UP-ideal) of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are either empty or UP-subalgebras (resp., near UP-filter, UP-filter, UP-ideal) of X.*

Theorem 4.3 ([49])**.** *An IVNS $\mathbf{A}$ in X is an interval-valued neutrosophic UP-subalgebra (resp., interval-valued neutrosophic near UP-filter, interval-valued neutrosophic UP-filter, interval-valued neutrosophic UP-ideal) of X if and only if for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $U(A_T;\tilde{a}), L(A_I;\tilde{b})$, and $U(A_F;\tilde{c})$ are either empty or UP-subalgebras (resp., near UP-filter, UP-filter, UP-ideal) of X.*

Combining Theorems 3.13, 4.2, and 4.3, we have the following corollary.

Corollary 4.4. *A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal) of X if and only if for all $[s_{T_1}, s_{T_2}], [s_{I_1}, s_{I_2}], [s_{F_1}, s_{F_2}] \in [[0,1]]$ and $t_T, t_I, t_F \in [0,1]$, the sets $U(A_T;[s_{T_1}, s_{T_2}]), L(A_I;[s_{I_1}, s_{I_2}])$,
$U(A_F;[s_{F_1}, s_{F_2}]), L(\lambda_T;t_T), U(\lambda_I;t_I)$, and $L(\lambda_F;t_F)$ are either empty or UP-sub- algebras (resp., near UP-filter, UP-filter, UP-ideal) of X.*

Theorem 4.5 ([48])**.** *A NS Λ in X is a special neutrosophic strong UP-ideal of X if and only if the sets $E(\lambda_T, \lambda_T(0)), E(\lambda_I, \lambda_I(0))$, and $E(\lambda_F, \lambda_F(0))$ are strong UP-ideals of X.*

Theorem 4.6 ([49])**.** *An IVNS $\mathbf{A}$ in X is an interval-valued neutrosophic strong UP-ideal of X if and only if the sets $E(A_T;A_T(0)), E(A_I;A_I(0))$, and $E(A_F;A_F(0))$ are strong UP-ideals of X.*

Combining Theorems 3.13, 4.5, and 4.6, we have the following corollary.

Corollary 4.7. *A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is a neutrosophic cubic strong UP-ideal of X if and only if the sets $E(A_T;A_T(0)), E(A_I;A_I(0)), E(A_F;A_F(0))$,
$E(\lambda_T, \lambda_T(0)), E(\lambda_I, \lambda_I(0))$, and $E(\lambda_F, \lambda_F(0))$ are strong UP-ideals of X.*

5. Conclusions and Future Work

In this paper, we have introduced the concepts of neutrosophic cubic UP-subalgebras, neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, and neutrosophic cubic strong UP-ideals of UP-algebras and investigated some of their important properties. Then, we have the diagram of generalization of NCSs in UP-algebras as shown in Figure 1.

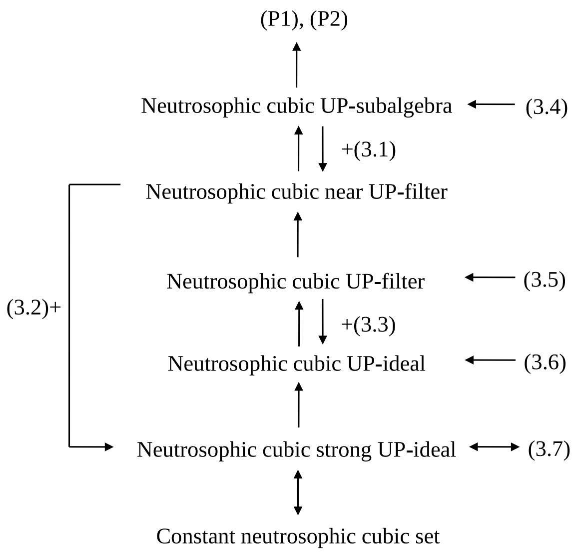

Figure 1. NCSs in UP-algebras

In our future study, we will apply this concept/results to other type of NCSs in UP-algebras. Also, we will study the soft set theory of neutrosophic cubic UP-subalgebras, neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, and neutrosophic cubic strong UP-ideals.

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