



Dedicated to Prof. Suthep Suantai on the occasion of his 60<sup>th</sup> anniversary

# Exponential Projective Synchronization of Neural Networks via Hybrid Adaptive Intermittent Control with Mixed Time-Varying Delays

Patarawadee Prasertsang<sup>1</sup> and Thongchai Botmart<sup>2,\*</sup>

<sup>1</sup>*Department of General Science, Kasetsart University, Chalermprakiat Sakon Nakhon Province Campus, Sakon Nakhon 47000, Thailand*  
e-mail : [patarawadee.s@ku.th](mailto:patarawadee.s@ku.th)

<sup>2</sup>*Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand*  
e-mail : [thongbo@kku.ac.th](mailto:thongbo@kku.ac.th)

**Abstract** This study investigated the exponential function projective synchronization between two neural networks with discrete interval and distributed time-varying delays via a new hybrid adaptive periodically intermittent control, composed of nonlinear and periodically adaptive intermittent feedback control. Based on the construction of improving piecewise Lyapunov-Krasovskii functionals merged with Leibniz-Newton's formula, applying the piecewise analysis method, adaptive periodically intermittent control and mathematical induction, some sufficient conditions for the exponential function projective synchronization of these networks were developed in terms of LMIs. Finally, a numerical example was produced to demonstrate the effectiveness of this proposed theoretical research.

**MSC:** 34D20; 34K20; 37C75

**Keywords:** exponential function projective synchronization; drive-response neural networks; mixed time-varying delays; adaptive periodically intermittent control

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## 1. INTRODUCTION

A neural network is a model of computation motivated by the biological neural networks in the brain with combination of neurons and synapses. The knowledge in term of a neural network has been applied in various fields such as computer science, psychology, physics, biology, artificial intelligence, electrical engineering and mathematics. Moreover, a neural network is derived from the connecting of neurons with synchronization which

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\*Corresponding author.

is important and interesting in natural science, biological and engineering networks. Applications of neural networks have been used in processing units and learning algorithms. Mathematical definitions of a neural network are proposed in [1], from the point of graph theory which defines a neural network as a directed graph. [2–14], neural networks draw extensive attention from many researchers due to their important applications in various branches including image processing, associative memory, nonlinear programming, pattern recognition, robotics and optimization problems.

It is widely realized that time delays are always necessary in neural networks because the finite exchange speed of neurons and amplifiers, may lead to network instability or inconstancy. Thus, neural networks with time delay have attracted a lot of research attentions, as reported in [2–6, 8–10, 12–14]. Moreover, time-varying delays of networks have been considered a valuable subject converting interval time-varying delays [4, 8], distributed time-varying delay [2, 3, 5, 9, 10, 12–15], discrete time-varying delays [2, 3, 5, 9, 10, 12–14], mixed time-varying delays [2, 3, 5, 6, 8–10, 12–14], and coupling delays [16–19]. This paper studies mixed time-varying delays, including both discrete and distributed time-varying delay. [3] investigated the issue of lag exponential synchronization between two delayed neural networks via adaptive intermittent control. [9] studied the topic of discrete-time stochastic recurrent neural networks with multiple time-varying delays in the leakage term and impulsive effects. [13] investigated the lag synchronization of chaotic neural networks with discrete and distributed delays, via periodically intermittent control.

Up to now, synchronization has been considered an active issue with regard to neural networks, which can be used to explain process in nature, society, technology, physical and biological systems. There are many potential applications in various engineering fields including harmonic oscillation generation, secure communication, image processing, mechanical engineering. Synchronization can be used to define processes relating to weaving the threads of execution of several tasks, without destroying the shared data but preventing deadlocks and race conditions; in addition synchronization occurs between the network nodes to make sure the send and receive streams operate correctly and to prevent the collision of information. Several types of synchronization have been defined: projective synchronization [4, 6, 7, 10, 11, 16, 18, 20–26], cluster synchronization [17, 27–29], quasi-synchronization [19], complete synchronization [30], lag synchronization [3, 13, 19, 31], finite-time synchronization [15, 32] and outer synchronization [25, 33]. Among them, projective synchronization characterized by a scaling factor  $\alpha$ , has been widely proposed to acquire a general type of proportional relationships between the drive and response systems. [20] studied the problem of exponential synchronization of a couple of chaotic systems with delays via intermittent control. In [7], Xing and et.al. provided periodically exponential synchronization for delayed recurrent neural networks via intermittent control by applying Lyapunov functional theory, mathematical induction and an inequality technique. To the best of our knowledge, there have been no publications of the proposed exponential projective synchronization of a neural network.

In the past few years, the drive and response systems of the networks have received more attention to derive some criteria conditions of networks, for example: complex networks [15, 22, 33], neural networks [3, 6, 13], dynamic networks [33] and chaotic networks [23, 31]. Moreover, several kinds of control have been broadly considered in various networks, including: feedback control [6, 16, 20, 23, 28, 29], intermittent control [3, 7, 11, 13, 15, 17–20, 22–26, 28, 31–34], adaptive control [6, 7, 10, 15–18, 20–22, 24, 25, 28, 33, 34], pinning

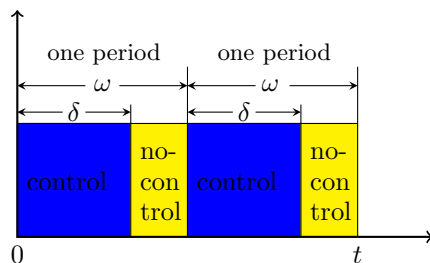


FIGURE 1. The sketch map of the intermittent controls for periodically.

control [6, 10, 17, 18, 21, 22, 24, 29, 34], impulsive control [32] and open-loop control [6]. Intermittent control has become a strategy of an increasing attention because of its wide use in engineering applications. A sketch map of the intermittent control as shown in Figure 1. Periodically intermittent control, as a discontinuous feedback control with both fixed constant and width of the control [7, 13, 19, 32]. On the other hand, nonperiodically (aperiodically) intermittent control, allows as both the constant and width of the control to be variable [3, 26, 34]. Clearly, aperiodically intermittent control is more possible than intermittent controls for periodically.

The topic of pinning adaptive complex synchronization between two delayed dynamical networks was studied in [22], based on the Lyapunov stability theory and periodically intermittent control. [28] proposed adaptive pinning cluster synchronization of directed heterogeneous dynamic networks via intermittent control by applying the Lyapunov function and the analysis technique where the adaptive update law for each controlled node was based on the information of the synchronous state of the controlled node. In [24], the issue of adaptive synchronization control for direct networks with node balance was investigated via intermittent control by constructing a piecewise auxiliary function and utilizing the piecewise analysis method and series theory. Based on the construction of a piecewise continuous Lyapunov function, the problem of exponential adaptive pinning cluster synchronization for directed community networks was investigated via aperiodically intermittent control as in [27]. [10] presented the problem of function projective synchronization of neural networks with asymmetric coupling involving discrete intervals and distributed time-varying delays and uncertainties via adaptive pinning controls and adaptive control. In [3], the topic of synchronization of drive and response complex networks with time-delay was studied by utilizing the Lyapunov stability theory, the periodically adaptive intermittent pinning control technique, the differential inequality method and mathematical induction to derive the theoretical results. Later, the adaptive finite-time hybrid projective synchronization of complex networks with distributed delay was studied via intermittent control in [15].

Based on the discussion above, the current study investigated the adaptive exponential function projective synchronization of drive-response neural networks between two delays via hybrid intermittent feedback control. The main works of this research are:

(i) The mixed time-varying delays, including both discrete interval and distributed time-varying delays are considered as continuous functions that belong to the specified intervals, which mean that the upper and lower boundaries for these delays exist while the support necessary to find the function derivative is similar to the time-delay in

[2, 3, 5, 6, 9, 10, 12–14].

(ii) For the control technique, exponential function projective synchronization is studied using hybrid feedback control and adaptive periodically intermittent control [20].

Based on the construction of improving piecewise Lyapunov-Krasovskii functionals merged with Leibniz-Newton's formula [2, 4, 5, 8, 9, 12, 14, 15, 29, 32] and applying the piecewise analysis method [3], adaptive periodically pinning intermittent control [15, 19, 24, 34] and mathematical induction [7, 17, 22, 23], some sufficient conditions for exponential function projective synchronization of these systems are first produced in terms of LMIs. Numerical simulation is given to describe the usefulness of this proposed result.

The arrangement of this study is organized as follows. Section 2 gives some model description and mathematical preliminaries. The problem of exponential function projective synchronization between two neural networks delays are derived in Section 3. In Section 4, numerical simulation is given to demonstrate the effectiveness of this proposed theoretical result. The conclusion is provided in Section 5 and references are cited.

**Notations.** This paper will be uses the following notation:  $\mathbb{R}^n$  denotes the  $n$ -dimensional space and  $\| \cdot \|$  denotes the Euclidean vector norm;  $A^T$  denotes the transpose of matrix  $A$ ;  $A$  is symmetric if  $A = A^T$ .

Consider the drive-response neural networks as follows:

$$\begin{aligned} \dot{z}(t) &= -Az(t) + Bf(z(t)) + Cg(z(t-h(t))) + D \int_{t-d(t)}^t h(z(s))ds, \\ z(t) &= \varphi(t), t \in [-\tau, 0], \tau = \max\{h_1, d_M\}, \end{aligned} \quad (1.1)$$

$$\begin{aligned} \dot{y}(t) &= -Ay(t) + Bf(y(t)) + Cg(y(t-h(t))) + D \int_{t-d(t)}^t h(y(s))ds + \mathcal{U}(t), \\ y(t) &= \phi(t), t \in [-\tau, 0], \tau = \max\{h_1, d_M\}, \end{aligned} \quad (1.2)$$

where  $z(t) = [z_1(t), z_2(t), \dots, z_n(t)] \in \mathbb{R}^n$  and  $y(t) = [y_1(t), y_2(t), \dots, y_n(t)] \in \mathbb{R}^n$  are the drive system's state vector and the response system's state vector of the neural networks,  $f(\cdot), g(\cdot)$  and  $h(\cdot)$  are the neuron activation functions,  $A = \text{diag}(a_1, a_2, \dots, a_n) > 0$  is the state feedback coefficient matrix,  $B, C$  and  $D$  are connection weight matrices,  $\mathcal{U}(t) \in \mathbb{R}^n$  is the control input,  $\omega$  is a period width, and  $n$  is the number of these neural networks.

The initial conditions  $\varphi(t)$  and  $\phi(t)$  denote continuous vector-valued initial functions of  $t \in [-\tau, 0]$ .

To prove the main theorem, we need the following Assumptions and Definition.

**Assumption 1.** The time-varying delay functions  $h(t)$  and  $d(t)$  satisfy the condition  $0 \leq h(t) \leq h_1$ ,  $0 \leq d(t) \leq d_M$  and  $0 \leq \dot{h}(t) \leq \beta < 1$ .

**Assumption 2.** ([10]) Let us denote

$$\begin{aligned} F(x(t)) &= f(y(t)) - f(\alpha(t)z(t)), \\ G(x(t-h(t))) &= g(y(t-h(t))) - g(\alpha(t)z(t-h(t))), \\ H(x(t)) &= h(y(t)) - h(\alpha(t)z(t)). \end{aligned}$$

The activation functions  $f(\cdot)$ ,  $g(\cdot)$  and  $h(\cdot)$  satisfy the Lipschitz constants  $f > 0, g > 0$  and  $h > 0$  such that

$$\begin{aligned} |F(x(t))| &\leq f|y(t) - \alpha(t)z(t)|, \\ |G(x(t-h(t)))| &\leq g|y(t-h(t)) - \alpha(t)z(t-h(t))|, \\ |H(x(t))| &\leq h|y(t) - \alpha(t)z(t)|, \end{aligned}$$

where  $F, G$  and  $H$  are positive constant matrices and

$$\begin{aligned} \bar{F} &= \text{diag}\{f_i, i = 1, 2, \dots, n\}, \\ \bar{G} &= \text{diag}\{g_i, i = 1, 2, \dots, n\}, \\ \bar{H} &= \text{diag}\{h_i, i = 1, 2, \dots, n\}. \end{aligned}$$

**Definition 1.1.** ([6]) Network (1.3) with mixed time-varying delays, which including both discrete interval and distributed time-varying delay is said to achieve exponential function projective synchronization (EFPS) if there exist  $\mathcal{M} \geq 1, \delta > 0$ , a continuously differentiable scaling function  $\alpha(t)$  such that

$$\lim_{t \rightarrow \infty} \|x(t)\| = \lim_{t \rightarrow \infty} \|y(t) - \alpha(t)z(t)\| \leq \mathcal{M}\|\phi - \alpha\omega\|e^{-\delta t}, \quad \forall t > 0, \quad i = 1, 2, \dots, N,$$

where  $\|\cdot\|$  stands for the Euclidean vector norm.

To verify the stability of the synchronized states, we establish the synchronized error  $x(t)$  in the form  $x(t) = y(t) - \alpha(t)z(t)$ , where the continuous function  $\alpha(t) \neq 0$  is bounded and a differentiable function. Then, the neural networks with mixed time-varying delays of synchronized error between the drive-response neural networks given in (1.1) and (1.2) can be written by

$$\begin{aligned} \dot{x}(t) &= \dot{y}(t) - \alpha(t)\dot{z}(t) - \dot{\alpha}(t)z(t), \\ &= -A[y(t) - \alpha(t)z(t)] + B[f(y(t)) - \alpha(t)f(z(t))] \\ &\quad + C[g(y(t-h(t))) - \alpha(t)g(z(t-h(t)))] - \dot{\alpha}(t)z(t) \\ &\quad + D \left[ \int_{t-d(t)}^t h(y(s))ds - \alpha(t) \int_{t-d(t)}^t h(z(s))ds \right] + \mathcal{U}(t) \end{aligned} \tag{1.3}$$

The state hybrid feedback controller  $\mathcal{U}(t)$  defines the following equation:

$$\mathcal{U}(t) = \mathcal{U}_1(t) + \mathcal{U}_2(t), \tag{1.4}$$

where

$$\begin{aligned} \mathcal{U}_1(t) &= \dot{\alpha}(t)z(t) + B\alpha(t)f(z(t)) - Bf(\alpha(t)z(t)) + C\alpha(t)g(z(t-h(t))) \\ &\quad - Cg(\alpha(t)z(t-h(t))) + D\alpha(t) \int_{t-d(t)}^t h(z(s))ds - D \int_{t-d(t)}^t h(\alpha(s)z(s))ds, \\ \mathcal{U}_2(t) &= -K_1(t)e^{-\mu(t-t_n)}x(t) - K_2(t)e^{-\mu(t-t_n)}x(t-h(t)) \\ &\quad - K_3(t)e^{-\mu(t-t_n)} \int_{t-d(t)}^t x(s)ds. \end{aligned}$$

Then, substituting (1.4) into (1.3), we have the following:

$$\begin{aligned} \dot{x}(t) &= -Ax(t) + BF(x(t)) + CG(x(t - h(t))) + D \int_{t-d(t)}^t h(x(s))ds \\ &\quad -K_1(t)e^{-\mu(t-t_n)}x(t) - K_2(t)e^{-\mu(t-t_n)}x(t - h(t)) \\ &\quad -K_3(t)e^{-\mu(t-t_n)} \int_{t-d(t)}^t x(s)ds. \end{aligned} \tag{1.5}$$

The adaptive intermittent control gain is given by  $K_i(t)$ ,  $i = 1, 2$  and  $3$  where

$$K_i(t) = \begin{cases} K_i(0), & t = 0, \\ K_i(n\omega + \delta), & t \in [n\omega, n\omega + \delta], \\ 0, & t \in (n\omega + \delta, (n + 1)\omega), \end{cases} \tag{1.6}$$

with the updating law

$$\begin{aligned} \dot{K}_1(t) &= q_1x^T(t)x(t), & t \in [n\omega, n\omega + \delta], \\ \dot{K}_2(t) &= q_2x^T(t)x(t - h(t)), & t \in [n\omega, n\omega + \delta], \\ \dot{K}_3(t) &= q_3x^T(t) \int_{t-d(t)}^t x(s)ds, & t \in [n\omega, n\omega + \delta], \end{aligned} \tag{1.7}$$

where  $q_i, i = 1, 2, 3$  are constants.

Next, we need the following Lemmas to complete the proof of our result.

**Lemma 1.2** ([35]). *For any constant symmetric positive definite matrix  $M > 0$ , scalar  $\gamma > 0$ , and vector function  $\omega : [0, \gamma] \rightarrow \mathcal{R}^n$  such that the integrations concerned are well defined, the following inequality holds:*

$$\left( \int_0^\gamma \omega(s)ds \right)^T M \left( \int_0^\gamma \omega(s)ds \right) \leq \gamma \left( \int_0^\gamma \omega^T(s)M\omega(s)ds \right),$$

**Lemma 1.3.** ([36, Cauchy inequality]) *For any symmetric positive definite matrix  $N \in M^{n \times n}$  and  $x, y \in \mathbb{R}^n$ , we have*

$$\pm 2x^T y \leq x^T N x + y^T N^{-1} y.$$

**Lemma 1.4.** ([37, Schur complement]) *Consider constant symmetric matrices  $X, Y, Z$  where  $X = X^T$  and  $0 < Y = Y^T$ , then  $X + Z^T Y^{-1} Z < 0$  if and only if*

$$\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0, \text{ or } \begin{bmatrix} -Y & Z \\ Z^T & X \end{bmatrix} < 0.$$

## 2. MAIN RESULTS

Let us denote

$$\begin{aligned} \delta_1^* &= \lambda_{max} \left\{ A - 2\alpha I - \frac{1}{2}\varepsilon_1 BB^T - \frac{1}{2}\varepsilon_1^{-1} \bar{F}^T \bar{F} - \frac{1}{2}\varepsilon_2 CC^T - \frac{1}{2}\varepsilon_3 DD^T \right. \\ &\quad \left. - \frac{1}{2(1-\beta)} I - \frac{d_M^2}{2} I - \frac{\varepsilon_4^2}{2} (e^{-2\alpha h_1} I - \varepsilon_2^{-1} \bar{G}^T \bar{G}) - \frac{\varepsilon_5^2}{2} (e^{-2\alpha d_M} I - \varepsilon_3^{-1} \bar{H}^T \bar{H}) \right\}, \\ \delta_2^* &= \lambda_{max} [e^{\mu\delta} \varepsilon_4 (e^{-2\alpha h_1} I - \varepsilon_2^{-1} \bar{G}^T \bar{G})], \\ \delta_3^* &= \lambda_{max} [e^{\mu\delta} \varepsilon_5 (e^{-2\alpha d_M} I - \varepsilon_3^{-1} \bar{H}^T \bar{H})], \\ t_m &= m\omega. \end{aligned}$$

**Theorem 2.1.** For some given synchronization scaling function  $\alpha(t)$ ,  $0 < \alpha < \frac{1}{2}\mu$ ,  $0 < \alpha < \frac{1}{2}\gamma$ , the drive-response neural networks (1.5) satisfying Assumptions 1 and 2 and the purpose system can realize exponential function projective synchronization (EFPS) by the adaptive intermittent control law as shown in (1.6)-(1.7) if there exist positive constants  $\varepsilon_i$ ,  $i = 1, 2, \dots, 5$  and by taking appropriate  $K_1^*$ ,  $K_2^*$  and  $K_3^*$  such that

$$K_1^* > \delta_1^*, \tag{2.1}$$

$$K_2^* < \delta_2^*, \tag{2.2}$$

$$K_3^* < \delta_3^*, \tag{2.3}$$

$$\begin{bmatrix} -A + 2\alpha I + \frac{1}{2}\varepsilon_1 BB^T + \frac{1}{2}\varepsilon_2 CC^T + \frac{1}{2}\varepsilon_3 DD^T + \frac{1}{2(1-\beta)}I + \frac{d_M^2}{2}I & \bar{F}^T \\ * & -\varepsilon_1 I \end{bmatrix} < 0, \tag{2.4}$$

$$\begin{bmatrix} -\frac{1}{2}e^{2\alpha h_1}I & \bar{G}^T \\ * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{2.5}$$

$$\begin{bmatrix} -\frac{1}{2}e^{2\alpha d_M}I & \bar{H}^T \\ * & -\varepsilon_3 I \end{bmatrix} < 0, \tag{2.6}$$

$$\alpha(\rho_1 - \rho_2) - 2(\gamma - \alpha)(1 - \rho_1) > 0. \tag{2.7}$$

Then, the neural network (1.5) is an exponential function projective synchronization.

*Proof.* Let us consider the Lyapunov-Krasovskii functional as follow:

$$V(t, x(t)) = V_1(t) + V_2(t) + V_3(t) + W(t),$$

where

$$V_1(t) = \frac{1}{2}x^T(t)x(t),$$

$$V_2(t) = \frac{1}{2(1-\beta)} \int_{t-h(t)}^t e^{2\alpha(s-t)} x^T(s)x(s)ds,$$

$$V_3(t) = \frac{d_M}{2} \int_{-d_M}^0 \int_{t+s}^t e^{2\alpha(\theta-t)} x^T(\theta)x(\theta)d\theta ds,$$

$$W(t) = \begin{cases} \frac{1}{2q_1}e^{-\mu(t-t_n)}(K_1(t) - K_1^*)^2 + \frac{1}{2q_2}e^{-\mu(t-t_n)}(K_2(t) - K_2^*)^2 \\ + \frac{1}{2q_3}e^{-\mu(t-t_n)}(K_3(t) - K_3^*)^2, & n\omega \leq t \leq n\omega + \delta, \\ \frac{1}{2q_1}e^{-\gamma(t-(t_n+\delta))}(K_1(t_n + \delta) - K_1^*)^2 + \frac{1}{2q_2}e^{-\gamma(t-(t_n+\delta))}(K_2(t_n + \delta) - K_2^*)^2 \\ + \frac{1}{2q_3}e^{-\gamma(t-(t_n+\delta))}(K_3(t_n + \delta) - K_3^*)^2, & n\omega + \delta < t < (n + 1)\omega. \end{cases}$$

Using the derivatives of  $V_1(t)$  along the trajectories of system (1.5) yields

$$\begin{aligned} \dot{V}_1(t) &= -x^T(t)Ax(t) + x^T(t)BF(x(t)) + x^T(t)CG(x(t-h(t))) \\ &\quad + x^T(t)D \int_{t-d(t)}^t H(x(s))ds - x^T(t)K_1(t)e^{-\mu(t-t_n)}x(t) \\ &\quad - x^T(t)K_2(t)e^{-\mu(t-t_n)}x(t-h(t)) - x^T(t)K_3(t)e^{-\mu(t-t_n)} \int_{t-d(t)}^t x(s)ds. \end{aligned}$$

Applying Lemma 1.3, it follows that

$$\begin{aligned} x^T(t)BF(x(t)) &\leq \frac{1}{2} [\varepsilon_1 x^T(t)BB^T x(t) + \varepsilon_1^{-1} x^T(t)\bar{F}^T \bar{F}x(t)], \\ x^T(t)CG(x(t-h(t))) &\leq \frac{1}{2} [\varepsilon_2 x^T(t)CC^T x(t) + \varepsilon_2^{-1} x^T(t-h(t))\bar{G}^T \bar{G}x(t-h(t))], \\ x^T(t)D \int_{t-d(t)}^t H(x(s))ds &\leq \frac{1}{2} [\varepsilon_3 x^T(t)DD^T x(t) \\ &\quad + \varepsilon_3^{-1} \left( \int_{t-d(t)}^t x(s)ds \right)^T \bar{H}^T \bar{H} \left( \int_{t-d(t)}^t x(s)ds \right)]. \end{aligned}$$

By taking the derivatives of  $V_2(t)$  and  $V_3(t)$  along the trajectories of system (1.5), it follows that:

$$\begin{aligned} \dot{V}_2(t) &\leq -2\alpha V_2 + \frac{1}{2(1-\beta)} x^T(t)x(t) - \frac{1}{2} e^{-2\alpha h_1} x^T(t-h(t))x(t-h(t)), \\ \dot{V}_3(t) &\leq -2\alpha V_3 + \frac{d_M^2}{2} x^T(t)x(t) - \frac{d_M}{2} e^{-2\alpha d_M} \int_{t-d(t)}^t x^T(s)x(s)ds. \end{aligned}$$

Applying Lemma 1.2,

$$d_M \int_{t-d(t)}^t x^T(s)x(s)ds \geq \left( \int_{t-d(t)}^t x(s)ds \right)^T \left( \int_{t-d(t)}^t x(s)ds \right),$$

then,

$$-\frac{d_M}{2} e^{-2\alpha d_M} \int_{t-d(t)}^t x^T(s)x(s)ds \leq -\frac{1}{2} e^{-2\alpha d_M} \left( \int_{t-d(t)}^t x(s)ds \right)^T \left( \int_{t-d(t)}^t x(s)ds \right).$$

By using the derivatives of  $W(t)$  along the trajectories of system (1.5), when  $n\omega \leq t \leq n\omega + \delta$ ,  $n = 0, 1, 2, \dots$ , we have the following:

$$\begin{aligned} \dot{W}(t) &\leq -\mu W(t) + e^{-\mu(t-t_n)} K_1(t)x^T(t)x(t) - e^{-\mu\delta} K_1^* x^T(t)x(t) \\ &\quad + e^{-\mu(t-t_n)} K_2(t)x^T(t)x(t-h(t)) - e^{-\mu\delta} K_2^* x^T(t)x(t-h(t)) \\ &\quad + e^{-\mu(t-t_n)} K_3(t)x^T(t) \int_{t-d(t)}^t x(s)ds - e^{-\mu\delta} K_3^* x^T(t) \int_{t-d(t)}^t x(s)ds. \end{aligned}$$

Applying Lemma 1.3 yields

$$\begin{aligned} -K_2^* e^{-\mu\delta} x^T(t)x(t-h(t)) &\leq \frac{K_2^*}{2} e^{-\mu\delta} [\varepsilon_4 x^T(t)x(t) + \varepsilon_4^{-1} x^T(t-h(t))x(t-h(t))], \\ -K_3^* e^{-\mu\delta} x^T(t) \int_{t-d(t)}^t x(s)ds &\leq \frac{K_3^*}{2} e^{-\mu\delta} [\varepsilon_5 x^T(t)x(t) \\ &\quad + \varepsilon_5^{-1} \left( \int_{t-d(t)}^t x(s)ds \right)^T \left( \int_{t-d(t)}^t x(s)ds \right)]. \end{aligned}$$



Now, we have

$$\begin{aligned} \dot{V}(t) + 2\alpha V(t) \leq & x^T(t)[-A + 2\alpha I + \frac{1}{2}\varepsilon_1 BB^T + \frac{1}{2}\varepsilon_1^{-1} \bar{F}^T \bar{F} + \frac{1}{2}\varepsilon_2 CC^T + \frac{1}{2}\varepsilon_3 DD^T \\ & + \frac{1}{2(1-\beta)}I + \frac{d_M^2}{2}I - K_1^* e^{-\mu\delta}I + \frac{K_2^*}{2}\varepsilon_4 e^{-\mu\delta}I + \frac{K_3^*}{2}\varepsilon_5 e^{-\mu\delta}I]x(t) \\ & + x^T(t-h(t))[\frac{1}{2}\varepsilon_2^{-1} \bar{G}^T \bar{G} - \frac{1}{2}e^{-2\alpha h_1}I + \frac{K_2^*}{2}\varepsilon_4^{-1} e^{-\mu\delta}I]x(t-h(t)) \\ & + \left(\int_{t-d(t)}^t x(s)ds\right)^T \left[\frac{1}{2}\varepsilon_3^{-1} \bar{H}^T \bar{H} - \frac{1}{2}e^{-2\alpha d_M}I\right. \\ & \left. + \frac{K_3^*}{2}\varepsilon_5^{-1} e^{-\mu\delta}I\right] \left(\int_{t-d(t)}^t x(s)ds\right) + (2\alpha - \mu)W(t). \end{aligned}$$

According to (2.1)-(2.6), we get

$$\dot{V}(t) + 2\alpha V(t) \leq 0. \tag{2.8}$$

Integrating both sides of (2.8) from 0 to  $t$ , we have

$$V(x(t)) \leq e^{-2\alpha t}, \quad \forall t \geq 0.$$

Moreover,

$$V(x(t)) \leq e^{-2\alpha(t-n\omega)}, \quad t \in [n\omega, n\omega + \delta]. \tag{2.9}$$

By using the derivatives of  $W(t)$  along the trajectories of system (1.5), when  $n\omega + \delta < t < (n+1)\omega$ ,  $n = 0, 1, 2, \dots$ , we have the following:

$$\begin{aligned} \dot{V}(t) - 2(\gamma - \alpha)V(t) \leq & x^T(t)[-A + 2\alpha I + \frac{1}{2}\varepsilon_1 BB^T + \frac{1}{2}\varepsilon_1^{-1} \bar{F}^T \bar{F} + \frac{1}{2}\varepsilon_2 CC^T \\ & + \frac{1}{2}\varepsilon_3 DD^T + \frac{1}{2(1-\beta)}I + \frac{d_M^2}{2}I]x(t) \\ & + x^T(t-h(t))[\frac{1}{2}\varepsilon_2^{-1} \bar{G}^T \bar{G} - \frac{1}{2}e^{-2\alpha h_1}I]x(t-h(t)) \\ & + \left(\int_{t-d(t)}^t x(s)ds\right)^T \left[\frac{1}{2}\varepsilon_3^{-1} \bar{H}^T \bar{H} - \frac{1}{2}e^{-2\alpha d_M}I\right] \times \\ & \left(\int_{t-d(t)}^t x(s)ds\right) + (2\alpha - \gamma)W(t), \end{aligned}$$

so,

$$\dot{V}(t) - 2(\gamma - \alpha)V(t) \leq 0. \tag{2.10}$$

Integrating both sides of (2.10) with respect to  $t$  from 0 to  $t$ , we have

$$V(x(t)) \leq e^{2(\gamma-\alpha)t}, \quad \forall t \geq 0,$$

consequently,

$$V(x(t)) \leq e^{2(\gamma-\alpha)(t-n\omega-\delta)}, \quad n\omega + \delta < t < (n+1)\omega. \tag{2.11}$$

It follows from (2.9) and (2.11) that

$$\begin{aligned} V(x(t)) &\leq \max_{n\omega - \tau \leq \theta \leq n\omega} V(\theta) e^{-2\alpha(t-n\omega)} \\ &= \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-2\alpha t}, \quad n\omega \leq t \leq n\omega + \delta, \end{aligned}$$

and

$$V(x(t)) \leq \max_{n\omega + \delta - \tau \leq \theta \leq n\omega + \delta} V(\theta) e^{2(\gamma-\alpha)(t-n\omega-\delta)}, \quad n\omega + \delta < t < (n+1)\omega.$$

To estimate the value of  $V(x(t))$  from above discussion, we set  $n = 0$ , from (2.9) and when  $0 \leq t \leq \delta$ ,

$$\begin{aligned} V(x(t)) &\leq \max_{0 - \tau \leq \theta \leq 0} V(\theta) e^{-2\alpha(t-0)} \\ &= \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-2\alpha t}, \end{aligned}$$

when  $\delta < t < \omega$ ,

$$\begin{aligned} V(x(t)) &\leq \max_{0 + \delta - \tau \leq \theta \leq 0 + \delta} V(\theta) e^{2(\gamma-\alpha)(t-0-\delta)} \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{2(\gamma-\alpha)(\omega-\delta) - 2\alpha(\delta-h_1)}, \end{aligned}$$

when  $\omega \leq t \leq \omega + \delta$ ,

$$\begin{aligned} V(x(t)) &\leq \max_{\omega - \tau \leq \theta \leq \omega} V(\theta) e^{-2\alpha(t-\omega)} \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-2\alpha(t-\omega) + 2(\gamma-\alpha)(\omega-\delta) - 2\alpha(\delta-h_1)}, \end{aligned}$$

when  $\omega + \delta < t < 2\omega$ ,

$$\begin{aligned} V(x(t)) &\leq \max_{\omega + \delta - \tau \leq \theta \leq \omega + \delta} V(\theta) e^{2(\gamma-\alpha)(t-\omega-\delta)} \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{2(\gamma-\alpha)(t-\omega-\delta) + 2(\gamma-\alpha)(\omega-\delta) - 4\alpha(\delta-h_1)}, \end{aligned}$$

when  $2\omega \leq t \leq 2\omega + \delta$ ,

$$\begin{aligned} V(x(t)) &\leq \max_{2\omega - \tau \leq \theta \leq 2\omega} V(\theta) e^{-2\alpha(t-2\omega)} \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-2\alpha(t-2\omega) + 4(\gamma-\alpha)(\omega-\delta) - 4\alpha(\delta-h_1)}, \end{aligned}$$

when  $2\omega + \delta < t < 3\omega$ ,

$$\begin{aligned} V(x(t)) &\leq \max_{2\omega + \delta - \tau \leq \theta \leq 2\omega + \delta} V(\theta) e^{2(\gamma-\alpha)(t-2\omega-\delta)} \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{2(\gamma-\alpha)(t-2\omega-\delta) + 4(\gamma-\alpha)(\omega-\delta) - 6\alpha(\delta-h_1)}. \end{aligned}$$

By induction, we approximate the value of  $V(x(t))$  for the integer  $n$ .

When  $n\omega \leq t \leq n\omega + \delta$ ,

$$\begin{aligned} V(x(t)) &\leq \max_{n\omega - \tau \leq \theta \leq n\omega} V(\theta) e^{-2\alpha(t-n\omega)} \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-2\alpha(t-n\omega) + 2n(\gamma-\alpha)(\omega-\delta) - 2n\alpha(\delta-h_1)}. \end{aligned} \quad (2.12)$$

From (2.11) and when  $n\omega + \delta < t < (n + 1)\omega$ , we get

$$\begin{aligned} V(x(t)) &\leq \max_{n\omega + \delta - \tau \leq \theta \leq n\omega + \delta} V(\theta) e^{2(\gamma - \alpha)(t - n\omega - \delta)} \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{2(\gamma - \alpha)(t - n\omega - \delta) + 2n(\gamma - \alpha)(\omega - \delta) - 2(n + 1)\alpha(\delta - h_1)}. \end{aligned} \tag{2.13}$$

Substituting  $\delta = \rho_1\omega$  and  $h_1 = \rho_2\omega$  into (2.12) and (2.13), we obtain the following: when  $n\omega \leq t \leq n\omega + \delta$ ,

$$\begin{aligned} V(x(t)) &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-2\alpha(t - n\omega) + 2n(\gamma - \alpha)(\omega - \delta) - 2n\alpha(\delta - h_1)}, \\ &= \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-2\alpha(t - n\omega) + 2n(\gamma - \alpha)(\omega - \rho_1\omega) - 2n\alpha(\rho_1\omega - \rho_2\omega)}, \\ &= \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-2\alpha(t - n\omega) + 2(\gamma - \alpha)(1 - \rho_1)n\omega - 2\alpha(\rho_1 - \rho_2)n\omega}, \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-2\alpha(t - n\omega) + 2(\gamma - \alpha)(1 - \rho_1)\omega + 2\alpha(\rho_1 - \rho_2)(-t + \delta)}, \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{2(\gamma - \alpha)(1 - \rho_1)t + 2\alpha(\rho_1 - \rho_2)(-t + \delta)}, \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{2\alpha(\rho_1 - \rho_2)\delta + 2(\gamma - \alpha)(1 - \rho_1)t - 2\alpha(\rho_1 - \rho_2)t}, \\ &= \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-[2\alpha(\rho_1 - \rho_2) - 2(\gamma - \alpha)(1 - \rho_1)]t + 2\alpha(\rho_1 - \rho_2)\delta}, \end{aligned}$$

when  $n\omega + \delta < t < (n + 1)\omega$ ,

$$\begin{aligned} V(x(t)) &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{2(\gamma - \alpha)(t - n\omega - \delta) + 2n(\gamma - \alpha)(\omega - \delta) - 2(n + 1)\alpha(\delta - h_1)}, \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{2(\gamma - \alpha)(t - n\omega - \rho_1\omega) + 2n(\gamma - \alpha)(\omega - \rho_1\omega) - 2(n + 1)\alpha(\rho_1\omega - \rho_2\omega)}, \\ &= \max_{-\tau \leq \theta \leq 0} V(\theta) e^{2(\gamma - \alpha)t - 2(n + 1)(\gamma - \alpha)\rho_1\omega - 2(n + 1)\alpha(\rho_1 - \rho_2)\omega}, \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{2(\gamma - \alpha)t - 2(\gamma - \alpha)\rho_1t - 2\alpha(\rho_1 - \rho_2)t}, \\ &= \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-[2\alpha(\rho_1 - \rho_2) - 2(\gamma - \alpha)(1 - \rho_1)]t}, \\ &\leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-[2\alpha(\rho_1 - \rho_2) - 2(\gamma - \alpha)(1 - \rho_1)]t + 2\alpha(\rho_1 - \rho_2)\delta}. \end{aligned}$$

Therefore, for any  $t \geq 0$ , we have

$$V(x(t)) \leq \max_{-\tau \leq \theta \leq 0} V(\theta) e^{-[2\alpha(\rho_1 - \rho_2) - 2(\gamma - \alpha)(1 - \rho_1)]t + 2\alpha(\rho_1 - \rho_2)\delta}.$$

From (2.7), it means that

$$\|x(t)\|_2 \leq \left( 2 \max_{-\tau \leq \theta \leq 0} V(\theta) \right)^{\frac{1}{2}} e^{-[\alpha(\rho_1 - \rho_2) - (\gamma - \alpha)(1 - \rho_1)]t + \alpha(\rho_1 - \rho_2)\delta}.$$

The proof is completed. ■

### 3. NUMERICAL SIMULATION

**Example 3.1.** Consider the drive-respond neural networks (1.5) with the parameters

$$A = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 & -0.1 \\ 0.1 & 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} -0.4 & 0.1 \\ 0.3 & -0.4 \end{bmatrix}, \quad D = \begin{bmatrix} 0.3 & -0.2 \\ -0.3 & 0.2 \end{bmatrix},$$

$$F = G = H = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

where  $h_1 = 0.3, d_M = 0.2, \alpha = 0.41, \beta = 0.1, \mu = 1.7$  and  $\gamma = 1.79$ .  
 For the period width  $\omega = 2$  and  $\delta = 1.65, \rho_1 = 0.8250$  and  $\rho_2 = 0.1500$ ,  
 consider the condition in (2.7) yields

$$\alpha(\rho_1 - \rho_2) - (\gamma - \alpha)(1 - \rho_1) = 0.0352 > 0.$$

Therefore, using the Matlab LMI Control Toolbox to solve the LMI in (2.4)-(2.6), we have the solution as follows:

$$\varepsilon_1 = 8.7961, \varepsilon_2 = 2.5600, \varepsilon_3 = 2.3586.$$

By letting  $\varepsilon_4 = \varepsilon_5 = 1$ , we have

$$\delta_1^* = -11.5488, \delta_2^* = 6.4670, \delta_3^* = 7.0202.$$

A numerical simulation is achieved by utilizing the explicit Runge-Kutta-like method (dde45) and interpolation and extrapolation by spline of the third order. Figure 2 shows the chaotic behavior of the drive system (1.1) and response system (1.2) with the time-varying scaling function  $\alpha(t) = 0.8 + 0.2 \sin \frac{0.5\pi t}{15}$ . Figure 3 show the trajectories of  $x_1(t)$  and  $x_2(t)$  of the drive-response neural networks. Figure 4 shows the function projective synchronization error trajectories of  $x_1(t)$  and  $x_2(t)$  of the drive-response neural networks. The function projective synchronization error trajectories of  $x_1(t)$ , and  $x_2(t)$  of the drive-response neural networks with intermittent control are shown in Figure 5.

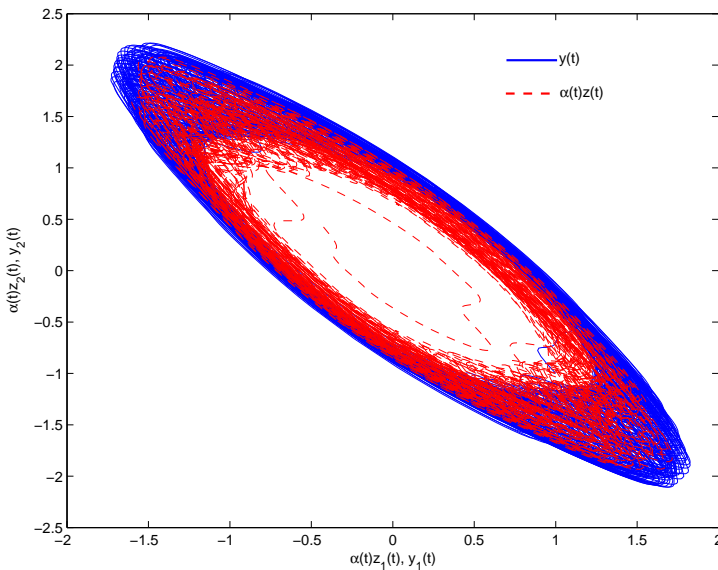


FIGURE 2. Chaotic behavior of drive system (1.1) and response system (1.2) with the time-varying scaling function  $\alpha(t) = 0.8 + 0.2 \sin \frac{0.5\pi t}{15}$ .

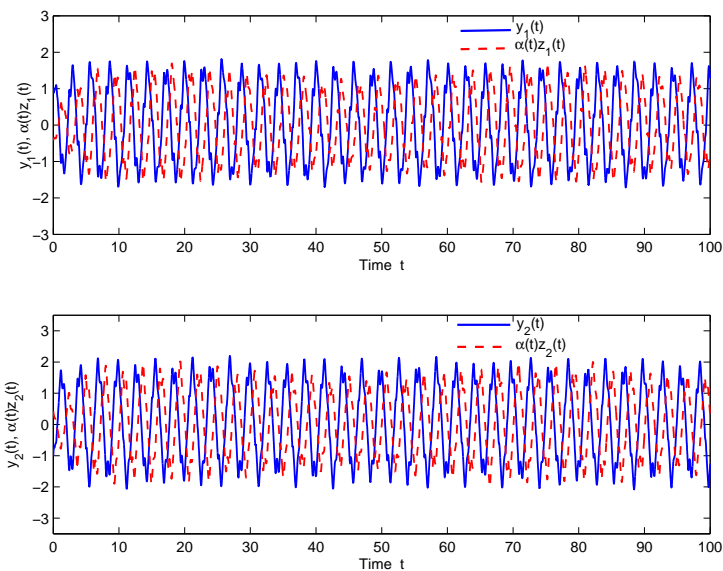


FIGURE 3. The trajectories of  $x_1(t)$  and  $x_2(t)$  of the drive-response neural networks.

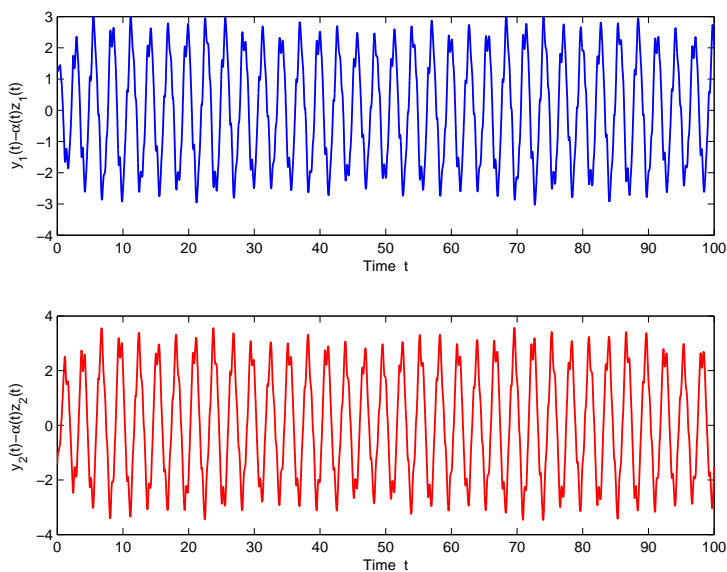


FIGURE 4. The function projective synchronization error trajectories of  $x_1(t)$ , and  $x_2(t)$  of the drive-response neural networks.

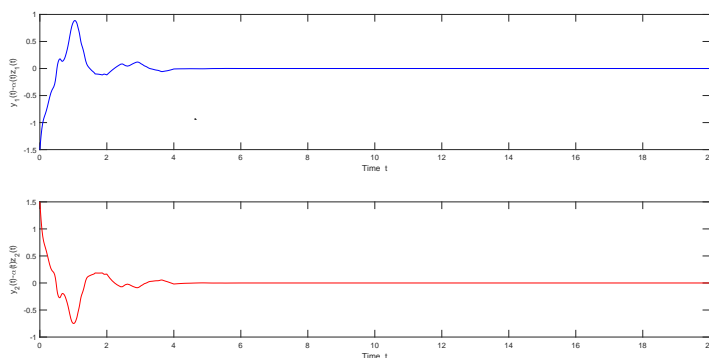


FIGURE 5. The function projective synchronization error trajectories of  $x_1(t)$ , and  $x_2(t)$  of the drive-response neural networks with intermittent control.

#### 4. CONCLUSIONS

The adaptive periodically intermittent control of exponential function projective synchronization of drive-response neural networks was investigated between discrete interval and distributed time-varying delays. Based on the improving piecewise Lyapunov-Krasovskii functionals combined with Leibniz-Newton's formula and applying the piecewise analysis method, adaptive periodically intermittent control and mathematical induction, some sufficient conditions for exponential function projective synchronization of drive-response neural systems were derived in terms of LMIs. Finally, a numerical simulation was provided to demonstrate the effectiveness of this proposed theoretical results.

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