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Dedicated to Prof. Suthep Suantai on the occasion of his 60^{th} anniversary

New Delay-Dependent Exponential Passivity Analysis of Neutral-Type Neural Networks with Discrete and Distributed Time-Varying Delays

Peerapongpat Singkibud[1,*], Narongsak Yotha[1] and Kanit Mukdasai[2]

[1] *Department of Applied Mathematics and Statistics, Faculty of Science and Liberal Arts, Rajamangala University of Technology Isan, Nakhon Ratchasima 30000, Thailand*
e-mail : peerapongpat.si@rmuti.ac.th (P. Singkibud); narongsak.yo@rmuti.ac.th (N. Yotha)

[2] *Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand*
e-mail : kanit@kku.ac.th (K. Mukdasai)

Abstract This work focuses on the problem of exponential passivity analysis for neutral-type neural networks with discrete and distributed time-varying delays by employing the mixed model transformation approach. The delays are discrete, neutral and distributed time-varying delays that the upper bounds for the time-varying delays are available. The restrictions on the derivatives of the distributed time-varying delays are removed, which mean that a fast distributed time-varying delay is allowed. Based on a appropriate Lyapunov-Krasovskii functional, application of zero equations and using various inequalities, such as the famous Jensen inequality, Wirtinger-based integral inequality, Peng-Park's integral inequality, etc. A novel delay-dependent criterion is established to ensure the exponential passivity of the systems considered. Moreover, the exponential passivity criterion is presented in terms of linear matrix inequalities (LMIs). Finally, numerical examples are given to show the superiority of the proposed method and capability of results over another research as compared with the least upper bounds of delay as well.

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1. Introduction

Since neural networks (NNs) appered. NNs have received extensive attention and have been applied successfully in many areas such as signal processing, pattern recognition,

*Corresponding author.

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associative memory and optimization problems [1]. Many scholars have paid their attentions to NNs which possess many advantages, including paralel computation, learning ability, function approximation, fault tolerance, etc. Most of these applications require that the equilibrium points of the designed network should be stable. So, it is important to study the stability of NNs. In reality, time-delay systems are frequently encountered in NNs, where a time delay is often a source of instability and oscillations. During the past few decades, both delay-independent and delay-dependent sufficient conditions have been proposed to verify the asymptotical or exponential stability of delay NNs and the references cited therein. Consequently, NNs with time delay has an important issue in control theory and has been extensively studied [1–10]. Moreover, the NNs containing the information of past state derivatives are called neutral-type [11–13] neural networks (NTNNs). Futhermore, neutral-type time-delay in the system models are usually encountered in many practical applications, such as population ecology, heat exchangers, water pipes, chemical reactors and robots in contact with rigid environments [14]. The existing work on the state estimator of NTNNs with mixed delays are only [15, 16] at present. Recently, study of NTNNs with delays has become one of impressive research topics and has been widely studied by many researchers. Therefore, it is necessary and important to investigate the NTNNs with delays.

The passivity theory plays an important role in the analysis of the stability of dynamical system, complexity [17], signal processing [18], chaos control, design of linear and nonlinear systems, especially for high-order systems [19]. In the first place, many systems need to be passive in order to attenuate noises effectively. In the second place, the robustness measure(such as robust stability or robust performace) of a system often reduces to a subsystem or a modified system that is passive. The essence of the passivity theory is that the passive properties of a system can keep the system internal stability. Thus, the passivity analysis approach has attracted a lot of research attentions [4–10, 20–24] and the references cited therein. Recently, the exponential passivity of NNs with time-varying delays has been studied in [2, 5, 6]. In the present, the passivity analysis have been studied several researchers [2, 25–27]. Moreover, The exponentially passivity condition for delayed NNs was obtained in [2]. In [26], the issue of robust passivity conditions for NNs with distributed and discrete delays has been extensively studied. Then, improved result on passivity analysis of NTNNs with time-varying delays [10] is presented. However, no result has been obtained for exponentially passive condition of NTNNs with discrete and distributed time-varying delays

Motivated by the above discussions, this paper involved with the analysis problem for the exponential passivity of NTNNs with discrete and distributed time-varying delays. By constructing novel augments Lyapunov-Krasovskii functional, using various inequalities, such as Jensen's inequality, Wirtinger-based integral inequality, Peng-Park's integral inequality, etc. Moreover, applying descriptor model transformation, Leibniz-Newton formula and application of zero equations. Then, a novel delay-dependent exponential passivity criterion for NTNNs with discrete and distributed time-varying delays is presented. As a result, a novel delay-dependent criterion is established in term of LMIs. Finally, three numerical examples are illustrated to show the usefulness of the proposed criteria.

2. Problem Formulation and Preliminaries

We introduce some notations and lemmas that will be used throughout the paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional space with the vector norm $\|\cdot\|$; $\|x\|$ denotes the Euclidean vector norm of $x \in R^n$; $R^{n\times r}$ denotes the set $n \times r$ real matrices; A^T denotes the transpose of the matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; matrix A is called semi-positive definite ($A \geq 0$) if $x^TAx \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $x^TAx > 0$ for all $x \neq 0$; matrix B is called semi-negative definite ($B \leq 0$) if $x^TBx \leq 0$, for all $x \in R^n$; B is negative definite ($B < 0$) if $x^TBx < 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$ ($B - A < 0$); $A \geq B$ means $A - B \geq 0$ ($B - A \leq 0$); $C([-\bar{h}, 0], R^n)$ denotes the space of all continuous vector functions mapping $[-\bar{h}, 0]$ into R^n when $\bar{h} = \max\{d_M, \rho_M, r_M\}$, $d_M, \rho_M, r_M \in R^+$; $x_t = x(t+s), s \in [-\bar{h}, 0]$; $*$ represents the elements below the main diagonal of a symmetric matrix.

Consider the following continuous NTNNs with mixed time-varying delays:

$$\begin{cases} \dot{\xi}(t) = -A\xi(t) + Wf(\xi(t)) + W_1 f(\xi(t - d(t))) + W_2 \int_{t-\rho(t)}^{t} f(\xi(s))ds \\ \qquad\quad +W_3\dot{\xi}(t - r(t)) + u(t), \\ z(t) = f(\xi(t)) + f(\xi(t - d(t))) + \dot{\xi}(t - r(t)) + u(t), \\ \xi(t) = \phi(t),\ t \in [-\tau_{\max}, 0],\ \tau_{\max} = \max\{d_M, \rho_M, r_M\}, \end{cases} \tag{2.1}$$

where $\xi(t) = [\xi_1(t), \xi_2(t), ..., \xi_n(t)] \in \mathcal{R}^n$ is the neural state vector. The diagonal matrix A is a self-feedback connection weight matrix. W, W_1, W_2 and W_3 are are the connection weight matrices between neurons with appropriate dimensions. $f(\cdot) = (f_1(\cdot), f_2(\cdot), \ldots, f_n(\cdot))^T$ represent the activation functions. $u(t)$ and $z(t)$ represent the input and output vectors, respectively; $\phi(t)$ is an initial condition. The variables $d(t)$ is the discrete time-varying delay, $\rho(t)$ is the distributed time-varying delay and $r(t)$ is the neutral time-varying delay are satisfying

$$0 \leq d(t) \leq d_M, \quad 0 \leq \dot{d}(t) \leq d_d, \tag{2.2}$$

$$0 \leq \rho(t) \leq \rho_M, \tag{2.3}$$

$$0 \leq r(t) \leq r_M, \quad 0 \leq \dot{r}(t) \leq r_d, \tag{2.4}$$

where d_M, ρ_M and r_M are positive real constants. The neural activation functions $f_k(\cdot), k = 1, 2, \cdots, n$ satisfy $f_k(0) = 0$ and for $s_1, s_2 \in \mathcal{R}, s_1 \neq s_2$,

$$l_k^- \leq \frac{f_k(s_1) - f_k(s_2)}{s_1 - s_2} \leq l_k^+, \tag{2.5}$$

where l_k^-, l_k^+, are known real scalars. Moreover, we denote $L^+ = \mathrm{diag}(l_1^+, l_2^+, \cdots, l_n^+)$, $L^- = \mathrm{diag}(l_1^-, l_2^-, \cdots, l_n^-)$.

Definition 2.1. [2] The neural networks are said to be exponential passive from input $u(t)$ to output $z(t)$, if there exist an exponential Lyapunov function (or, called the exponential storage function) V defined on R^n, and a constant $\beta > 0$ such that for all $u(t)$, all initial conditions $\xi(0)$, all $t \geq t_0$, the following inequality holds:

$$\dot{V}(t) + \beta V(t) \leq 2z^T(t)u(t), \quad t \geq 0, \tag{2.6}$$

where $\dot{V}(t)$ denotes the total derivative of $V(t)$ along the state trajectories $\xi(t)$, $t \geq 0$, of (2.1).

Lemma 2.2. *(Jensen's inequality) For any symmetric positive definite matrix Q, positive real number h, and vector function $\dot{x} : [-h, 0] \to R^n$ such that the following integral is well defined, then*

$$-h\int_{-h}^{0}\dot{x}^T(s+t)Q\dot{x}(s+t)ds \le -\Big(\int_{-h}^{0}\dot{x}(s+t)ds\Big)^T Q\Big(\int_{-h}^{0}\dot{x}(s+t)ds\Big).$$

Lemma 2.3. *(Wirtinger-based integral inequality)* [28] *For any matrix $Z > 0$, the following inequality holds for all continuously differentiable function $\dot{x} : [\alpha, \beta] \to R^n$*

$$-(\beta-\alpha)\int_{\alpha}^{\beta}\dot{x}^T(s)Z\dot{x}(s)ds \;\; \le \;\; \omega^T\Phi\omega,$$

where $\omega = [x^T(\beta), x^T(\alpha), \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta}x^T(s)ds]^T$ *and* $\Phi = \begin{bmatrix} -4Z & -2Z & 6Z \\ * & -4Z & 6Z \\ * & * & -12Z \end{bmatrix}.$

Lemma 2.4. *(Peng-Park's integral inequality)* [29, 30] *For any matrix $\begin{bmatrix} Z & S \\ * & Z \end{bmatrix} \ge 0$, positive scalars τ and $\tau(t)$ satisfying $0 < \tau(t) < \tau$, vector function $\dot{x} : [-\tau, 0] \to R^n$ such that the concerned integrations are well defined, then*

$$-\tau\int_{t-\tau}^{t}\dot{x}^T(s)Z\dot{x}(s)ds \;\; \le \;\; \xi^T\Theta\xi,$$

where $\xi = [x^T(t), x^T(t-\tau(t)), x^T(t-\tau)]^T$ *and* $\Theta = \begin{bmatrix} -Z & Z-S & S \\ * & -2Z+S+S^T & Z-S \\ * & * & -Z \end{bmatrix}.$

Lemma 2.5. [31] *For a positive matrix M, the following inequality holds:*

$$-\frac{(\alpha-\beta)^2}{2}\int_{\beta}^{\alpha}\int_{s}^{\alpha}x^T(u)Mx(u)duds \;\; \le \;\; -\Big(\int_{\beta}^{\alpha}\int_{s}^{\alpha}x(u)duds\Big)^T M\Big(\int_{\beta}^{\alpha}\int_{s}^{\alpha}x(u)duds\Big).$$

Lemma 2.6. [32] *For any constant symmetric positive definite matrix $Q \in R^{n\times n}$, $h(t)$ is discrete time-varying delays with (2.3), vector function $\omega : [-h_M, 0] \to R^n$ such that the integrations concerned are well defined, then*

$$\begin{aligned} & -h_M\int_{-h_M}^{0}\omega^T(s)Q\omega(s)ds \\ \le \; & -\int_{-h(t)}^{0}\omega^T(s)dsQ\int_{-h(t)}^{0}\omega(s)ds - \int_{-h_M}^{-h(t)}\omega^T(s)dsQ\int_{-h_M}^{-h(t)}\omega(s)ds. \end{aligned}$$

Lemma 2.7. [32] *For any constant matrices $Q_1, Q_2, Q_3 \in R^{n\times n}$, $Q_1 \ge 0, Q_3 > 0$, $\begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \ge 0$, $h(t)$ is discrete time-varying delays with (2.3) and vector function $\dot{x}$:*

$[-h_M, 0] \to R^n$ *such that the following integration is well defined, then*

$$-h_M \int_{t-h_M}^{t} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds$$
$$\leq \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \\ \int_{t-h(t)}^{t} x(s)ds \\ \int_{t-h_M}^{t-h(t)} x(s)ds \end{bmatrix}^T \begin{bmatrix} -Q_3 & Q_3 & 0 & -Q_2^T & 0 \\ * & -Q_3 - Q_3^T & Q_3 & Q_2^T & -Q_2^T \\ * & * & -Q_3 & 0 & Q_2^T \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \\ \int_{t-h(t)}^{t} x(s)ds \\ \int_{t-h_M}^{t-h(t)} x(s)ds \end{bmatrix}.$$

Lemma 2.8. [32] *Let $x(t) \in R^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any constant matrices $X, M_i \in R^{n\times n}, i = 1, 2, \ldots, 5$ and $h(t)$ is discrete time-varying delays with (2.3),*

$$-\int_{t-h_M}^{t} \dot{x}^T(s)X\dot{x}(s)ds$$
$$\leq \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}^T \begin{bmatrix} M_1 + M_1^T & -M_1^T + M_2 & 0 \\ * & M_1 + M_1^T - M_2 - M_2^T & -M_1^T + M_2 \\ * & * & -M_2 - M_2^T \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}$$
$$+h_M \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}^T \begin{bmatrix} M_3 & M_4 & 0 \\ * & M_3 + M_5 & M_4 \\ * & * & M_5 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix},$$

where

$$\begin{bmatrix} X & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \geq 0.$$

3. Main Results

In this section, we display our main results. We introduce the following notations for later use:

$$\sum = [\Omega_{(i,j)}]_{18\times 18}, \tag{3.1}$$

$$\begin{aligned}
\Omega_{(1,1)} &= \alpha P_1 + \alpha P_1^T - Q_2^T A - A^T Q_2 + Q_3 + Q_3^T + \alpha P_2 + \alpha P_2^T + P_3 + R_1 + R_4 \\
&\quad + e^{-2\alpha d_M}(M_1 + M_1^T) + d_M e^{-2\alpha d_M} M_3 - 4e^{-2\alpha d_M} P_6 - e^{-2\alpha d_M} P_7 + d_M^2 R_4 \\
&\quad - e^{-2\alpha d_M} R_6 + \frac{d_M^4}{4} P_9 - 2d_M^2 e^{-2\alpha d_M} P_{10} + d_M^2 P_4, \quad \Omega_{(1,2)} = P_1, \\
\Omega_{(1,3)} &= -A^T Q_5 - Q_3^T + Q_6 + Q_4^T + e^{-2\alpha d_M}(-M_1^T + M_2) + d_M e^{-2\alpha d_M} M_4 \\
&\quad + e^{-2\alpha d_M} P_7 - e^{-2\alpha d_M} S + e^{-2\alpha d_M} R_6, \\
\Omega_{(1,4)} &= -A^T Q_{11} + Q_{12} - Q_4^T - 2e^{-2\alpha d_M} P_6 + e^{-2\alpha d_M} S, \quad \Omega_{(1,5)} = Q_2^T W_3, \\
\Omega_{(1,5)} &= Q_2^T W_3, \quad \Omega_{(1,6)} = -\sqrt{2} d_M e^{-2\alpha d_M} P_{10}, \quad \Omega_{(1,7)} = -A^T Q_8 + Q_9 - e^{-2\alpha d_M} R_5^T,
\end{aligned}$$

$$\begin{aligned}
\Omega_{(1,8)} &= Q_1 - Q_2^T - A^T Q_{14} + Q_{15} + d_M^2 R_5, \quad \Omega_{(1,10)} = -Q_3^T, \\
\Omega_{(1,11)} &= 2\alpha C^T + Q_2^T W + R_2 + R_5 + A^T, \quad \Omega_{(1,12)} = Q_2^T W_1 + 2\alpha C^T, \\
\Omega_{(1,14)} &= Q_2^T W_2 + A^T, \quad \Omega_{(1,15)} = \frac{6}{d_M} e^{-2\alpha d_M} P_6, \quad \Omega_{(1,17)} = Q_2^T, \quad \Omega_{(1,18)} = -Q_4^T,
\end{aligned}$$

$$\begin{aligned}
\Omega_{(2,2)} &= -Q_1 - Q_1^T + P_{12} + 2K_1, \quad \Omega_{(2,8)} = Q_1 - Q_{17}^T + K_2 - K_1^T, \\
\Omega_{(2,11)} &= C^T + I, \quad \Omega_{(2,14)} = I, \\
\Omega_{(3,3)} &= -Q_6^T - Q_6 + Q_7^T + Q_7 - e^{-2\alpha d_M}(1-d_d)R_1 + e^{-2\alpha d_M}(M_1 + M_1^T - M_2 - M_2^T) \\
&\quad + d_M e^{-2\alpha d_M}(M_3 + M_5) - 2e^{-2\alpha d_M} P_7 + e^{-2\alpha d_M}(S + S^T) - e^{-2\alpha d_M}(R_6 + R_6^T), \\
\Omega_{(3,4)} &= -Q_{12} - Q_7^T + Q_{13} + e^{-2\alpha d_M}(-M_1^T + M_2) + d_M e^{-2\alpha d_M} M_4 \\
&\quad + e^{-2\alpha d_M} P_7 - e^{-2\alpha d_M} S + e^{-2\alpha d_M} R_6, \\
\Omega_{(3,5)} &= Q_5^T W_3, \quad \Omega_{(3,7)} = -Q_9 + Q_{10} + e^{-2\alpha d_M} R_5^T, \quad \Omega_{(3,8)} = -Q_5^T - Q_{15} + Q_{16}, \\
\Omega_{(3,9)} &= -e^{-2\alpha d_M} R_5^T, \quad \Omega_{(3,10)} = -Q_6^T, \quad \Omega_{(3,11)} = Q_5^T W, \\
\Omega_{(3,12)} &= Q_5^T W_1 - e^{-2\alpha d_M}(1-d_d)R_2, \quad \Omega_{(3,14)} = Q_5^T W_2, \\
\Omega_{(3,17)} &= Q_5^T, \quad \Omega_{(3,18)} = -Q_7^T, \\
\Omega_{(4,4)} &= -Q_{13}^T - Q_{13} - e^{-2\alpha d_M} P_3 - e^{-2\alpha d_M} R_4 + e^{-2\alpha d_M}(-M_2 - M_2^T) \\
&\quad + d_M e^{-2\alpha d_M} M_5 - 4e^{-2\alpha d_M} P_6 - e^{-2\alpha d_M} P_7 - e^{-2\alpha d_M} R_6, \\
\Omega_{(4,5)} &= Q_{11}^T W_3, \quad \Omega_{(4,7)} = -Q_{10}, \quad \Omega_{(4,8)} = -Q_{11}^T - Q_{16}, \quad \Omega_{(4,9)} = e^{-2\alpha d_M} R_5^T, \\
\Omega_{(4,10)} &= -Q_{12}^T, \quad \Omega_{(4,11)} = -Q_{11} W, \quad \Omega_{(4,12)} = Q_{11}^T W_1, \quad \Omega_{(4,13)} = -e^{-2\alpha d_M} R_5, \\
\Omega_{(4,14)} &= Q_{11}^T W_2, \quad \Omega_{(4,15)} = \frac{6}{d_M} e^{-2\alpha d_M} P_6, \quad \Omega_{(4,17)} = Q_{11}^T, \quad \Omega_{(4,18)} = -Q_{13}^T, \\
\Omega_{(5,5)} &= -e^{-2\alpha r_M} P_{12} + r_d P_{12}, \quad \Omega_{(5,7)} = W_3^T Q_8, \quad \Omega_{(5,8)} = W_3^T Q_{14}, \\
\Omega_{(5,11)} &= -W_3^T, \Omega_{(5,14)} = -W_3^T, \quad \Omega_{(5,17)} = -I, \quad \Omega_{(6,6)} = -e^{-2\alpha d_M} P_{10}, \\
\Omega_{(7,7)} &= -d_M e^{-2\alpha d_M} P_4 - e^{-2\alpha d_M} R_4, \quad \Omega_{(7,8)} = Q_8^T, \quad \Omega_{(7,10)} = -Q_9^T, \\
\Omega_{(7,11)} &= Q_8 W, \quad \Omega_{(7,12)} = Q_8^T W_1, \quad \Omega_{(7,14)} = Q_8^T W_2, \\
\Omega_{(7,17)} &= Q_8^T, \quad \Omega_{(7,18)} = -Q_{10}^T, \quad \Omega_{(8,8)} = -Q_{14}^T + Q_{14} + Q_{17} + Q_{17}^T + d_M P_5 \\
&\quad + d_M^2 P_6 + d_M^2 P_7 + d_M^2 R_6 + d_M^2 P_8 + \frac{d_M^4}{2} P_{10} - 2K_2, \\
\Omega_{(8,10)} &= -Q_{15}^T, \quad \Omega_{(8,11)} = Q_{14}^T W, \quad \Omega_{(8,12)} = Q_{14}^T W_1, \quad \Omega_{(8,14)} = Q_{14}^T W_2, \\
\Omega_{(8,17)} &= Q_{14}^T, \quad \Omega_{(8,18)} = -Q_{16}^T, \\
\Omega_{(9,9)} &= -d_M e^{-2\alpha d_M} P_4 - e^{-2\alpha d_M} R_4, \quad \Omega_{(10,10)} = -e^{-2\alpha d_M} P_8, \\
\Omega_{(10,18)} &= -e^{-2\alpha d_M} P_8, \quad \Omega_{(11,11)} = R_3 + R_6 - 2W + \rho_M^2 P_{11}, \quad \Omega_{(11,12)} = -W_1, \\
\Omega_{(11,14)} &= -W_2 - W^T, \quad \Omega_{(11,17)} = -2I, \quad \Omega_{(12,12)} = -e^{-2\alpha d_M} R_3(1-d_d), \\
\Omega_{(12,14)} &= -W_1^T, \quad \Omega_{(12,17)} = -I, \quad \Omega_{(13,13)} = -e^{-2\alpha d_M} R_6, \\
\Omega_{(14,14)} &= -2W_2 - e^{-2\alpha \rho_M} P_{11}, \quad \Omega_{(14,17)} = -I, \\
\Omega_{(15,15)} &= -\frac{12}{d_M^2} e^{-2\alpha d_M} P_6, \quad \Omega_{(16,16)} = -e^{-2\alpha d_M} P_9, \\
\Omega_{(17,17)} &= -2I, \quad \Omega_{(18,18)} = -e^{-2\alpha d_M} P_8,
\end{aligned}$$

and the other terms are 0.

Theorem 3.1. *The delayed NTNNs* (2.1) *are exponentially passive, if there exist positive definite matrices* Q_1, R_4, R_6 P_i, $i \in \{1, 2, ..., 12\}$, *any appropriate dimensional matrices* Q_m *and* $m = 1, 2, ..., 17$ *such that the following symmetric linear matrix inequalities hold*

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_7 & R_8 \\ * & R_9 \end{bmatrix} \geq 0, \tag{3.2}$$

$$\begin{bmatrix} P_7 & S \\ * & P_7 \end{bmatrix} \geq 0, \quad \begin{bmatrix} P_5 & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \geq 0, \tag{3.3}$$

$$\sum < 0. \tag{3.4}$$

Proof. From model transformation method, we rewrite the system (2.1) in the following system

$$\dot{\xi}(t) = y(t), \tag{3.5}$$

$$\begin{aligned} 0 &= -y(t) - A\xi(t) + Wf(\xi(t)) + W_1 f(\xi(t - d(t))) \\ &\quad + W_2 \int_{t-\rho(t)}^{t} f(\xi(s))ds + W_3 \dot{\xi}(t - r(t)) + u(t). \end{aligned} \tag{3.6}$$

Construct a Lyapunov-Krasovskii functional candidate for the system (3.5)-(3.6) of the form

$$V(t) = \sum_{i=1}^{10} V_i(t), \tag{3.7}$$

where

$$\begin{aligned} V_1(t) &= \xi^T(t)P_1\xi(t) + 2\sum_{i=1}^{N} c_i \int_0^{\xi_i(t)} f(s)ds, \\ V_2(t) &= \zeta^T(t)EP_2\zeta(t) + 2\sum_{i=1}^{N} c_i \int_0^{\xi_i(t)} f(s)ds, \\ V_3(t) &= \int_{t-d_M}^{t} e^{2\alpha(s-t)}\xi^T(s)P_3\xi(s)ds + \int_{t-d(t)}^{t} e^{2\alpha(s-t)} \begin{bmatrix} \xi(s) \\ f(\xi(s)) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \\ &\quad \times \begin{bmatrix} \xi(s) \\ f(\xi(s)) \end{bmatrix} ds + \int_{t-d_M}^{t} e^{2\alpha(s-t)} \begin{bmatrix} \xi(s) \\ f(\xi(s)) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} \begin{bmatrix} \xi(s) \\ f(\xi(s)) \end{bmatrix} ds, \\ V_4(t) &= d_M \int_{-d_M}^{0} \int_{t+s}^{t} e^{2\alpha(\theta-t)}\xi^T(\theta)P_4\xi(\theta)d\theta ds \\ &\quad + \int_{-d_M}^{0} \int_{t+s}^{t} e^{2\alpha(\theta-t)} y^T(\theta)P_5 y(\theta)d\theta ds, \end{aligned}$$

$$
\begin{aligned}
V_5(t) &= d_M \int_{-d_M}^{0} \int_{t+s}^{t} e^{2\alpha(\theta - t)} y^T(\theta) P_6 y(\theta) d\theta ds \\
&\quad + d_M \int_{-d_M}^{0} \int_{t+s}^{t} e^{2\alpha(\theta - t)} y^T(\theta) P_7 y(\theta) d\theta ds, \\
V_6(t) &= d_M \int_{-d_M}^{0} \int_{t+s}^{t} e^{2\alpha(\theta - t)} \begin{bmatrix} \xi(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} R_7 & R_8 \\ * & R_9 \end{bmatrix} \begin{bmatrix} \xi(\theta) \\ y(\theta) \end{bmatrix} d\theta ds, \\
V_7(t) &= d_M \int_{-d_M}^{0} \int_{t+s}^{t} e^{2\alpha(\theta - t)} y^T(\theta) P_8 y(\theta) d\theta ds, \\
V_8(t) &= \frac{d_M^2}{2} \int_{-d_M}^{0} \int_{\theta}^{0} \int_{t+s}^{t} e^{2\alpha(\theta + s - t)} \xi^T(\theta) P_9 \xi(\theta) d\theta ds d\lambda \\
&\quad + d_M^2 \int_{-d_M}^{0} \int_{\theta}^{0} \int_{t+s}^{t} e^{2\alpha(\theta + s - t)} y^T(\theta) P_{10} y(\theta) d\theta ds d\lambda, \\
V_9(t) &= -\rho_M \int_{-\rho_M}^{0} \int_{t+s}^{t} e^{2\alpha(\theta - t)} f(\xi(\theta))^T P_{11} f(\xi(\theta)) d\theta ds, \\
V_{10}(t) &= \int_{t-r(t)}^{t} e^{2\alpha(s-t)} \dot{\xi}^T(s) P_{12} \dot{\xi}(s) ds,
\end{aligned}
$$

with

$$
E = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 \\ Q_2 & Q_5 & Q_8 & Q_{11} & Q_{14} \\ Q_3 & Q_6 & Q_9 & Q_{12} & Q_{15} \\ Q_4 & Q_7 & Q_{10} & Q_{13} & Q_{16} \end{bmatrix}, \quad \zeta(t) = \begin{bmatrix} \xi(t) \\ \xi(t - d(t)) \\ \int_{t-d(t)}^{t} \xi(s) ds \\ \xi(t - d_M) \\ y(t) \end{bmatrix}.
$$

The time derivative of $V(t)$ along the trajectory of system (3.5)-(3.6) is given by

$$
\dot{V}(t) = \sum_{i=1}^{10} \dot{V}_i(t). \tag{3.8}
$$

The time derivative of $V_1(t)$ is calculated as

$$
\begin{aligned}
\dot{V}_1(t) &\leq 2\xi^T(t) P_1 \dot{\xi}(t) + 2f^T(\xi(t)) C \dot{\xi}(t) + 4\alpha f^T(\xi(t)) C \xi(t) + 2\alpha \xi^T(t) P_1 \xi(t) \\
&\quad - 2\alpha V_1(t).
\end{aligned} \tag{3.9}
$$

It is noted that $\zeta^T(t) E P_2 \zeta(t)$ is really $\xi^T(t) Q_1 \xi(t)$. Then the time derivative of $V_2(t)$ is calculated as

$$
\begin{aligned}
\dot{V}_2(t) &= 2\xi^T(t) Q_1 \dot{\xi}(t) + 2\dot{\xi}^T(t) Q_1 \big[-\dot{\xi}(t) + y(t) \big] + 4\alpha f^T(\xi(t)) C \xi(t) + 2\alpha \xi^T(t) P_2 \xi(t) \\
&\quad - 2\alpha V_2(t) \\
&= 2\zeta^T(t) P_2^T \begin{bmatrix} \dot{\xi}(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} + 2\dot{\xi}^T(t) Q_1 \big[-\dot{\xi}(t) + y(t) \big] + 4\alpha f^T(\xi(\theta)) C \xi(t) \\
&\quad + 2\alpha \xi^T(t) P_2 \xi(t) - 2\alpha V_2(t)
\end{aligned} \tag{3.10}
$$

$$
\begin{aligned}
&= 2\begin{bmatrix} \xi(t) \\ \xi(t-d(t)) \\ \int_{t-d(t)}^{t}\xi(s)ds \\ \xi(t-d_M) \\ y(t) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2^T & Q_3^T & Q_4^T \\ 0 & Q_5^T & Q_6^T & Q_7^T \\ 0 & Q_8^T & Q_9^T & Q_{10}^T \\ 0 & Q_{11}^T & Q_{12}^T & Q_{13}^T \\ 0 & Q_{14}^T & Q_{15}^T & Q_{16}^T \end{bmatrix} \begin{bmatrix} \dot{\xi}(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} + 2\dot{\xi}^T(t)Q_1\big[-\dot{\xi}(t)+y(t)\big] \\
&\quad +4\alpha f(\xi(\theta))^T C\xi(t) + 2\alpha\xi^T(t)P_2\xi(t) - 2\alpha V_2(t) \\
&= 2\xi^T(t)Q_1 y(t) + 2\dot{\xi}^T(t)Q_1\big[-\dot{\xi}(t)+y(t)\big] \\
&\quad +2\Big[\xi^T(t)Q_2^T + \xi^T(t-d(t))Q_5^T + (\int_{t-d(t)}^{t}\xi(s)ds)^T Q_8^T + \xi^T(t-d_M)Q_{11}^T \\
&\quad +y^T(t)Q_{14}^T\Big] \times \Big[-y(t) - A\xi(t) + Wf(\xi(t)) + W_1 f(\xi(t-d(t))) \\
&\quad +W_2\int_{t-\rho(t)}^{t} f(\xi(s))ds + W_3\dot{\xi}(t-r(t)) + u(t)\Big] \\
&\quad +2[\xi^T(t)Q_3^T + \xi^T(t-d(t))Q_6^T + \int_{t-d(t)}^{t}\xi(s)ds)^T Q_9^T + \xi^T(t-d_M)Q_{12}^T \\
&\quad +y^T(t)Q_{15}^T\Big] \times \Big[\xi(t) - \xi(t-d(t)) - \int_{t-d(t)}^{t} y(s)ds\Big] \\
&\quad +2\Big[\xi^T(t)Q_4^T + \xi^T(t-d(t))Q_7^T + (\int_{t-d(t)}^{t}\xi(s)ds)^T Q_{10}^T + \xi^T(t-d_M)Q_{13}^T \\
&\quad +y^T(t)Q_{16}^T\Big] \times \Big[\xi(t-d(t)) - \xi(t-d_M) - \int_{t-d_M}^{t-d(t)} y(s)ds\Big] \\
&\quad +2y^T(t)Q_{17}\big[-\dot{\xi}(t)+y(t)\big] + 2f^T(\xi(t))C\dot{\xi}(t) + 4\alpha f^T(\xi(t))C\xi(t) \\
&\quad +2\alpha\xi^T(t)P_2\xi(t) - 2\alpha V_2(t).
\end{aligned} \tag{3.11}
$$

Differentiating $V_3(t)$, we have

$$
\begin{aligned}
\dot{V}_3(t) &= \xi^T(t)P_3\xi(t) - e^{-2\alpha d_M}\xi^T(t-d_M)P_3\xi(t-d_M) \\
&\quad + \begin{bmatrix} \xi(t) \\ f(\xi(t)) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \begin{bmatrix} \xi(t) \\ f(\xi(t)) \end{bmatrix} \\
&\quad -e^{-2\alpha d_M}(1-d_d)\begin{bmatrix} \xi(t-d(t)) \\ f(\xi(t-d(t))) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \begin{bmatrix} \xi(t-d(t)) \\ f(\xi(t-d(t))) \end{bmatrix} \\
&\quad + \begin{bmatrix} \xi(t) \\ f(\xi(t)) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \begin{bmatrix} \xi(t) \\ f(\xi(t)) \end{bmatrix} \\
&\quad -e^{-2\alpha d_M}\begin{bmatrix} \xi(t-d_M) \\ f(\xi(t-d_M)) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \begin{bmatrix} \xi(t-d_M) \\ f(\xi(t-d_M)) \end{bmatrix} - 2\alpha V_3(t).
\end{aligned} \tag{3.12}
$$

Using Lemma 2.6 and Lemma 2.8, $V_4(t)$ is calculated as

$$
\begin{aligned}
\dot{V}_4(t) &= d_M\int_{-d_M}^{0}\Big(\xi^T(t)P_4\xi(t) - e^{2\alpha s}\xi^T(t+s)P_4\xi(t+s)\Big)ds \\
&\quad + \int_{-d_M}^{0}\Big(y^T(t)P_5 y(t) - e^{2\alpha s}y^T(t+s)P_5 y(t+s)\Big)ds - 2\alpha V_4(t)
\end{aligned} \tag{3.13}
$$

$$
\begin{aligned}
&= d_M\int_{-d_M}^{0}\xi^T(t)P_4\xi(t)ds - d_M\int_{-d_M}^{0}e^{2\alpha s}\xi^T(t+s)P_4\xi(t+s)ds \\
&\quad +\int_{-d_M}^{0}y^T(t)P_5y(t)ds - \int_{-d_M}^{0}e^{2\alpha s}y^T(t+s)P_5y(t+s)ds - 2\alpha V_4(t) \\
&= d_M^2\xi^T(t)P_4\xi(t) - d_Me^{-2\alpha d_M}\int_{t-d_M}^{t}\xi^T(s)P_4\xi(s)ds \\
&\quad +d_My^T(t)P_5y(t) - e^{-2\alpha d_M}\int_{t-d_M}^{t}\dot{\xi}^T(s)P_5\dot{\xi}(s)ds - 2\alpha V_4(t) \\
&\leq d_M^2\xi^T(t)P_4\xi(t) + d_My^T(t)P_5y(t) \\
&\quad -d_Me^{-2\alpha d_M}\int_{t-d(t)}^{t}\xi^T(s)dsP_4\int_{t-d(t)}^{t}\xi(s)ds \\
&\quad -d_Me^{-2\alpha d_M}\int_{t-d_M}^{t-d(t)}\xi^T(s)dsP_4\int_{t-d_M}^{t-d(t)}\xi(s)ds \\
&\quad +e^{-2\alpha d_M}\int_{t-d(t)}^{t}\begin{bmatrix}\xi(t)\\ \xi(t-d(t))\\ \xi(t-d_M)\end{bmatrix}^T \\
&\quad \times\begin{bmatrix}M_1+M_1^T & -M_1^T+M_2 & 0\\ -M_1+M_2^T & M_1+M_1^T-M_2-M_2^T & -M_1^T+M_2\\ 0 & -M_1+M_2^T & -M_2-M_2^T\end{bmatrix}\begin{bmatrix}\xi(t)\\ \xi(t-d(t))\\ \xi(t-d_M)\end{bmatrix} \\
&\quad +d_Me^{-2\alpha d_M}\int_{t-d(t)}^{t}\begin{bmatrix}\xi(t)\\ \xi(t-d(t))\\ \xi(t-d_M)\end{bmatrix}^T\begin{bmatrix}M_3 & M_4 & 0\\ M_4^T & M_3+M_5 & M_4\\ 0 & M_4^T & M_5\end{bmatrix}\begin{bmatrix}\xi(t)\\ \xi(t-d(t))\\ \xi(t-d_M)\end{bmatrix} \\
&\quad -2\alpha V_4(t). \qquad (3.14)
\end{aligned}
$$

Using Lemma 2.3 (Wirtinger-base integral inequality) and Lemma 2.4 (Peng-Park's integral inequality), an upper bound of $V_5(t)$ can be obtained as

$$
\begin{aligned}
\dot{V}_5(t) &= d_M^2y^T(t)P_6y(t) - d_M\int_{t-d_M}^{t}e^{2\alpha(s-t)}\dot{\xi}^T(s)P_6\dot{\xi}(s)ds - 2\alpha V_5(t) \\
&\quad +d_M^2y^T(t)P_7y(t) - d_M\int_{t-d_M}^{t}e^{2\alpha(s-t)}\dot{\xi}^T(s)P_7\dot{\xi}(s)ds - 2\alpha V_5(t) \\
&\leq d_M^2y^T(t)P_6y(t) + d_M^2y^T(t)P_7y(t) \\
&\quad +e^{-2\alpha d_M}\begin{bmatrix}\xi^T(t) & \xi^T(t-d_M) & \frac{1}{d_M}\int_{t-d_M}^{t}\xi^T(s)ds\end{bmatrix}\begin{bmatrix}-4P_6 & -2P_6 & 6P_6\\ -2P_6^T & -4P_6 & 6P_6\\ 6P_6^T & 6P_6^T & -12P_6\end{bmatrix} \\
&\quad \times\begin{bmatrix}\xi(t)\\ \xi(t-d_M)\\ \frac{1}{d_M}\int_{t-d_M}^{t}\xi(s)ds\end{bmatrix} + e^{-2\alpha d_M}\begin{bmatrix}\xi^T(t) & \xi^T(t-d(t)) & \xi^T(t-d_M)\end{bmatrix} \\
&\quad \times\begin{bmatrix}-P_7 & P_7-S & S\\ P_7^T-S^T & -2P_7+S+S^T & P_7-S\\ S^T & P_7^T-S^T & -P_7\end{bmatrix}\begin{bmatrix}\xi(t)\\ \xi(t-d(t))\\ \xi(t-d_M)\end{bmatrix} - 2\alpha V_5(t). \qquad (3.15)
\end{aligned}
$$

It is from Lemma 2.7 that we have

$$\begin{aligned}
\dot{V}_6(t) &= d_M\int_{-d_M}^{0}\Big(\begin{bmatrix}\xi(t)\\ y(t)\end{bmatrix}^T\begin{bmatrix}R_4 & R_5\\ R_5^T & R_6\end{bmatrix}\begin{bmatrix}\xi(t)\\ y(t)\end{bmatrix}\\
&\quad -e^{2\alpha s}\begin{bmatrix}\xi(t+s)\\ y(t+s)\end{bmatrix}^T\begin{bmatrix}R_4 & R_5\\ R_5^T & R_6\end{bmatrix}\begin{bmatrix}\xi(t+s)\\ y(t+s)\end{bmatrix}\Big)ds-2\alpha V_6(t)\\
&= d_M^2\begin{bmatrix}\xi(t)\\ y(t)\end{bmatrix}^T\begin{bmatrix}R_4 & R_5\\ R_5^T & R_6\end{bmatrix}\begin{bmatrix}\xi(t)\\ y(t)\end{bmatrix}\\
&\quad -d_M\int_{t-d_M}^{t}e^{2\alpha(s-t)}\begin{bmatrix}\xi(s)\\ y(s)\end{bmatrix}^T\begin{bmatrix}R_4 & R_5\\ R_5^T & R_6\end{bmatrix}\begin{bmatrix}\xi(s)\\ y(s)\end{bmatrix}ds-2\alpha V_6(t)\\
&\le d_M^2\begin{bmatrix}\xi(t)\\ y(t)\end{bmatrix}^T\begin{bmatrix}R_4 & R_5\\ R_5^T & R_6\end{bmatrix}\begin{bmatrix}\xi(t)\\ y(t)\end{bmatrix}\\
&\quad +e^{-2\alpha d_M}\begin{bmatrix}\xi(t)\\ \xi(t-d(t))\\ \xi(t-d_M)\\ \int_{t-d(t)}^{t}\xi(s)ds\\ \int_{t-d_M}^{t-d(t)}\xi(s)ds\end{bmatrix}^T\begin{bmatrix}-R_6 & R_6 & 0 & -R_5^T & 0\\ R_6^T & -R_6-R_6^T & R_6 & R_5^T & -R_5^T\\ 0 & R_6^T & -R_6 & 0 & R_5^T\\ -R_5 & R_5 & 0 & -R_4 & 0\\ 0 & -R_5 & R_5 & 0 & -R_4\end{bmatrix}\\
&\quad \times\begin{bmatrix}\xi(t)\\ \xi(t-d(t))\\ \xi(t-d_M)\\ \int_{t-d(t)}^{t}\xi(s)ds\\ \int_{t-d_M}^{t-h(t)}\xi(s)ds\end{bmatrix}-2\alpha V_6(t). \qquad (3.16)
\end{aligned}$$

Using Lemma 2.2 (Jensen's inequality) that we have

$$\begin{aligned}
\dot{V}_7(t) &\le d_M^2 y^T(t)P_8y(t)-e^{-2\alpha d_M}\int_{t-d_M}^{t}y^T(s)dsP_8\int_{t-d_M}^{t}y(s)ds-2\alpha V_7(t)\\
&\le d_M^2 y^T(t)P_8y(t)\\
&\quad -\Big[\int_{t-d(t)}^{t}y^T(s)ds+\int_{t-d_M}^{t-d(t)}y^T(s)ds\Big]e^{-2\alpha d_M}P_8\Big[\int_{t-d(t)}^{t}y(s)ds\\
&\quad +\int_{t-d_M}^{t-d(t)}y(s)ds\Big]-2\alpha V_7(t) \qquad (3.17)
\end{aligned}$$

By Lemma 2.5, we can obtain $\dot{V}_8(t)$ as follows

$$\begin{aligned}
\dot{V}_8(t) &= \frac{d_M^2}{2}\Big(\frac{d_M^2}{2}\xi^T(t)P_9\xi(t)-\int_{t-d_M}^{t}\int_{u}^{t}e^{2\alpha(\theta+s-t)}\xi^T(\lambda)P_9\xi(\lambda)d\lambda du\Big)\\
&\quad +d_M^2\Big(\frac{d_M^2}{2}y^T(t)P_{10}y(t)-\int_{t-d_M}^{t}\int_{u}^{t}e^{2\alpha(\theta+s-t)}\dot{\xi}^T(\lambda)P_{10}\dot{\xi}(\lambda)d\lambda du\Big)-2\alpha V_8(t)\\
&= \frac{d_M^4}{4}\xi^T(t)P_9\xi(t)-\frac{d_M^2}{2}\int_{t-d_M}^{t}\int_{u}^{t}e^{2\alpha(\theta+s-t)}\xi^T(\lambda)P_9\xi(\lambda)d\lambda du\\
&\quad +\frac{d_M^4}{2}y^T(t)P_{10}y(t)-d_M^2\int_{t-d_M}^{t}\int_{u}^{t}e^{2\alpha(\theta+s-t)}\dot{\xi}^T(\lambda)P_{10}\dot{\xi}(\lambda)d\lambda du-2\alpha V_8(t)
\end{aligned}$$

$$
\begin{aligned}
\leq\ & \frac{d_M^4}{4}\xi^T(t)P_9\xi(t) - e^{-2\alpha d_M}\int_{t-d_M}^{t}\int_{u}^{t}\xi^T(\lambda)d\lambda du P_9\int_{t-h_M}^{t}\int_{u}^{t}\xi(\lambda)d\lambda du \\
& -2\alpha V_8(t) \\
& +\frac{d_M^4}{2}y^T(t)P_{10}y(t) - 2e^{-2\alpha d_M}\int_{t-d_M}^{t}\int_{u}^{t}\dot{\xi}^T(\lambda)d\lambda du P_{10}\int_{t-h_M}^{t}\int_{u}^{t}\dot{\xi}(\lambda)d\lambda du \\
=\ & \frac{d_M^4}{4}\xi^T(t)P_9\xi(t) + \frac{d_M^4}{2}y^T(t)P_{10}y(t) \\
& -e^{-2\alpha d_M}\int_{t-d_M}^{t}\int_{u}^{t}\xi^T(\lambda)d\lambda du P_9\int_{t-d_M}^{t}\int_{u}^{t}\xi(\lambda)d\lambda du \\
& -e^{-2\alpha d_M}\Big[\sqrt{2}d_M\xi^T(t) - \sqrt{2}\int_{t-d_M}^{t}\xi^T(u)du\Big]P_{10}\Big[\sqrt{2}d_M\xi(t) \\
& -\sqrt{2}\int_{t-d_M}^{t}\xi(u)du\Big] - 2\alpha V_8(t). \qquad (3.18)
\end{aligned}
$$

Calculating $\dot{V}_9(t)$ leads to

$$
\begin{aligned}
\dot{V}_9(t)\ \leq\ & \rho_M^2 f(\xi^T(t))P_{11}f(\xi(t)) \\
& -e^{-2\alpha\rho_M}\int_{t-\rho(t)}^{t}f(\xi^T(s))dsP_{11}\int_{t-\rho(t)}^{t}f(\xi(s))ds - 2\alpha V_9(t). \qquad (3.19)
\end{aligned}
$$

Taking the time derivative of $V_{10}(t)$, we obtain

$$
\begin{aligned}
\dot{V}_{10}(t)\ =\ & \dot{\xi}^T(t)P_{12}\dot{\xi}(t) - (1-\dot{r}(t))e^{-2\alpha r(t)}\dot{\xi}^T(t-r(t))P_{12}\dot{\xi}(t-r(t)) - 2\alpha V_{10}(t) \\
\leq\ & \dot{\xi}^T(t)P_{12}\dot{\xi}(t) - e^{-2\alpha r_M}\dot{\xi}^T(t-r(t))P_{12}\dot{\xi}(t-r(t)) \\
& +r_d\dot{\xi}^T(t-r(t))P_{12}\dot{\xi}(t-r(t)) - 2\alpha V_{10}(t). \qquad (3.20)
\end{aligned}
$$

From (2.5), we obtain for any positive real constants ϵ_1 and ϵ_2,

$$
\begin{bmatrix}\xi(t)\\ f(\xi(t))\end{bmatrix}^T\begin{bmatrix}-2H_1\epsilon_1 & H_1\epsilon_2\\ \epsilon_2^T H_1^T & -2H_1\end{bmatrix}\begin{bmatrix}\xi(t)\\ f(\xi(t))\end{bmatrix}\geq 0, \qquad (3.21)
$$

$$
\begin{bmatrix}\xi(t-d(t))\\ f(\xi(t-d(t)))\end{bmatrix}^T\begin{bmatrix}-2H_2\epsilon_1 & H_2\epsilon_2\\ \epsilon_2^T H_2^T & -2H_2\end{bmatrix}\begin{bmatrix}\xi(t-d(t))\\ f(\xi(t-d(t)))\end{bmatrix}\geq 0. \qquad (3.22)
$$

From (2.1), we have

$$
[\dot{\xi}(t) - y^T(t)]\times[2K_1\dot{\xi}(t) + 2K_2y(t)]) = 0, \qquad (3.23)
$$

$$
\begin{aligned}
& 2f(\xi(t))\times[\dot{\xi}(t) + A\xi(t) - Wf(\xi(t)) - W_1f(\xi(t-d(t))) - W_2\int_{t-\rho(t)}^{t}f(\xi(s))ds \\
& -W_3\dot{\xi}(t-r(t)) - u(t)] = 0, \qquad (3.24)
\end{aligned}
$$

$$
\begin{aligned}
& 2\int_{t-\rho(t)}^{t}f(\xi(s))ds\times[\dot{\xi}(t) + A\xi(t) - Wf(\xi(t)) - W_1f(\xi(t-d(t))) \\
& -W_2\int_{t-\rho(t)}^{t}f(\xi(s))ds - W_3\dot{\xi}(t-r(t)) - u(t)] = 0. \qquad (3.25)
\end{aligned}
$$

By utilization of zero equation, the following equations are true for any real constant matrices K_i, $i = 3, 4$ with appropriate dimensions

$$\begin{aligned}&\left[2K_3 f^T(\xi(t)) + 2K_4 u^T(t)\right] \times \\ &[-\dot{\xi}(t) - A\xi(t) + Wf(\xi(t)) + W_1 f(\xi(t-d(t))) + W_2 \int_{t-\rho(t)}^{t} f(\xi(s))ds \\ &+W_3\dot{\xi}(t-r(t)) + u(t)] = 0. \end{aligned} \tag{3.26}$$

According to (3.9)-(3.26) with (2.1), it is straightforward to see that

$$\dot{V}(t) + 2\alpha V(t) - 2z^T(t)u(t) \leq \eta^T(t) \sum \eta(t) \tag{3.27}$$

where

$$\begin{aligned}\eta(t) \quad = \quad & col \Bigg\{ \xi(t),\ \dot{\xi}(t),\ \xi(t-d(t)),\ \xi(t-d_M),\ \dot{\xi}(t-r(t)),\ \sqrt{2}\int_{t-d_M}^{t-d(t)} \xi(u)du, \\ & \int_{t-d(t)}^{t} \xi(s)ds,\ \ y(t),\ \int_{t-d_M}^{t-d(t)} \xi(s)ds,\ \int_{t-d(t)}^{t} y(s)ds,\ f(\xi(t)),\ f(\xi(t-d(t))), \\ & f(\xi(t-d_M)),\ \int_{t-\rho(t)}^{t} f(\xi(s))ds,\ \int_{t-d_M}^{t} \xi(s)ds,\ \int_{t-d_M}^{t}\int_{u}^{t} \xi(\lambda)d\lambda du,\ u(t), \\ & \int_{t-d_M}^{t-d(t)} y(s)du \Bigg\}.\end{aligned}$$

Since $\sum$ is negative definite and the conditions (3.2)-(3.3) hold, then

$$\dot{V}(t) + 2\alpha V(t) \leq 2z^T(t)u(t),\ \forall t \in \mathbb{R}^+. \tag{3.28}$$

Therefore, system (2.1) is exponentially passive from Definition (2.1). The proof of theorem is complete. ∎

Now the system (2.1) when $u(t) = 0$ and $z(t) \equiv 0$ are presented. We define a new parameter

$$\widehat{\sum} = \left[\hat{\Omega}_{(i,j)}\right]_{17\times 17}, \tag{3.29}$$

$$
\begin{aligned}
\hat{\Omega}_{(1,1)} &= \alpha P_1 + \alpha P_1^T - Q_2^T A - A^T Q_2 + Q_3 + Q_3^T + \alpha P_2 + \alpha P_2^T + P_3 + R_1 + R_4 \\
&\quad + e^{-2\alpha d_M}(M_1 + M_1^T) + d_M e^{-2\alpha d_M} M_3 - 4e^{-2\alpha d_M} P_6 - e^{-2\alpha d_M} P_7 + d_M^2 R_4 \\
&\quad - e^{-2\alpha d_M} R_6 + \frac{d_M^4}{4} P_9 - 2d_M^2 e^{-2\alpha d_M} P_{10} + d_M^2 P_4, \\
\hat{\Omega}_{(1,2)} &= P_1, \quad \hat{\Omega}_{(1,3)} = -A^T Q_5 - Q_3^T + Q_6 + Q_4^T + e^{-2\alpha d_M}(-M_1^T + M_2) \\
&\quad + d_M e^{-2\alpha d_M} M_4 + e^{-2\alpha d_M} P_7 - e^{-2\alpha d_M} S + e^{-2\alpha d_M} R_6, \\
\hat{\Omega}_{(1,4)} &= -A^T Q_{11} + Q_{12} - Q_4^T - 2e^{-2\alpha d_M} P_6 + e^{-2\alpha d_M} S, \quad \hat{\Omega}_{(1,5)} = Q_2^T W_3, \\
\hat{\Omega}_{(1,6)} &= -\sqrt{2} d_M e^{-2\alpha d_M} P_{10}, \quad \hat{\Omega}_{(1,7)} = -A^T Q_8 + Q_9 - e^{-2\alpha d_M} R_5^T, \\
\hat{\Omega}_{(1,8)} &= Q_1 - Q_2^T - A^T Q_{14} + Q_{15} + d_M^2 R_5, \quad \hat{\Omega}_{(1,10)} = -Q_3^T, \\
\hat{\Omega}_{(1,11)} &= 2\alpha C^T + Q_2^T W + R_2 + R_5 + A^T, \quad \hat{\Omega}_{(1,12)} = Q_2^T W_1 + 2\alpha C^T, \\
\hat{\Omega}_{(1,14)} &= Q_2^T W_2 + A^T, \quad \hat{\Omega}_{(1,15)} = \frac{6}{d_M} e^{-2\alpha d_M} P_6, \quad \hat{\Omega}_{(1,17)} = -Q_4^T, \\
\hat{\Omega}_{(2,2)} &= -Q_1 - Q_1^T + P_{12} + 2K_1, \quad \hat{\Omega}_{(2,8)} = Q_1 - Q_{17}^T + K_2 - K_1^T, \\
\hat{\Omega}_{(2,11)} &= C^T + I, \quad \hat{\Omega}_{(2,14)} = I, \\
\hat{\Omega}_{(3,3)} &= -Q_6^T - Q_6 + Q_7^T + Q_7 - e^{-2\alpha d_M}(1 - d_d) R_1 + e^{-2\alpha d_M}(M_1 + M_1^T - M_2 - M_2^T) \\
&\quad + d_M e^{-2\alpha d_M}(M_3 + M_5) - 2e^{-2\alpha d_M} P_7 + e^{-2\alpha d_M}(S + S^T) - e^{-2\alpha d_M}(R_6 + R_6^T), \\
\hat{\Omega}_{(3,4)} &= -Q_{12} - Q_7^T + Q_{13} + e^{-2\alpha d_M}(-M_1^T + M_2) + d_M e^{-2\alpha d_M} M_4 + e^{-2\alpha d_M} P_7 \\
&\quad - e^{-2\alpha d_M} S + e^{-2\alpha d_M} R_6, \\
\hat{\Omega}_{(3,5)} &= Q_5^T W_3, \quad \hat{\Omega}_{(3,7)} = -Q_9 + Q_{10} + e^{-2\alpha d_M} R_5^T, \\
\hat{\Omega}_{(3,8)} &= -Q_5^T - Q_{15} + Q_{16}, \quad \hat{\Omega}_{(3,9)} = -e^{-2\alpha d_M} R_5^T, \quad \hat{\Omega}_{(3,10)} = -Q_6^T, \\
\hat{\Omega}_{(3,11)} &= Q_5^T W, \quad \hat{\Omega}_{(3,12)} = Q_5^T W_1 - e^{-2\alpha d_M}(1 - d_d) R_2, \\
\hat{\Omega}_{(3,14)} &= Q_5^T W_2, \quad \hat{\Omega}_{(3,17)} = -Q_7^T, \\
\hat{\Omega}_{(4,4)} &= -Q_{13}^T - Q_{13} - e^{-2\alpha d_M} P_3 - e^{-2\alpha d_M} R_4 + e^{-2\alpha d_M}(-M_2 - M_2^T) \\
&\quad + d_M e^{-2\alpha d_M} M_5 - 4e^{-2\alpha d_M} P_6 - e^{-2\alpha d_M} P_7 - e^{-2\alpha d_M} R_6, \\
\hat{\Omega}_{(4,5)} &= Q_{11}^T W_3, \quad \hat{\Omega}_{(4,7)} = -Q_{10}, \quad \hat{\Omega}_{(4,8)} = -Q_{11}^T - Q_{16}, \quad \hat{\Omega}_{(4,9)} = e^{-2\alpha d_M} R_5^T, \\
\hat{\Omega}_{(4,10)} &= -Q_{12}^T, \quad \hat{\Omega}_{(4,11)} = -Q_{11} W, \quad \hat{\Omega}_{(4,12)} = Q_{11}^T W_1, \quad \hat{\Omega}_{(4,13)} = -e^{-2\alpha d_M} R_5, \\
\hat{\Omega}_{(4,14)} &= Q_{11}^T W_2, \quad \hat{\Omega}_{(4,15)} = \frac{6}{d_M} e^{-2\alpha d_M} P_6, \quad \hat{\Omega}_{(4,17)} = -Q_{13}^T, \\
\hat{\Omega}_{(5,5)} &= -e^{-2\alpha r_M} P_{12} + r_d P_{12}, \quad \hat{\Omega}_{(5,7)} = W_3^T Q_8, \quad \hat{\Omega}_{(5,8)} = W_3^T Q_{14}, \\
\hat{\Omega}_{(5,11)} &= -W_3^T, \quad \hat{\Omega}_{(5,14)} = -W_3^T, \quad \hat{\Omega}_{(6,6)} = -e^{-2\alpha d_M} P_{10}, \\
\hat{\Omega}_{(7,7)} &= -d_M e^{-2\alpha d_M} P_4 - e^{-2\alpha d_M} R_4, \\
\hat{\Omega}_{(7,8)} &= Q_8^T, \quad \hat{\Omega}_{(7,10)} = -Q_9^T, \quad \hat{\Omega}_{(7,11)} = Q_8 W, \\
\hat{\Omega}_{(7,12)} &= Q_8^T W_1, \quad \hat{\Omega}_{(7,14)} = Q_8^T W_2, \\
\hat{\Omega}_{(7,17)} &= -Q_{10}^T, \quad \hat{\Omega}_{(8,8)} = -Q_{14}^T + Q_{14} + Q_{17} + Q_{17}^T + d_M P_5 + d_M^2 P_6 + d_M^2 P_7 \\
&\quad + d_M^2 R_6 + d_M^2 P_8 + \frac{d_M^4}{2} P_{10} - 2K_2,
\end{aligned}
$$

$$\begin{aligned}
\hat{\Omega}_{(8,10)} &= -Q_{15}^T, \quad \hat{\Omega}_{(8,11)} = Q_{14}^T W, \quad \hat{\Omega}_{(8,12)} = Q_{14}^T W_1, \quad \hat{\Omega}_{(8,14)} = Q_{14}^T W_2, \\
\hat{\Omega}_{(8,17)} &= -Q_{16}^T, \quad \hat{\Omega}_{(9,9)} = -d_M e^{-2\alpha d_M} P_4 - e^{-2\alpha d_M} R_4, \quad \hat{\Omega}_{(10,10)} = -e^{-2\alpha d_M} P_8, \\
\hat{\Omega}_{(10,17)} &= -e^{-2\alpha d_M} P_8, \quad \hat{\Omega}_{(11,11)} = R_3 + R_6 - 2W + \rho_M^2 P_{11}, \quad \hat{\Omega}_{(11,12)} = -W_1, \\
\hat{\Omega}_{(11,14)} &= -W_2 - W^T, \quad \hat{\Omega}_{(12,12)} = -e^{-2\alpha d_M} R_3(1-d_d), \quad \hat{\Omega}_{(12,14)} = -W_1^T, \\
\hat{\Omega}_{(13,13)} &= -e^{-2\alpha d_M} R_6, \quad \hat{\Omega}_{(14,14)} = -2W_2 - e^{-2\alpha \rho_M} P_{11}, \\
\hat{\Omega}_{(15,15)} &= -\frac{12}{d_M^2} e^{-2\alpha d_M} P_6, \quad \hat{\Omega}_{(16,16)} = -e^{-2\alpha d_M} P_9, \quad \hat{\Omega}_{(17,17)} = -e^{-2\alpha d_M} P_8,
\end{aligned}$$

and the other terms are 0.

Corollary 3.2. *The delayed NTNNs (2.1) with $u(t) = 0$ and $z(t) \equiv 0$ are exponential stability, if there exist positive definite matrices Q_1, R_4, R_6 P_i, $i \in \{1, 2, ..., 12\}$, any appropriate dimensional matrices Q_m and $m = 1, 2, ..., 17$ such that the following symmetric linear matrix inequalities hold*

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_7 & R_8 \\ * & R_9 \end{bmatrix} \geq 0, \tag{3.30}$$

$$\begin{bmatrix} P_7 & S \\ * & P_7 \end{bmatrix} \geq 0, \quad \begin{bmatrix} P_5 & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \geq 0, \tag{3.31}$$

$$\widehat{\sum} < 0. \tag{3.32}$$

4. Numerical Examples

Example 4.1. Consider the following NTNNs with discrete and distributed time-varying (2.1). We consider exponential passivity of system (2.1) by using Theorem 3.1. The system (2.1) is specified as follow:

$$\begin{aligned}
A &= \begin{bmatrix} 2 & 0 \\ 0 & 3.5 \end{bmatrix}, \quad W = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1 \end{bmatrix}, \quad W_1 = \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \\
W_2 &= \begin{bmatrix} -0.2 & 0 \\ 0 & -0.2 \end{bmatrix}, \quad W_3 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\epsilon_1^- &= \epsilon_2^- = -0.1, \quad \epsilon_1^+ = \epsilon_2^+ = 0.5, \quad \epsilon_1 = 0.5, \quad \epsilon_2 = 0.6, \\
d(t) &= |\cos(t)|, \quad \rho(t) = \cos^2(0.5t), \quad r(t) = \sin^2(0.6t), \quad \phi(t) = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, t \in [-1, 0].
\end{aligned}$$

It can be seen that α=0.1, d_M=0.5, d_d=0.3, ρ_M=0.1, r_M=0.2 and r_d=0.6. By using LMI Toolbox in MATLAB, we use (3.2)-(3.3) in Theorem 3.1. This example shows that the solutions of LMIs are given as follows:

$$
\begin{aligned}
P_1 &= 10^7 \times \begin{bmatrix} 3.5519 & -0.1404 \\ -0.1404 & 3.9082 \end{bmatrix}, \quad P_2 = 10^7 \times \begin{bmatrix} 9.7581 & -0.2916 \\ -0.2916 & 9.9934 \end{bmatrix}, \\
P_3 &= 10^7 \times \begin{bmatrix} 9.1721 & -0.8127 \\ -0.8127 & 9.6510 \end{bmatrix}, \quad P_4 = 10^7 \times \begin{bmatrix} 8.5699 & -0.1276 \\ -0.1276 & 8.7457 \end{bmatrix}, \\
P_5 &= 10^7 \times \begin{bmatrix} 1.8712 & 0.1883 \\ 0.1883 & 1.4839 \end{bmatrix}, \quad P_6 = 10^6 \times \begin{bmatrix} 4.6315 & -0.1781 \\ -0.1781 & 4.8429 \end{bmatrix}, \\
P_7 &= 10^7 \times \begin{bmatrix} 3.4789 & 0.2827 \\ 0.2827 & 3.0053 \end{bmatrix}, \quad P_8 = 10^7 \times \begin{bmatrix} 2.70803 & 0.1675 \\ 0.1675 & 1.8440 \end{bmatrix}, \\
P_9 &= 10^8 \times \begin{bmatrix} 1.0420 & -0.0013 \\ -0.0013 & 1.0430 \end{bmatrix}, \quad P_{10} = 10^7 \times \begin{bmatrix} 6.7326 & 0.1965 \\ 0.1965 & 6.3537 \end{bmatrix}, \\
P_{11} &= 10^8 \times \begin{bmatrix} 1.0980 & 0.0135 \\ 0.0135 & 1.0908 \end{bmatrix}, \quad P_{12} = 10^7 \times \begin{bmatrix} 1.6175 & 0.1818 \\ 0.1818 & 1.2890 \end{bmatrix}, \\
R_1 &= 10^8 \times \begin{bmatrix} 0.9724 & -0.0068 \\ -0.0068 & 1.2022 \end{bmatrix}, \quad R_2 = 10^7 \times \begin{bmatrix} -3.7603 & -0.7745 \\ -0.7745 & -5.5326 \end{bmatrix}, \\
R_3 &= 10^8 \times \begin{bmatrix} 0.9709 & 0.0557 \\ 0.0557 & 1.0034 \end{bmatrix}, \quad R_4 = 10^7 \times \begin{bmatrix} 8.5227 & -0.2556 \\ -0.2556 & 8.8240 \end{bmatrix}, \\
R_5 &= 10^7 \times \begin{bmatrix} -1.0405 & -0.2097 \\ -0.2097 & -1.1793 \end{bmatrix}, \quad R_6 = 10^7 \times \begin{bmatrix} 3.5334 & 0.2353 \\ 0.2353 & 3.0431 \end{bmatrix}, \\
R_7 &= 10^7 \times \begin{bmatrix} 9.8891 & 0 \\ 0 & 9.8891 \end{bmatrix}, \quad R_8 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad R_9 = 10^7 \times \begin{bmatrix} 9.8891 & 0 \\ 0 & 9.98891 \end{bmatrix}, \\
Q_1 &= 10^8 \times \begin{bmatrix} 1.0165 & 0.1465 \\ 0.1465 & 1.3396 \end{bmatrix}, \quad Q_2 = 10^8 \times \begin{bmatrix} 1.0718 & 0.0270 \\ 0.0270 & 0.8474 \end{bmatrix}, \\
Q_3 &= 10^6 \times \begin{bmatrix} 4.6723 & -2.6712 \\ -2.6712 & 3.0289 \end{bmatrix}, Q_4 = 10^6 \times \begin{bmatrix} 4.6723 & -2.6712 \\ -2.6712 & 3.0289 \end{bmatrix}, \\
Q_5 &= 10^7 \times \begin{bmatrix} 1.7435 & 0.2196 \\ 0.2196 & 2.0436 \end{bmatrix}, \quad Q_6 = 10^6 \times \begin{bmatrix} 5.2395 & 0.0850 \\ 0.0850 & 4.7565 \end{bmatrix}, \\
Q_7 &= 10^6 \times \begin{bmatrix} 5.2395 & 0.0850 \\ 0.0850 & 4.7565 \end{bmatrix}, \quad Q_8 = 10^6 \times \begin{bmatrix} 2.0029 & 0.9240 \\ 0.9240 & 2.2569 \end{bmatrix}, \\
Q_9 &= 10^5 \times \begin{bmatrix} -6.3908 & 0.5473 \\ 0.5473 & -1.7135 \end{bmatrix}, \quad Q_{10} = 10^5 \times \begin{bmatrix} -6.3908 & 0.5473 \\ 0.5473 & -1.7135 \end{bmatrix}, \\
Q_{11} &= 10^6 \times \begin{bmatrix} 7.1846 & 2.6367 \\ 2.6367 & -5.2159 \end{bmatrix}, \quad Q_{12} = 10^6 \times \begin{bmatrix} -8.6050 & 0.5348 \\ 0.5348 & 0.2678 \end{bmatrix}, \\
Q_{13} &= 10^6 \times \begin{bmatrix} -8.6050 & 0.5348 \\ 0.5348 & -5.2159 \end{bmatrix}, \quad Q_{14} = 10^7 \times \begin{bmatrix} 5.0385 & 0.3796 \\ 0.3796 & 4.1523 \end{bmatrix}, \\
Q_{15} &= 10^5 \times \begin{bmatrix} -3.7693 & 1.5629 \\ 1.5629 & 1.0577 \end{bmatrix}, \quad Q_{16} = 10^5 \times \begin{bmatrix} -3.7693 & 1.5629 \\ 1.5629 & 1.0577 \end{bmatrix}, \\
Q_{17} &= 10^7 \times \begin{bmatrix} -2.8374 & 0.4131 \\ 0.4131 & -2.4522 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
M_3 &= 10^8 \times \begin{bmatrix} 1.0212 & -0.0436 \\ -0.0436 & 1.0503 \end{bmatrix}, \quad M_4 = 10^7 \times \begin{bmatrix} -2.4579 & -0.5579 \\ -0.5579 & -1.8902 \end{bmatrix},
\end{aligned}
$$

$$M_5 = 10^8 \times \begin{bmatrix} 1.0513 & -0.0061 \\ -0.0061 & 1.0638 \end{bmatrix}, \quad S = 10^6 \times \begin{bmatrix} 3.5607 & 1.4465 \\ 1.4465 & 2.4570 \end{bmatrix},$$

$$C = 10^7 \times \begin{bmatrix} 1.2510 & -2.9252 \\ -2.9252 & 0.1721 \end{bmatrix}, \quad H_1 = 10^8 \times \begin{bmatrix} 3.2629 & -0.2210 \\ -0.2210 & 2.9761 \end{bmatrix},$$

$$H_2 = 10^7 \times \begin{bmatrix} 7.3534 & -1.1825 \\ -1.1825 & 8.6257 \end{bmatrix}, \quad K_1 = 10^7 \times \begin{bmatrix} 2.7519 & 2.1237 \\ 2.1237 & 7.0642 \end{bmatrix},$$

$$K_2 = 10^7 \times \begin{bmatrix} 2.8374 & -0.4131 \\ -0.4131 & 2.4522 \end{bmatrix}, \quad K_3 = 10^7 \times \begin{bmatrix} -9.0197 & -0.7793 \\ -0.7793 & -6.8279 \end{bmatrix},$$

$$K_4 = 10^6 \times \begin{bmatrix} -4.5184 & 0.7955 \\ 0.7955 & -1.8073 \end{bmatrix},$$

Example 4.2. We focus on system (2.1) with W_3=0, and $\dot{\xi}(t - r(t)) \equiv 0$, that means neural networks system with discrete and distributed time-varying delay:

$$\begin{cases} \dot{\xi}(t) = -A\xi(t) + Wf(\xi(t)) + W_1 f(\xi(t - d(t))) + W_2 \int_{t-\rho(t)}^{t} f(\xi(s))ds + u(t), \\ z(t) = f(\xi(t)) + f(\xi(t - d(t))) + u(t), \\ \xi(t) = \phi(t), \ t \in [-\tau_{\max}, 0], \ \tau_{\max} = \max\{d_M, \rho_M\}, \end{cases} \tag{4.1}$$

with the parameters

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3.5 \end{bmatrix}, \ W = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1 \end{bmatrix}, \ W_1 = \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 2 & 0 \\ 0 & 3.5 \end{bmatrix}, \ \epsilon_1^- = \epsilon_2^- = -0.1, \quad \epsilon_1^+ = \epsilon_2^+ = 0.5,$$

$$\text{for } d(t) = 0.1 + \frac{\sin^2(t)}{3}, \quad \rho(t) = 0.3 + \frac{|\cos(t)|}{3}.$$

In this example, we interested in the exponential passivity for system (4.1). Table 1 provides the calculated allowable upper bound d_M.

Table 1. Calculated delay upper bound d_M for fixed $\rho_M = 0.7$ and different d_d and α of Example 4.2.

d_d	$\alpha = 0.1$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
0	0.1023	0.0050	0.2121	1.2020
0.1	1.0021	0.0211	1.0010	1.1010
0.3	0.0022	0.0201	0.0210	1.0201
0.5	0.0030	0.1110	0.1401	0.0230

Example 4.3. Consider the system (2.1) with $W_2 = W_3$=0 and $u(t) = z(t) \equiv 0$, that means neural networks system with time-varying delay:

$$\begin{cases} \dot{\xi}(t) = -A\xi(t) + Wf(\xi(t)) + W_1 f(\xi(t - d(t))), \\ \xi(t) = \phi(t), \ t \in [-d_M, 0], \end{cases} \tag{4.2}$$

with the parameters

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3.5 \end{bmatrix}, \ W_1 = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1 \end{bmatrix}, \ W_2 = \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix},$$

$$\epsilon_1^- = \epsilon_2^- = 0, \quad \epsilon_1^+ = \epsilon_2^+ = 1.$$

Table 2 lists the comparison of exponential convergence rates of system (4.2) by different methods. It is clear that our results are superior to those in [33–36].

Table 2. Allowable exponential convergence rate α for various d_d and $d_m = 1$ of Example 4.3.

Method	$d_d = 0.8$	$d_d = 0.9$	Unknown d_d
Wu (2008) [33]	0.8643	0.8344	0.8169
Ji (2014) [34]	0.8696	0.8354	0.8169
Ji (2015) [35]	0.8784	0.8484	0.8217
He (2016) [36]	0.8841	0.8570	0.8260
Theorem 3.1	1.0214	1.2010	1.1011

5. Conclusions

In this paper, we propose the delay-dependent exponentially passive conditions for NTNNs with discrete and distributed time-varying delays by using descriptor model transformation, new class of augmented Lyapunov-Krasovskii functional, Leibniz-Newton formula, improved integral inequalities, utilization of zero equation, Wirtinger-based integral inequality, and Peng-Park's integral inequality. Then, we represented the delay-dependent exponential passivity criterion for NTNNs with time-varying delays. Moreover, we obtained exponential stability criterion for considered system. Finally, three numerical results verified the improvement and effectiveness of the proposed exponential passivity criteria.

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