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Majorization Results on Multivalently Meromorphic Functions Involving Wright Hypergeometric Functions

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Abstract The main object of this present paper is to investigate the problem of majorization of certain class of meromorphic starlike functions of complex order. Moreover we point out some new or known consequences of our main result.

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1. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are *analytic* in the open unit disc

$$\mathbb{U} = \{z : |z| < 1\}.$$

Denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting functions of the form (1.1) that are *normalized* by

$$f(0) = 0 = f'(0) - 1$$

and are *univalent* in $\mathbb{U}$.

If f and g are analytic functions in $\mathbb{U} = \{z : |z| < 1\}$, following MacGregor [1], we say that f is majorized by g in $\mathbb{U}$ that is $f(z) \ll g(z), (z \in \mathbb{U})$ if there exists a function $\phi(z)$, analytic in $\mathbb{U}$, such that

$$|\phi(z)| < 1 \text{ and } f(z) = \phi(z)g(z),\ z \in \mathbb{U}.$$

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It is interested to note that the notation of majorization is closely related to the concept of quasi-subordination between analytic functions.

For two analytic functions $f, g \in \mathcal{A}$ we say that f is subordinate to g denoted by $f \prec g$ if there exists a Schwar'z function $\omega(z)$ which is analytic in $\mathbb{U}$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ for all $z \in \mathbb{U}$, such that $f(z) = g(\omega(z))$ and $z \in \mathbb{U}$.

Note that, if the function g is univalent in $\mathbb{U}$, due to Miller and Mocanu [2] we have

$$f(z) \prec g(z) \Longleftrightarrow f(0) = g(0) \text{ and } f(\mathbb{U}) \subset g(\mathbb{U}).$$

Denote by $\mathcal{S}^*(\gamma)$ and $\mathcal{C}(\gamma)$ the class of starlike and convex functions of complex order $\gamma(\gamma \in \mathbb{C} \setminus \{0\}$, satisfying the following conditions

$$\frac{f(z)}{z} \neq 0 \text{ and } \Re\left(1 + \frac{1}{\gamma}\left[\frac{zf'(z))}{f(z)} - 1\right]\right) > 0$$

and

$$f'(z) \neq 0 \text{ and } \Re\left(1 + \frac{1}{\gamma}\left[\frac{zf''(z))}{f'(z)}\right]\right) > 0, (z \in \mathbb{U})$$

respectively. Further,

$$\mathcal{S}^*((1-\alpha)cos\lambda\ e^{-i\lambda}) = \mathcal{S}^*(\alpha, \lambda),\ |\lambda| < \frac{\pi}{2};\ 0 \leq \alpha \leq 1$$

and

$$\mathcal{S}^*(cos\lambda\ e^{-i\lambda}) = \mathcal{S}^*(\lambda),\ |\lambda| < \frac{\pi}{2};\ 0 \leq \alpha \leq 1$$

where denotes $\mathcal{S}^*(\alpha, \lambda)$ the class of $\lambda-$ Spiral-like function of order α investigated by Libera [3] and $\mathcal{S}^*(\lambda)$ the class of Spiral-like functions introduced by Spacek [4] (see [5]).

We recall the Wright generalized hypergeometric function [6]

$$\begin{aligned} &{}_l\Psi_s[(\alpha_1, A_1), ..., (\alpha_l, A_l); (\beta_1, B_1), ..., (\beta_s, B_s); z] \\ &= {}_l\Psi_s[(\alpha_m, A_m)_{1,l}\,; (\beta_m, B_m)_{1,s}\,; z] \end{aligned}$$

is defined by

$$\begin{aligned} &{}_l\Psi_s[(\alpha_m, A_m)_{1,l}\,; (\beta_m, B_m)_{1,s}\,; z] \\ &= \sum_{n=0}^{\infty}\left(\prod_{m=1}^{l}\Gamma(\alpha_m + nA_m)\right)\left(\prod_{m=1}^{s}\Gamma(\beta_m + nB_m)\right)^{-1}\frac{z^n}{n!}, \quad (z \in \mathbb{U}). \end{aligned} \tag{1.2}$$

If $A_m = 1(m = 1, ..., l)$ and $B_m = 1(m = 1, .., s)$, we have the relationship

$$\Omega\ {}_l\Psi_s[(\alpha_{n,1})_{1,l}\ ;\ (\beta_{n,1})_{1,s}\ ;\ z]\ =\ {}_lF_s(\alpha_1, ..., \alpha_l; \beta_1, ..., \beta_s; z), \tag{1.3}$$

where, ${}_lF_s(\alpha_1, ..., \alpha_l\,; \beta_1, ..., \beta_s\,; z)$ is the generalized hypergeometric function and

$$\Omega = \left(\prod_{m=1}^{l}\Gamma(\alpha_m)\right)^{-1}\left(\prod_{m=1}^{s}\Gamma(\beta_m)\right). \tag{1.4}$$

Let $\mathcal{M}_p$ be the class of meromorphic functions which are analytic in the punctured open unit disk $\mathbb{U}^* = \{z : z \in \mathbb{C} : 0 < |z| < 1\}$ of the form

$$f(z) = \frac{1}{z^p} + \sum_{n=1}^{\infty} a_n z^{n-p}. \tag{1.5}$$

For given $g(z) = \frac{1}{z^p} + \sum\limits_{n=1}^{\infty} b_n z^{n-p} \in \mathcal{M}_p$, the Hadamard product of f and g is denoted by

$$(f * g)(z) = \frac{1}{z^p} + \sum_{n=1}^{\infty} a_n b_n z^{n-p} = (g * f)(z), \tag{1.6}$$

note that $f * g \in \mathcal{M}_p$ which are meromorphic in the open disc $\mathbb{U}^*$.

Recently, following Dziok and Raina [7] Murugusundaramoorthy and Aouf [8] defined a linear operator for $p-$valently meromorphic functions as

$$\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z) = \mathcal{W}_p[(\alpha_1, A_1), ..., (\alpha_l, A_l); (\beta_1, B_1), ..., (\beta_s, B_s)] : \mathcal{M}_p \to \mathcal{M}_p \tag{1.7}$$

defined by the Hadamard product

$$\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z) := \mathcal{W}_p[(\alpha_m, A_m)_{l,s}; (\beta_m, B_m)_{l,s}; z] * f(z). \tag{1.8}$$

If $f \in \mathcal{M}_p$ and is given by (1.5), then we have

$$\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z) = \frac{1}{z^p} + \Omega \sum_{n=1}^{\infty} \frac{\prod_{m=1}^{l} \Gamma(\alpha_m + nA_m)}{\prod_{m=1}^{s} \Gamma(\beta_m + nB_m) n!} \, a_n z^{n-p}, \quad z \in \mathbb{U}^*, \tag{1.9}$$

where Ω is given in (1.4). It is easy to verify that

$$zA_1(\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z))' = \alpha_1 \mathcal{W}_p^{l,s}[\alpha_1 + 1, A_1]f(z) - (\alpha_1 + pA_1)\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z). \tag{1.10}$$

From (1.10), we have the following recurrence relation for the operator $\mathcal{W}_p^{l,s}$

$$\begin{aligned} &zA_1(\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z))^{j+1} \\ &= \alpha_1(\mathcal{W}_p^{l,s}[\alpha_1 + 1, A_1]f(z))^j - (\alpha_1 + A_1(j + p))(\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z))^j. \end{aligned} \tag{1.11}$$

In particular, for $A_m = B_m = 1$ $(m = 1, ..., l, m = 1, ..., s)$, we get the linear operator

$$\mathcal{H}_p^{l,s}[\alpha_1]f(z) = \frac{1}{z^p} + \sum_{n=1}^{\infty} \frac{\prod_{m=1}^{l} (\alpha_m)_n}{\prod_{m=1}^{s} (\beta_m)_n n!} a_n z^{n-p} \tag{1.12}$$

introduced and studied by Liu and Srivastava [9]. It is easy to verify from (1.12) that

$$z(\mathcal{H}_p^{l,s}[\alpha_1]f(z))' = \alpha_1 \mathcal{H}_p^{l,s}[\alpha_1 + 1]f(z) - (\alpha_1 + p)\mathcal{H}_p^{l,s}[\alpha_1]f(z). \tag{1.13}$$

It is of interest to note that for the suitable choices of l, s in turn it includes various operators (also see [10–12]).Obviously, for $l = 2, s = 1, \alpha_2 = 1$ and $p = 1$, we get

$$\mathcal{L}[\alpha_1, \beta_1]f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{(\alpha_1)_n}{(\beta_1)_n} a_n z^{n-1}. \tag{1.14}$$

Majorization problems for the class $\mathcal{S}^* = \mathcal{S}^*(0)$ had been investigated by MacGregor [1], further Altintas et al. [13] investigated a majorization problem for the class $\mathcal{S}(\gamma)$ and recently using linear operators by [14]. Very recently Goyal and Goswami [15] extended these results for meromorphic functions . In the present paper we introduce a new subclass of $p-$valently meromorphic starlike functions of complex order associated with Wright hypergeometric functions given by (1.9) and investigate a majorization problem for the function class .

2. Majorization Problem for the Class $M_{p,j}^{l,s}(\gamma)$

Definition 2.1. A function $f(z) \in \mathcal{M}_p$ is said to in the class $M_{p,j}^{l,s}(\gamma)$ of univalent function of complex order $\gamma \neq 0$ in $\mathbb{U}^*$ if and only if

$$\Re\left(1 - \frac{1}{\gamma}\left[\frac{z(\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z))^{j+1}}{\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z)^j} + j + p\right]\right) > 0 \tag{2.1}$$

where $z \in \mathbb{U}^*, j \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, l \leq s+1, \gamma \in \mathbb{C} \setminus \{0\}$.

Theorem 2.2. *Let $f(z) \in \mathcal{M}_p$ and $g(z) \in M_{p,j}^{l,s}(\gamma)$ if $(\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z))^j$ is majorized by $(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^j$ in $\mathbb{U}^*$ then*

$$|(\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z))^j| \leq |(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^j|, \quad |z| \leq r_1, \tag{2.2}$$

where $r_1 = r_1(A, B, \alpha_1, A_1, \gamma, \rho)$ is the smallest root of the equation

$$|\alpha_1 B - A_1\gamma(A-B)|r^3 - [\alpha_1 + 2\rho A_1|B|]r^2 - [|\alpha_1 B - A_1\gamma(A-B)| + 2\rho A_1]r + \alpha_1 = 0. \tag{2.3}$$

Proof. Let

$$h(z) = 1 - \frac{1}{\gamma}\left[\frac{z(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^{j+1}}{\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z)^j} + j + p\right]. \tag{2.4}$$

Since $g(z) \in M_{p,j}^{l,s}(\gamma)$, we have $\Re(h(z)) > 0$ and

$$h(z) = \frac{1 + A\omega(z)}{1 + B\omega(z)}, \tag{2.5}$$

where

$$\omega(z) = c_1 z + c_2 z^2 + ...$$

and w denotes the well known class of bounded analytic functions in $\mathbb{U}^*$ and satisfies the conditions $\omega(0) = 0$, and $|\omega(z)| \leq |z|$, $(z \in \mathbb{U}^*)$ making use of (2.4) and (2.5), we get

$$\frac{z(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^{j+1}}{(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^j} = -\frac{(B(j+p) + \gamma(A-B))(w(z)) + (j+p)}{1 + B(z)}. \tag{2.6}$$

Hence

$$|(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^j| \leq \frac{A_1(1 + |B||z|)}{\alpha_1 - |\alpha_1 B - (A_1\gamma(A-B)||z|}\left|\frac{\alpha_1}{A_1}(\mathcal{W}_p^{l,s}[\alpha_1 + 1, A_1]g(z))^j\right|. \tag{2.7}$$

Since $(\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z))^j$ is majorized by $(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^j$ in $\mathbb{U}^*$, then

$$(\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z))^j = \phi(z)(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^j \tag{2.8}$$

and

$$z((\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z))^{j+1} = z\phi'(z)(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^j + z\phi(z)(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^{j+1}.$$

Noting that the Schwarz function $\phi(z)$ satisfies

$$\left|\phi'(z)\right| \leq \frac{1 - |\phi(z)|^2}{1 - |z|^2} \tag{2.9}$$

and using (1.10), (2.7) and (2.9) in (2.8) we have

$$\left|\frac{\alpha_1}{A_1}(\mathcal{W}_p^{l,s}[\alpha_1+1,A_1]f(z))^j\right| \leq \left(|\phi(z)| + \tfrac{(1-|\phi(z)|^2)}{(1-|z|^2)}\tfrac{A_1(1+|B||z|)}{\alpha_1-|\alpha_1 B-(A_1\gamma(A-B)||z|}\right) \times \left|\frac{\alpha_1}{A_1}(\mathcal{W}_p^{l,s}[\alpha_1+1,A_1]g(z))^j\right|.$$

Setting $|z|=r$ and $|\phi(z)|=\rho$, $0\leq\rho\leq 1$

$$\left|\frac{\alpha_1}{A_1}(\mathcal{W}_p^{l,s}[\alpha_1+1,A_1]f(z))^j\right| \leq \frac{\psi(\rho)|\frac{\alpha_1}{A_1}(\mathcal{W}_p^{l,s}[\alpha_1+1,A_1]g(z))^j|}{(1-r^2)(\alpha_1-|\alpha_1 B-(A_1\gamma(A-B)|r)}, \tag{2.10}$$

where

$$\psi(\rho)=\rho(1-r^2)(\alpha_1-|\alpha_1 B-(A_1\gamma(A-B)|r)+(1-\rho^2)A_1(1+|B|r)r$$

takes its maximum value at $\rho=1$. Furthermore, if $0\leq\sigma\leq r_1$, the function $\varphi(\rho)$ defined by

$$\varphi(\rho)=\rho(1-\sigma^2)(\alpha_1-|\alpha_1 B-(A_1\gamma(A-B)|\sigma)+(1-\rho^2)A_1(1+|B|\sigma)\sigma$$

is an increasing function if

$$(1-\sigma^2)(\alpha_1-|\alpha_1 B-(A_1\gamma(A-B)|\sigma)>2\rho A_1(1+|B|\sigma)\sigma$$

$(0\leq\rho\leq 1)$, so that

$$\varphi(\rho)\leq\varphi(1)=(1-\sigma^2)(\alpha_1-|\alpha_1 B-(A_1\gamma(A-B)|\sigma),\ 0\leq\rho\leq 1,\ 0\leq\sigma\leq r_1. \tag{2.11}$$

Therefore, from this fact (2.10) gives the inequality (2.2). This completes the proof of Theorem 2.2. ∎

3. Consequences and Corollaries

By taking $A=1$ and $B=-1$ and $\rho=1$ in Theorem 2.2, we state the following corollary without proof.

Corollary 3.1. *Let the function $f\in\mathcal{M}_p$ and $g(z)\in M_{p,j}^{l,s}(\gamma)$ if $(\mathcal{W}_p^{l,s}[\alpha_1,A_1]f(z))^j$ is majorized by $(\mathcal{W}_p^{l,s}[\alpha_1,A_1]g(z))^j$ in $\mathbb{U}$ then*

$$|(\mathcal{W}_p^{l,s}[\alpha_1,A_1]f(z))^j|\leq|(\mathcal{W}_p^{l,s}[\alpha_1,A_1]g(z))^j|,\quad |z|\leq r_1,$$

where $r_1=r_1(\alpha_1,A_1,\gamma)$ is the smallest positive root of the equation

$$\{|\alpha_1+2A_1\gamma|\}r^3-\{\alpha_1+2A_1\}r^2-\{|\alpha_1+2A_1\gamma|+2A_1\}r+\alpha_1=0,$$

$$r_1=\frac{L_1-\sqrt{L_1^2-4\alpha_1|\alpha_1+2A_1\gamma|}}{2|\alpha_1+2A_1\gamma|} \tag{3.1}$$

and $L_1=\alpha_1+2A_1+|\alpha_1+2A_1\gamma|$.

Since, $\mathcal{W}_p^{l,s}[1,1]f(z)=f(z)$ from Corollary 3.1, we state the following corollary.

Corollary 3.2. *Let the function $f\in\mathcal{M}_p$ and $g(z)\in M_{p,j}(\gamma)$ if $(f(z))^{(j)}$ is majorized by $(g(z))^{(j)}$ in $\mathbb{U}^*$ then*

$$|(f(z))^{(j)}|\leq|(g(z))^{(j)}|,\quad |z|\leq r_2,$$

where $r_2 = r_2(1, 1, \gamma)$ *is the smallest positive root of the equation*

$$\{|1 + 2\gamma|\}r^3 - 3r^2 - \{|1 + 2\gamma| + 2\}r + 1 = 0,$$

$$r_2 = \frac{L_2 - \sqrt{L_2^2 - 4|1 + 2\gamma|}}{2|1 + 2\gamma|} \tag{3.2}$$

and $L_2 = 3 + |1 + 2\gamma|$.

By setting $\alpha_1 = 1, A_1 = 1$ and $\gamma = p - \delta$ in Corollary 3.1, we state the following corollary.

Corollary 3.3. *Let the function* $f \in \mathcal{M}_p$ *and* $g(z) \in M_{p,j}^{l,s}(\delta)$ *if* $(f(z))^{(j)}$ *is majorized by* $(g(z))^{(j)}$ *in* $\mathbb{U}^*$, *then*

$$|(f(z))^{(j)}| \leq |(g(z))^{(j)}|, \quad |z| \leq r_3,$$

where $r_3 = r_3(1, 1, (p - \delta))$ *is the smallest positive root of the equation*

$$|1 + 2(p - \delta)|r^3 - 1 + 2(p - \delta)r^2 - \{|1 + 2(p - \delta)| + 2\}r + 1 = 0,$$

$$r_3 = \frac{L_3 - \sqrt{L_3^2 - 4|1 + 2(p - \delta)|}}{2|1 + 2(p - \delta)|} \tag{3.3}$$

and $L_3 = 3 + |1 + 2(p - \delta)|$.

By taking $j = 1$, Corollary 3.3 yields results of Goyal and Gosami [15].
By taking $\gamma = (p - \delta)cos\ \lambda e^{-i\lambda}$ $((|\lambda| < \frac{\pi}{2}, \delta(0 \leq \delta < p),)$ in Corollary 3.1, we state the following corollary without proof.

Corollary 3.4. *Let* $f \in \mathcal{M}_p$ *and* $g(z) \in M_{p,j}^{l,s}(\lambda, \delta)$ *if* $(\mathcal{W}_p^{l,s}[\alpha_1, A_1]f(z))^{(j)}$ *is majorized by* $(\mathcal{W}_p^{l,s}[\alpha_1, A_1]g(z))^{(j)}$ *in* $\mathbb{U}^*$, *then*

$$|(\mathcal{W}_p^{l,s}f(z))^{(j)}| \leq |(\mathcal{W}_p^{l,s}g(z))^{(j)}|, \ |z| \leq r_4, \tag{3.4}$$

where $r_4 = r_4(L_4, \lambda)$ *is given by*

$$r_4 = \frac{L_4 - \sqrt{L_4^2 - 4\alpha_1|\alpha_1 + 2A_1(p - \delta)cos\lambda\ e^{-i\lambda}}}{2|\alpha_1 + 2A_1(p - \delta)cos\lambda\ e^{-i\lambda}|} \tag{3.5}$$

and

$$L_4 = (\alpha_1 + 2A_1) + |\alpha_1 + 2A_1(p - \delta)cos\lambda e^{-i\lambda}|,$$

the smallest positive root of the equation

$$|\alpha_1 + 2A_1(p - \delta)cos\lambda\ e^{-i\lambda}|r^3 - \alpha_1 + 2A_1r^2 - |\alpha_1 + 2A_1(p - \delta)cos\lambda\ e^{-i\lambda}| + 2A_1r + \alpha_1 = 0.$$

Concluding Remarks

Further specializing the parameters l, s one can define the various other interesting subclasses of $\mathcal{M}_p$ involving the various differential operators (see [9–12]) and the corresponding corollaries as mentioned above can be derived easily.

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References

[1] T.H. MacGregor, Majorization by univalent functions, Duke Math. J. 34 (1967) 95–102.

[2] S.S. Miller, P.T. Mocanu, Differential Subordinations, Monographs and Textbooks in Pure and Applied Mathematics 225, Dekker, New York, 2000.

[3] R.J. Libera, Univalent α spiral-like functions, Canadian. J. Math. Soc. 19 (1967) 449–456.

[4] L. Špaček, Příspěvek k teorii funkci prostýčh, Časopis Pěst Math. 63 (1933) 12–19.

[5] H.M. Srivastava, S. Owa, A note on certain class of spiral-like functions, Rend. Sem. Mat. Univ. Padavo. 80 (1988) 17–24.

[6] E.M. Wright, The asymptotic expansion of the generalized hypergeometric function, Proc. London Math. Soc. 46 (1940) 389–408.

[7] J. Dziok, R.K. Raina, Families of analytic functions associated with the Wright generalized hypergeometric function, Demonstratio Math. 37 (3) (2004) 533–542.

[8] G. Murugusundaramoorthy, M.K. Aouf, Families of meromorphic multivalent functions associated with the Dziok-Raina operator, International Journal of Analysis and Applications 2 (1) (2013) 1–18.

[9] J.L. Liu, H.M. Srivastava, Classes of meromorphically multivalent functions asoociated with the generalized hypergeometric function, Math. Comput. Modeling 39 (2004) 21–34.

[10] J. Patel, A.K. Patil, On certain subclasses of meromorphically multivalent functions associated with the generalized hypergeometric function, J. Inequal. Pure Appli. Math. 10 (1) (2009) art. 13.

[11] A.O. Mostafa, Applications of differential subordination to certain subclasses of p-valent meromorphic functions involving certain operator, Math. Comput. Modelling 54 (2011) 1486–1498.

[12] D.G. Yang, On new subclasses of meromorphic p-valent functions, J. Math. Res. Exposition 15 (1995) 7–13.

[13] O. Altintaş, Ö. Özkan, H.M. Srivastava, Majorization by starlike functions of complex order, Complex Variables Theory Appl. 46 (2001) (3) 207–218.

[14] G. Murugusundaramoorthy, K. Thilagavathi, Majorization of starlike and convex functions of complex order involving linear operators, Math. Slovaca. 66 (1) (2016) 127–134.

[15] S.P. Goyal, P. Goswami, Majorization for certain classes of meromorphic functions defined by integral operator, Annales. Univ. Marie-Curie-Sklod. Math. Lett. 66 (2012) (2) 57–62.