



# Economic Production Quantity Model for Deteriorating Items for Three Stage System with Partial Backlogging

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**Abstract** In this paper we develop an inventory model for deteriorating items with partial backlogging. This paper deals with the problem of deteriorating EOQ model for deteriorating items under three different situations. In the proposed model, shortages are allowed and partial backlogged. We also show that the total cost function is convex with time. The model is explained with the help of numerical examples. Finally, the sensitivity analysis is given to validate the proposed model. Mathematica software is used for finding numerical solutions.

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## 1. INTRODUCTION

It is important to find the optimal stock and best possible time to meet future demand. The main objective of inventory management is to minimize inventory carrying cost. In the past few decades inventory problems for deteriorating items have been studied widely. Some of the items damaged, vaporized, affected by some other factors and are not in a perfect condition to satisfy the demand. Food items, grains, vegetables, fruits, drugs, radioactive substances, electronic substances are few examples of such items in which sufficient deterioration can take place during the normal storage period. Therefore the loss due to the deterioration cannot be ignored.

Most of the classical inventory models did not take into account the effects of inflation and the time value of the money. It has been observed that in supermarkets that the demand rate is usually influenced by the amount of stock level i.e. the demand rate may go up and down with the on hand stock level. EOQ model was first introduced by Harris [1].

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The model was based on constant demand without any deterioration function. But in real life the demand may go up and down in the course of time. Goel [2] developed on Economic Order Quantity under Conditions of Permissible Delay in Payments by considering two different cases. Chand and Ward [3] analyzed Goel's problem [2] under assumptions of the classical economic order quantity model, obtaining different results. Chung [4], [5] developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Hwang and Shinn [6] modeled an inventory system for retailer's pricing and lot sizing policy for exponential deteriorating products under the condition of permissible delay in payments. Jamal *et al.* [7] and Sarker *et al.* [8] addressed the optimal payment time under permissible delay in payment with deterioration. Covert and Philips [9] developed the model with variable deterioration rate with two parameters Weibull distribution. Philip [10] then developed the inventory model with three parameters Weibull distribution rate without shortage. In real life most of the items would have a span of maintaining quality for some period. During that period there is no deterioration occurring. Wu *et al.* [11] developed an inventory model for non-instantaneous deteriorating items with stock dependent demand. Tripathy and Mishra [12] developed inventory model for deteriorating items in declining market when delay in payment is allowed the retailer to settle the account against the purchase made by him. Tripathy and Pradhan [13] developed an inventory model for weibull deteriorating items with constant demand when delay in payment is allowed to the retailer to settle the account against the purchases made.

In some business transactions shortages play an important role in the study of inventory systems. When shortages occur, it is assumed that it is either completely backlogged or completely lost. But in actual practice some customers are willing to wait for backorder and others would turn to buy from other sellers. Meher *et al.* [14] developed an inventory model with weibull deterioration rate under the delay in payment in demand declining market. Uthayakumar and Geetha [15] developed an inventory model for infinite planning horizon and non-instantaneous deteriorating items with stock dependent demand rate when shortages are allowed and partial backlogged. Singh *et al.* [16] developed an EOQ model for perishable items with power demand and partial backlogging by considering backlogging backlogging rate inversely proportional to the waiting time. Basu and Sinha [17] developed a general inventory model with due consideration to the factors of time dependent partial backlogging and time dependent deterioration. Patra *et al.* [18] developed a deterministic inventory model when the deterioration rate is time dependent and demand rate is a function of selling price and non linear by considering shortages and shortages are completely backlogged. Tripathi [19] developed an inventory model for items with an exponential demand rate under a permissible delay in payments with shortages. Researchers such as Park [20], Hollier and Mak [21] developed inventory models with partial backorder. Goyal and Giri [22] established production inventory model with shortages partially backlogged. Hou [23] established an inventory model for deteriorating items with stock dependent demand under inflation. He considered that shortages are completely backordered. Several research papers dealing with shortages such as Yang [24], Sana [25], Ye [26], Nasab [27], Konstantaras [28] and Wee [29] etc. are worth mentioning.

Many researchers considered time-varying demand like Donaldson [30]. Hariga [31]. Silver and Meal [32] etc. Skouri and Papachristos [33], Skouri *et al.* [34] considered either

a constant or exponential deterioration rate. Sana [35] extended optimal selling price and lot size with time dependent deterioration with partial backlogging.

Rest of the paper is framed as follows. In Section 2 assumptions and notations are given followed by mathematical formulations in Section 3. Numerical example is mentioned in Section 4. Sensitivity analysis for various parameters is given in Section 5. Conclusion and future direction is provided in the last section, Section 6.

## 2. ASSUMPTIONS AND NOTATIONS

The following assumptions are being made throughout the manuscript.

- (a) Demand rate of the items is constant.
- (b) Replenishment rate is infinite and lead time is zero.
- (c) System operates for a prescribed period of planning horizon.
- (d) Shortages are allowed only for a ' $\delta$ ' fraction of it is backlogged. The remaining fraction  $(1-\delta)$  is lost.
- (e) Inventory constrains three steps in a cycle.
- (f) Product has no deterioration for fresh product time i.e. first step.
- (g) Product transactions are followed by instantaneous cash flow.

In addition, the following notations are made all over the manuscript:

$f$	discount rate, representing the time value of money
$i$	inflation rate
$r = f - i$	net discount rate of inflation
$H$	planning horizon
$T$	replenishment cycle
$m$	number of replenishment during the planning horizon, $m = H/T$
$T_d$	length of time in which the product has no deteriorations (fresh product time)
$Q$	deteriorating rate
$A$	cost of replenishment \$/order
$I_m$	maximum inventory level
$I_b$	maximum amount of shortage to be backlogged
$p$	per unit cost of items \$/ unit
$h$	per unit inventory holding cost /unit time, \$/unit/unit time
$b$	per unit opportunity cost due to lost sales, \$/unit
$C_1$	present unit value of ordering cost
$H_1$	present value of holding cost
$s$	per unit shortage cost /unit time, (\$/ unit/unit time)
$S$	present value of shortage cost
$C_2$	present unit value of opportunity cost
$M$	present value of material cost
$Z(m, k)$	present value of total cost
$T(m, k)$	present value of total cost of the system over a finite planning horizon
$T_1$	time at which backlog starts
$I_1(t)$	inventory level at time $t$ , $0 \leq t \leq T_d$
$I_2(t)$	inventory level at time $t$ , $T_d \leq t \leq T_1$
$I_3(t)$	inventory level at time $t$ , $T_1 \leq t \leq T$ .

### 3. MATHEMATICAL FORMULATIONS

Let us consider the planning horizon  $H$  divided into  $m$  equal parts and length  $T = H/m$ . thus the order times over the planning horizon  $H$  are  $T_j = jT$  ( $J = 0, 1, 2, 3, \dots, m$ ). When the inventory is positive, demand rate is constant whereas for negative inventory, the demand is partially backlogged. The first replenishment lot size of  $I_m$  is replenished at  $T = 0$ . During the interval  $[0, T_d]$ , the inventory level decreases due to the constant demand rate. The inventory level drops to zero due to demand and deterioration during  $[T_d, T_1]$ . During the interval  $[T_1, T]$ , shortages occur and are accumulated unit  $t = T_1$ , before they are partially backlogged. The inventory system can be represented by the following equations:

$$\frac{dI_1(t)}{dt} = -D, \quad 0 \leq t \leq T_d \quad (3.1)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D, \quad T_d \leq t \leq T_1 \quad (3.2)$$

$$\frac{dI_3(t)}{dt} = -D\delta, \quad T_1 \leq t \leq T \quad (3.3)$$

$$I_1(0) = I_m, \quad I_2(T_1) = 0, \quad I_3(T_1) = 0. \quad (3.4)$$

The solution of the above differentiated equations after applying the boundary conditions (3.4) we obtain:

$$I_1(t) = I_m - D(t), \quad 0 \leq t \leq T_d \quad (3.5)$$

$$I_2(t) = \frac{D}{\theta} \left\{ e^{\theta(T_1-t)} - 1 \right\}, \quad T_d \leq t \leq T_1 \quad (3.6)$$

$$I_3(t) = -D\delta(t - T_1), \quad T_1 \leq t \leq T. \quad (3.7)$$

Coordinating the continuities of  $I(t)$  at  $T = T_d$  it follows that

$$I_1(T_d) = I_2(T_d) \text{ which implies that}$$

$$I_m = D \left\{ T_d + \frac{1}{\theta} \left( e^{\theta(T_1-T_d)} - 1 \right) \right\}. \quad (3.8)$$

Substituting (3.8) into (3.5) we get

$$I_1(t) = D \left\{ T_d + \frac{1}{\theta} \left( e^{\theta(T_1-T_d)} - 1 \right) \right\} - Dt. \quad (3.9)$$

Thus the maximum inventory level and maximum amount of shortage demand to be backlogged during the first replenishment cycle is:

$$I_m = D \left\{ T_d + \frac{1}{\theta} \left( e^{\theta\left(\frac{kH}{m} - T_d\right)} - 1 \right) \right\} \quad (3.10)$$

$$I_b = D\delta \left( \frac{H}{m} \right) (1 - k), \quad (3.11)$$

respectively.

There are  $m$  cycles during the planning horizon. Since inventory is assumed to start and end at zero, an extra replenishment at  $T_m = H$  is required to satisfy the backorders of the last cycle in the planning horizon. Therefore there are  $m + 1$  replenishment in the

entire planning horizon  $H$ .

$$Q = I_m + I_b \tag{3.12}$$

and the last or  $(m + 1)^{\text{th}}$  Replenishment lot size is  $I_b$ .

The present value of ordering cost during the first cycle is

$$C_1 = A. \tag{3.13}$$

The present value of holding cost during the first replenishment cycle is

$$\begin{aligned} H_1 &= h \left[ \int_0^{T_d} I_1(t)e^{-rt} dt + \int_{T_d}^{T_1} I_2(t)e^{-rt} dt \right] \\ &= hD \left[ \left\{ T_d + \frac{1}{\theta} \left( e^{\theta(T_1 - T_d)} - 1 \right) \right\} \frac{(1 - e^{-rT_d})}{r} + \frac{T_d - e^{-rT_d}}{r} - \frac{1 - e^{-rT_d}}{r^2} \right] \\ &\quad + \frac{hD}{\theta} \left[ e^{\theta T_1} + \frac{e^{-rT_1} - e^{-rT_d}}{r} \right]. \end{aligned} \tag{3.14}$$

Shortages are partially backlogged. The present value of shortage cost during the first replenishment cycle is:

$$\begin{aligned} S &= -s \int_{T_1}^T I_3(t)e^{-rt} dt = sD \int_{T_1}^T (t - T_1)e^{-rt} dt \\ &= \frac{-D\delta}{r} \left[ (T - T_1)e^{-rT} + \frac{e^{-rT} - e^{-rT_1}}{r} \right] \\ &= \frac{sD\delta}{r^2} \left[ \left\{ \frac{rH}{m}(k - 1) - 1 \right\} e^{-rH/m} + e^{-rkH/m} \right]. \end{aligned} \tag{3.15}$$

The present value of opportunity cost due to lost sales during the first replenishment cycle is:

$$\begin{aligned} C_2 &= b \int_{T_1}^T D(1 - \delta)e^{-rt} dt \\ &= \frac{bD(1 - \delta)}{r} (e^{-rT} - e^{-rT_1}) \\ &= \frac{bD(1 - \delta)}{r} (e^{-rkH/m} - e^{-rH/m}). \end{aligned} \tag{3.16}$$

Replenishment is done at  $t = 0$  and  $t = T$ . The present value of material cost during the first replenishment cycle is

$$\begin{aligned} M &= pI_m + pI_d e^{-rT} \\ &= pD \left\{ T_d + \frac{1}{\theta} \left( e^{\theta(\frac{kH}{m} - T_d)} - 1 \right) \right\} + pD\delta \left( \frac{H}{m} \right) (1 - k)e^{-rH/m}. \end{aligned} \tag{3.17}$$

The present value of the total cost of the system during a finite planning horizon  $H$  is:

$$Z(m, k) = C_1 + H_1 + S + C_2 + M. \tag{3.18}$$

The present value of total cost of the system over a finite planning horizon is:

$$T(m, k) = \sum_{i=0}^{m-1} Z(m, k)e^{-irT} + A.e^{-rH} = Z(m, k) \left( \frac{1 - e^{-rH}}{1 - e^{-rH/m}} \right) + A.e^{-rH}.$$

Truncated CityplaceTaylor series is used for exponential terms to find the closed form solution i.e.  $e^{-rT} \approx 1 - rT + \frac{r^2T^2}{2}$  (for finding closed form optimal solution)

$$H_1 = hDT_d \left[ \left\{ T_1 + \frac{\theta(T_1 - T_d)^2}{2} \right\} \left( 1 - \frac{rT_d}{2} \right) - \frac{T_d}{2} \left( 1 - \frac{rT_d}{2} \right) \right] \\ + \frac{hD}{\theta} (T_1 - T_d) \left[ \theta T_1 + \frac{\theta^2 T_1^2}{2} + \theta \left( \frac{\theta+r}{2} \right) T_1 (T_1 + T_d) (1 + \theta T_1) + \frac{r}{2} (T_1 + T_d) \right] \quad (3.19)$$

$$S = \frac{Ds\delta}{2} [(T^2 - T_1^2) - T(rT^2 - rT.T_1 + 2T_1)] \quad (3.20)$$

$$C_2 = \frac{HbD(1-\delta)(1-k)}{m} \left\{ 1 - \frac{rH}{2m}(1+k) \right\} \quad (3.21)$$

$$M = pD \left\{ T_1 + \frac{\theta(T_1 - T_d)^2}{2} \right\} + pD\delta(H/m)(1-k) \left( 1 - \frac{rH}{m} \right) \quad (3.22)$$

$$TC(m, k) = (C_1 + H_1 + S + C_2 + M) \left( \frac{1 - e^{-rH}}{1 - e^{-rH/m}} \right) + Ae^{-rH} \quad (3.23) \\ = (C_1 + H_1 + S + C_2 + M)G + Ae^{-rH} \left( T_1 = \frac{rH}{m}T = \frac{H}{m} \right).$$

For finding optimal solution taking partial derivative of  $TC(m, k)$  with respect to  $k$  and equating to zero we obtain  $\frac{\partial TC(m, k)}{\partial k} = 0$ . We obtain

$$\frac{HbD(1-\delta)}{m} \left( 1 - \frac{rHk}{m} \right) - \frac{Ds\delta H}{2m} (2T_1 + 2T - rT^2) \\ + \frac{hDT_d H}{m} \left[ \left\{ 1 + \theta(T_1 - T_d) \right\} \left( 1 - \frac{rT_d}{2} \right) \right] \\ + \frac{hDH}{\theta m} (T_1 - T_d) \left[ \theta T_1 + \frac{\theta^2 T_1^2}{2} + \theta \left( \frac{\theta+r}{2} \right) T_1 \cdot (T_1 + T_d) (1 + \theta T_1) + \frac{r}{2} (T_1 + T_d) \right] \\ + \frac{hDH}{\theta m} (T_1 - T_d) \left[ \theta + \theta^2 T_1 + \theta \left( \frac{\theta+r}{2} \right) \cdot \{ (2T_1 + T_d(1 + \theta T_1) + \theta T_1(T_1 + T_d)) \} + \frac{r}{2} \right] \\ + \frac{pDH}{m} \left[ 1 + \theta \left( \frac{kH}{m} - T_d \right) - \delta \left( 1 - \frac{rH}{m} \right) \right] = 0. \quad (3.24)$$

Taking second differential with respect to  $k$  we get  $\frac{\partial^2 TC(m, k)}{\partial k^2} > 0$ , which shows that the value of  $T_1 = T_1^*$  and  $TC(m, k) = TC^*(m, k)$  obtained from (3.23) and (3.24) is minimum.

#### 4. NUMERICAL EXAMPLE

Let us consider  $a = 250$ ,  $h = 1.2$ ,  $s = 2.2$ ,  $b = 1.8$ ,  $Q = 0.06$ ,  $T_d = 0.08$ ,  $\delta = 0.5$ ,  $H = 10$ ,  $r = 0.2$ ,  $D = 800$ ,  $T = H/m$  in appropriate units. We obtain Optimal value of  $k = k^* = 0.1$ ,  $T_1 = T_1^* = 0.0790479$ , optimal (minimum),  $Z = Z^* = \$1660.82$ .

#### 5. SENSITIVITY ANALYSIS

Taking the parameters as in above mentioned numerical example. The only variation is in the value of  $k$ .

TABLE 1. The variation of  $k$  keeping  $m = 2$  (constant)

$k$	$T_1 = T_1^*$	$TC(m, k) = TC^*(m, k)$	$k$	$T_1 = T_1^*$	$TC(m, k) = TC^*(m, k)$
0.2	0.0790753	1354.81	0.7	0.079192	364.736
0.3	0.0791012	1084.79	0.8	0.079212	274.724
0.4	0.0791258	850.775	0.9	0.079231	220.713
0.5	0.079149	652.761	1.0	0.0792491	202.702

TABLE 2. The variation of  $m$  keeping  $k = 0.1$  (constant).

$m$	$T_1 = T_1^*$	$T(m, k) = TC^*(m, k)$	$m$	$T_1 = T_1^*$	$TC(m, k) = TC^*(m, k)$
2	0.0790479	1660.82	7	0.0784899	2447.59
3	0.0788547	4019.84	8	0.0784395	2189.5
4	0.0787225	3695.81	9	0.0783965	1984.26
5	0.0786254	3196.22	10	0.0783589	1818.28

From Table 1, we observe that increase of fraction of scheduling constant lime ' $k$ ' results increase in optimal  $T_1 = T_1^*$  and decrease in optimal total cost  $TC(m, k) = TC^*(m, k)$ .

From Table 2, we observe that the increase in the value of number of replenishment during the planning horizon  $m = H/T$ , results in decrease in the optimal time  $T_1 = T_1^*$  and  $TC(m, k) = TC^*(m, k)$  increases from value  $m = 2$  to  $m = 3$  and again decreases from  $m = 4$  and onwards.

## 6. CONCLUSION

This paper has been developed for deterministic inventory model for deteriorating items with constant demand rate over a finite planning horizon. Shortages are allowed and partially backlogged. We have also considered the effect of inflation in formulating the inventory policy. We have given the numerical formulation of the problem to present an optimal solution. Truncated CityplaceTaylor series is used for finding numerical optimal closed form solution. Sensitivity analysis has been carried out with respect to various parameters. The result shows that the effect of inflation on present value of the total cost is more significant.

The paper can be extended in several ways, for instance we may extend the paper for exponential demand rate as well as quadratic time dependent demand rate. We could also generalize the model for time dependent holding cost etc.

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## APPENDIX

All partial derivatives with respect to  $k$  is given as:

$$\begin{aligned} \frac{\partial M}{\partial k} &= \frac{pDH}{m} \left[ 1 + \theta \left( \frac{kH}{m} - T_d \right) - \delta \left( 1 - \frac{rH}{m} \right) \right] \quad \text{and} \quad \frac{\partial^2 M}{\partial k^2} = \frac{\theta pDH^2}{m^2} \\ \frac{\partial C_2}{\partial k} &= \frac{HbD(1-\delta)}{m} \left( 1 - \frac{rHk}{m} \right) \quad \text{and} \quad \frac{\partial^2 C_2}{\partial k^2} = -\frac{bHD(1-\delta)}{m^2} \\ \frac{\partial S}{\partial k} &= -\frac{Ds\delta H}{2m} (2T_1 + 2T - rT^2) \quad \text{and} \quad \frac{\partial^2 S}{\partial k^2} = -\frac{Ds\delta H^2}{m^2} \\ \frac{\partial H_1}{\partial k} &= \frac{hDT_d H}{m} \left[ \{1 + \theta(T_1 - T_d)\} \left( 1 - \frac{rT_d}{2} \right) \right] \\ &\quad + \frac{hDH}{\theta m} (T_1 - T_d) \left[ \theta T_1 + \frac{\theta^2 T_1^2}{2} + \theta \left( \frac{\theta + r}{2} \right) T_1 \cdot (T_1 + T_d)(1 + \theta T_1) + \frac{r}{2} (T_1 + T_d) \right] \\ &\quad + \frac{hDH}{\theta m} (T_1 - T_d) \left[ \theta + \theta^2 T_1 + \theta \left( \frac{\theta + r}{2} \right) \{ (2T_1 + T_d)(1 + \theta T_1) + \theta T_1 (T_1 + T_d) \} + \frac{r}{2} \right] \\ \frac{\partial^2 H_1}{\partial k^2} &= \frac{hDT_d H}{m} \left[ \frac{\theta H}{m} \left( 1 - \frac{rT_d}{2} \right) \right] + \frac{hDH}{\theta m} \left[ \frac{\theta H}{m} + \frac{\theta^2 kH^2}{m^2} + \frac{\theta(\theta + r)}{2} \right. \\ &\quad \cdot \left. \left( \frac{2kH^2}{m^2} + \frac{HT_d}{m} \right) (1 + \theta T_1) + T_1 (T_1 + T_d) \frac{\theta H}{m} + \frac{rH}{2m} \right] \\ &\quad + \frac{hDH}{\theta m} \cdot \frac{H}{m} \left[ \theta + \theta^2 T_1 + \frac{\theta(\theta + r)}{2} \left\{ \frac{2H}{m} (1 + \theta T_1) + (2T_1 + T_d) \frac{\theta H}{m} + \frac{2kH^2}{m^2} + \frac{T_d H}{m} \right\} \right] \end{aligned}$$

**Note.** All partial derivatives have been taken after truncating the exponential terms.

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