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Numerical Solutions of Nonlinear Wave-Like Equations by Reduced Differential Transform Method

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Abstract This paper is presented to give numerical solutions of nonlinear wave-like equations with variable coefficients by using Reduced Differential Transform Method (RDTM). RDTM can be applied most of the physical, engineering, biological and etc. models as an alternative to obtain reliable and fastest converge, efficient approximations. Hence, our obtained results showed that RDTM is a very simple method and has a quite accuracy.

MSC: 35G20; 65M99; 68U01

Keywords: wave-like equations; reduced differential transform method; numerical approximation; nonlinear model

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1. Introduction

Many physical problems can be described by mathematical models that involve partial differential equations. Large varieties of physical, chemical and biological phenomena are governed by the partial differential equations. A mathematical model is a simplified description of physical reality expressed in mathematical terms. Additionally, nonlinear partial differential equations are central to research in many fields such as Hydrodynamics, Engineering, Quantum field theory, Optics, Plasma physics etc. They mostly do not have exact solutions and therefore they are approximated using numerical schemes.

By applying the Adomian Decomposition Method, M. Ghoreishi et al solved some types of nonlinear wave-like equation [1], V.G. Gupta and S. Gupta worked out by using Homotopy Perturbation Transform Method these types of equation tool [2], furthermore, A. Aslanov [3], F. Yin and et al [4] and A. Atangana and et al [5] researched for solving nonlinear heat and wave-like equation by using Homotopy Perturbation, Variational Iteration and Homotopy Decomposition Methods respectively. Moreover, various techniques, such as homotopy analysis, perturbations, decompositions, iterations, differential and

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laplace transformation techniques have been used to handle similar types of these wave-like and also heat-like problems numerically and analytically as in references [3, 6–12]. Recently, Rawashdeh and et al [13] studied to solve Telegraph and Cahn-Hilliard equation by applying Reduced differential transform method, Obeidat and et al [14] studied to find approximate solution of nonlinear partial differential equations by using Differential transform and Adomian decomposition methods.

The fundamental motivation of the present study is the extension of a recently developed technique which is called Reduced Differential Transform Method (RDTM) to tackle some of nonlinear wave-like equations as the following form

$$u_{tt} = \sum_{i,j=1}^{n} F_{1ij}\left(X,t,u\right) \frac{\partial^{k+m}}{\partial x_i^k \partial x_j^m} F_{2ij}\left(u_{x_i}, u_{x_j}\right) + S\left(X,t\right) + \sum_{i=1}^{n} G_{1i}\left(X,t,u\right) \frac{\partial^p}{\partial x_i^p} G_{2i}\left(u_{x_i}\right) + H\left(X,t,u\right) \tag{1.1}$$

with initial conditions

$$u(X,0) = a_0(X) \quad \text{and} \quad u_t(X,0) = a_1(X). \tag{1.2}$$

Here, $X = (x_1, x_2, ..., x_n)$ and F_{1ij},G_{1i} are nonlinear functions of X, t, u. F_{2ij} and G_{2i} are nonlinear functions of derivatives of x_i and x_j respectively. Also, H,S are nonlinear functions and k,m,p are integers. These kind of equations describe the evolution of stochastic systems for example, erratic motions of small particles that are immersed in fluids, fluctuations of the intensity of laser light, velocity distributions of fluid particles in turbulent flows and the stochastic behavior of exchange rates [1, 2].

Let's $v\left(x,t\right)$ is a two variables function and assume that it can be demonstrated as a product of two functions which are single variable $v\left(x,t\right) = y\left(x\right) z\left(t\right)$. By making use of differential transform properties, $v\left(x,t\right)$ can be written as

$$v(x,t) = \sum_{i=0}^{\infty} Y(i)x^i \sum_{j=0}^{\infty} Z(j)t^j = \sum_{k=0}^{\infty} V_k(x)t^k \tag{1.3}$$

where $V_k(x)$ called t-dimensional spectrum function of $v(x,t)$ [15–17]. If the function $v(x,t)$ is analytic and differentiable continuously with respect to time t and space x, so we can displayed $V_k(x)$ as

$$V_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} v(x,t)\right]_{t=0}. \tag{1.4}$$

Here, the lowercase $v(x,t)$ represents the original function and the uppercase $V_k(x)$ stand for the transformed. From (1.4), we define the differential inverse transform of $V_k(x)$

$$v(x,t) = \sum_{k=0}^{\infty} V_k(x)t^k \tag{1.5}$$

and then to compose (1.4) and (1.5), we get the numerical solution of $v(x,t)$ as below

$$v(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} v(x,t)\right]_{t=0} t^k. \tag{1.6}$$

In the way of our utilization for the rest of the paper, from [15–17], the basic mathematical theorems taken by RDTM can be obtained as follows and their proofs and other properties of RDTM are found in [18].

Theorem 1.1. *If $u(x,t)$ is two variables functions, then reduced differential transformation form is*

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0}.$$

Theorem 1.2. *If $w(x,t) = u(x,t) \pm v(x,t)$ and $w(x,t) = \alpha u(x,t)$ then transformation form are as follow respectively*

$$W_k(x) = U_k(x) \pm V_k(x)$$
$$W_k(x) = \alpha U_k(x),\ \alpha \text{ is constant.}$$

Theorem 1.3. *If $w(x,t) = x^m t^n$ and $w(x,t) = x^m t^n u(x,t)$, then transformation form are as follow respectively*

$$W_k(x) = x^m \delta(k-n),\ \delta(k-n) = \begin{cases} 1, k = 0 \\ 0, k \neq 0 \end{cases}$$
$$W_k(x) = x^m U_{k-n}(x).$$

Theorem 1.4. *If $w(x,t) = u(x,t)v(x,t)$ and $w(x,t) = \frac{\partial^r}{\partial t^r}u(x,t)$, then transformation form are as follow respectively*

$$W_k(x) = \sum_{r=0}^{k} U_r(x)V_{k-r}(x) = \sum_{r=0}^{k} V_r(x)U_{k-r}(x)$$
$$W_k(x) = (k+1)(k+2)...(k+r)U_{k+r}(x).$$

Theorem 1.5. *If $w(x,t) = \frac{\partial^2}{\partial x^2}u(x,t)$, then transformation form is as follow*

$$W_k(x) = \frac{d^2}{dx^2}U_k(x).$$

2. Implementations of Reduced Differential Transform Method

RDTM is a powerful numerical technique used various linear and nonlinear problems as seen some of [13, 15, 17] etc. Therefore in this part of paper, three nonlinear wave-like equations with variable coefficients as in type of (1.1)-(1.2) are solved by RDTM with showing errors, accuracy and efficiency of solutions.

Problem 1: Let's at first, consider the nonlinear two dimensional time-dependent wave-like equations which is a form of (1.1)-(1.2) with variable coefficients [1, 2],

$$v(x,y,t)_{tt} = \frac{\partial^2}{\partial x \partial y}\left(v(x,y,t)_{xx}v(x,y,t)_{yy}\right) - \frac{\partial^2}{\partial x \partial y}\left(xyv(x,y,t)_x v(x,y,t)_y\right) - v(x,y,t) \tag{2.1}$$

with initial conditions

$$v(x,y,0)=e^{xy}\text{and}v(x,y,0)_t=e^{xy}. \tag{2.2}$$

If we rearrange the equation (2.1), it can be written as following

$$\begin{aligned} v_{tt} &= v_{xxxy}v_{yy}+v_{xxy}v_{yyx}+v_{xxx}v_{yyy}+v_{xx}v_{yyyx} \\ &-v_xv_y-xv_{xx}v_y-xv_xv_{xy}-yv_{xy}v_y-xyv_{xxy}v_y \\ &-xy(v_{xy})^2-yv_xv_{yy}-xyv_{xx}v_{yy}-xyv_xv_{yyx}-v. \end{aligned} \tag{2.3}$$

By applying the RDTM process for equation (2.3) and from Theorem 1.1 to Theorem 1.5, we get reduced transformation form as below

$$\begin{aligned} (k+1)(k+2)V_{k+2}(x,y) &= \sum_{r=0}^{k}\frac{d^2}{dy^2}V_r(x,y)\frac{d^4}{dx^3dy}V_{k-r}(x,y) \\ +\sum_{r=0}^{k}\frac{d^3}{dx^2dy}V_r(x,y)\frac{d^3}{dy^2dx}V_{k-r}(x,y) &+\sum_{r=0}^{k}\frac{d^3}{dx^3}V_r(x,y)\frac{d^3}{dy^3}V_{k-r}(x,y) \\ +\sum_{r=0}^{k}\frac{d^2}{dx^2}V_r(x,y)\frac{d^4}{dy^3dx}V_{k-r}(x,y) &-\sum_{r=0}^{k}\frac{d}{dx}V_r(x,y)\frac{d}{dy}V_{k-r}(x,y) \\ -x\sum_{r=0}^{k}\frac{d^2}{dx^2}V_r(x,y)\frac{d}{dy}V_{k-r}(x,y) &-x\sum_{r=0}^{k}\frac{d}{dx}V_r(x,y)\frac{d^2}{dxdy}V_{k-r}(x,y) \\ -y\sum_{r=0}^{k}\frac{d}{dy}V_r(x,y)\frac{d^2}{dxdy}V_{k-r}(x,y) &-xy\sum_{r=0}^{k}\frac{d}{dy}V_r(x,y)\frac{d^3}{dx^2dy}V_{k-r}(x,y) \\ -xy\sum_{r=0}^{k}\frac{d^2}{dxdy}V_r(x,y)\frac{d^2}{dxdy}V_{k-r}(x,y) &-y\sum_{r=0}^{k}\frac{d}{dx}V_r(x,y)\frac{d^2}{dy^2}V_{k-r}(x,y) \\ -xy\sum_{r=0}^{k}\frac{d^2}{dx^2}V_r(x,y)\frac{d^2}{dy^2}V_{k-r}(x,y) &-xy\sum_{r=0}^{k}\frac{d}{dx}V_r(x,y)\frac{d^3}{dy^2dx}V_{k-r}(x,y) \\ &-V_k(x,y) \end{aligned} \tag{2.4}$$

and also the initial conditions of equation (2.2) are transformed

$$V_0(x,y)=e^{xy} \quad \text{and} \quad V_1(x,y)=e^{xy}. \tag{2.5}$$

By substituting initial conditions into (2.4) and therewithal we apply the RDTM process as in the (1.4)-(1.6), then some $V_k(x,y)$ values are obtained following

$$\begin{aligned} V_2(x,y)=-\frac{1}{2}e^{xy}, V_3(x,y)=-\frac{1}{6}e^{xy}, V_4(x,y)=\frac{1}{24}e^{xy}, \\ V_5(x,y)=\frac{1}{120}e^{xy}, V_6(x,y)=-\frac{1}{720}e^{xy}, V_7(x,y)=-\frac{1}{5040}e^{xy}, \ldots \end{aligned} \tag{2.6}$$

Also, we continue this process and also the inverse transformation of the set of $\{V_k(x,y)\}_{k=0}^{\infty}$ values and from (1.5)-(1.6), RDTM solution of $v(x,y,t)$ is obtained as

$$v(x,y,t)=\sum_{k=0}^{\infty}V_k(x,y)t^k=e^{xy}\begin{pmatrix} 1+t-\frac{1}{2}t^2 \\ -\frac{1}{6}t^3+\frac{1}{24}t^4+\frac{1}{120}t^5 \\ -\frac{1}{720}t^6-\frac{1}{5040}t^7\pm\cdots \end{pmatrix} \tag{2.7}$$

$$\approx e^{xy}(\sin(t)+\cos(t)).$$

t	$x=y$	Exact Soln	RDTM Soln	Absolute errors
0.0	0.0	1.000000000	1.000000000	0.0
	0.25	1.064494459	1.064494459	0.0
	0.5	1.284025417	1.284025417	0.0
	1.0	2.718281828	2.718281828	0.0
0.5	0.0	1.357008100	1.357008102	2×10^{-9}
	0.25	1.444527604	1.444527604	0.0
	0.5	1.742432892	1.742432893	1×10^{-9}
	1.0	3.688730460	3.688730461	1×10^{-9}
1.0	0.0	1.381773291	1.381773290	1×10^{-9}
	0.25	1.470890012	1.470890011	1×10^{-9}
	0.5	1.774232026	1.774232026	0.0
	1.0	3.756049227	3.756049229	2×10^{-9}
1.5	0.0	1.068232188	1.068232190	2×10^{-9}
	0.25	1.137127245	1.137127245	0.0
	0.5	1.371637281	1.371637284	3×10^{-9}
	1.0	2.903756146	2.903756149	3×10^{-9}
2.0	0.0	0.4931505903	0.4931505902	1×10^{-10}
	0.25	0.5249560708	0.5249560707	1×10^{-10}
	0.5	0.6332178927	0.6332178932	5×10^{-10}
	1.0	1.340522289	1.340522291	2×10^{-9}

TABLE 1. Comparison of Exact and 7 terms RDTM solution of eq. (2.1).

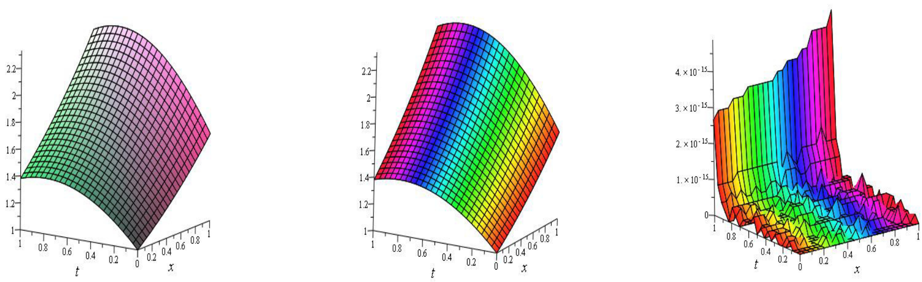

FIGURE 1. Comparison of **left:** Exact solution, **middle:** RDTM solution and **right:** absolute error for eq. (2.1) when $y=0.5$.

Thus, the approximate solution converges to the exact solution of (2.1) and also has higher accuracy as shown Table 1, Figure 1.

Problem 2: Secondly, we take into account of the nonlinear wave-like equations which is a form of (1.1) with variable coefficients [1, 2] as follows

$$v(x,t)_{tt} = v(x,t)^2 \frac{\partial^2}{\partial x^2} \left(v(x,t)_x v(x,t)_{xx} v(x,t)_{xxx} \right) + \left(\frac{\partial}{\partial x} v(x,t) \right)^2 \frac{\partial^2}{\partial x^2} \left(\left(\frac{\partial^2}{\partial x^2} v(x,t) \right)^3 \right) - 18 v(x,t)^5 + v(x,t) \tag{2.8}$$

with initial conditions

$$v(x,0) = e^x \ , \ v(x,0)_t = e^x. \tag{2.9}$$

Again, if we apply the (2.3) operations for the equation (2.8), it can be written as below

$$v_{tt} = v^2 \left(\begin{array}{c} 3v_{xx} \left(v_{xxx}\right)^2 + 2\left(v_{xx}\right)^2 v_{xxxx} \\ +3v_x v_{xxx} v_{xxxx} + v_x v_{xx} v_{xxxxx} \end{array} \right) + \left(v_x\right)^2 \left(\begin{array}{c} 6v_{xx}\left(v_{xxx}\right)^2 \\ +3\left(v_{xx}\right)^2 v_{xxxx} \end{array} \right) - 18v^5 + v. \tag{2.10}$$

Then, by using reduced differential transformation like (1.4)-(1.6) for on the equation (2.10) and from Theorem 1.1 to Theorem 1.5, we write down transformed form

$$\begin{aligned}(k+1)(k+2)V_{k+2}(x) = {} & 3 \left(\begin{array}{c} \sum_{r=0}^{k} \sum_{s=0}^{k-r} \sum_{m=0}^{k-r-s} \sum_{n=0}^{k-r-s-m} V_r(x) \\ V_s(x) \frac{d^2}{dx^2} V_m(x) \frac{d^3}{dx^3} V_n(x) \frac{d^3}{dx^3} V_{k-r-s-m-n}(x) \end{array} \right) \\ & +2 \left(\begin{array}{c} \sum_{r=0}^{k} \sum_{s=0}^{k-r} \sum_{m=0}^{k-r-s} \sum_{n=0}^{k-r-s-m} V_r(x) \\ V_s(x) \frac{d^2}{dx^2} V_m(x) \frac{d^2}{dx^2} V_n(x) \frac{d^4}{dx^4} V_{k-r-s-m-n}(x) \end{array} \right) \\ & +3 \left(\begin{array}{c} \sum_{r=0}^{k} \sum_{s=0}^{k-r} \sum_{m=0}^{k-r-s} \sum_{n=0}^{k-r-s-m} V_r(x) \\ V_s(x) \frac{d}{dx} V_m(x) \frac{d^3}{dx^3} V_n(x) \frac{d^4}{dx^4} V_{k-r-s-m-n}(x) \end{array} \right) \\ & + \left(\begin{array}{c} \sum_{r=0}^{k} \sum_{s=0}^{k-r} \sum_{m=0}^{k-r-s} \sum_{n=0}^{k-r-s-m} V_r(x) \\ V_s(x) \frac{d}{dx} V_m(x) \frac{d^2}{dx^2} V_n(x) \frac{d^5}{dx^5} V_{k-r-s-m-n}(x) \end{array} \right) \\ & +6 \left(\begin{array}{c} \sum_{r=0}^{k} \sum_{s=0}^{k-r} \sum_{m=0}^{k-r-s} \sum_{n=0}^{k-r-s-m} \frac{d}{dx} V_r(x) \\ \frac{d}{dx} V_s(x) \frac{d^2}{dx^2} V_m(x) \frac{d^3}{dx^3} V_n(x) \frac{d^3}{dx^3} V_{k-r-s-m-n}(x) \end{array} \right) \\ & +3 \left(\begin{array}{c} \sum_{r=0}^{k} \sum_{s=0}^{k-r} \sum_{m=0}^{k-r-s} \sum_{n=0}^{k-r-s-m} \frac{d}{dx} V_r(x) \\ \frac{d}{dx} V_s(x) \frac{d^2}{dx^2} V_m(x) \frac{d^2}{dx^2} V_n(x) \frac{d^4}{dx^4} V_{k-r-s-m-n}(x) \end{array} \right) \\ & -18 \left(\begin{array}{c} \sum_{r=0}^{k} \sum_{s=0}^{k-r} \sum_{m=0}^{k-r-s} \sum_{n=0}^{k-r-s-m} V_r(x) \\ V_s(x) V_m(x) V_n(x) V_{k-r-s-m-n}(x) \end{array} \right) + V_k(x) \end{aligned} \tag{2.11}$$

and initial conditions transform

$$V_0(x) = e^x \quad \text{and} \quad V_1(x) = e^x \ . \tag{2.12}$$

From reduced differential inverse transform process of (1.5)-(1.6), some $V_k(x)$ values are obtained following

$$V_2(x) = \frac{1}{2} e^x, V_3(x) = \frac{1}{6} e^x, V_4(x) = \frac{1}{24} e^x, \tag{2.13}$$

$$V_5(x) = \frac{1}{120} e^x, V_6(x) = \frac{1}{720} e^x, V_7(x) = \frac{1}{5040} e^x, \ldots$$

We perform the inverse transformation of the set of $\{V_k(x)\}_{k=0}^{\infty}$ values and from (1.5)-(1.6), RDTM solution of (2.8) is obtained as

$$v(x,t)=\sum_{k=0}^{\infty}V_k(x)t^k=e^x\left(\begin{array}{c}1+t+\frac{1}{2}t^2\\ +\frac{1}{6}t^3+\frac{1}{24}t^4+\frac{1}{120}t^5\\ +\frac{1}{720}t^6+\frac{1}{5040}t^7+\cdots\end{array}\right)\approx e^{x+t} \tag{2.14}$$

which converges efficiently to exact solution of (2.8) and has higher accuracy as shown Table 2, Figure 2.

Thus, the approximate solution converges to the exact solution of (2.1) and also has higher accuracy

t	x	Exact Soln	RDTM Soln	Absolute errors
0.0	0.0	1.000000000	1.000000000	0.0
	0.25	1.284025417	1.284025417	0.0
	0.5	1.648721271	1.648721271	0.0
	1.0	2.718281828	2.718281828	0.0
0.5	0.0	1.648721271	1.648721270	1×10^{-9}
	0.25	2.117000017	2.117000017	0.0
	0.5	2.718281828	2.718281829	1×10^{-9}
	1.0	4.481689070	4.481689069	1×10^{-9}
1.0	0.0	2.718281828	2.718281830	2×10^{-9}
	0.25	3.490342957	3.490342958	1×10^{-9}
	0.5	4.481689070	4.481689070	0.0
	1.0	7.389056099	7.389056099	0.0
1.5	0.0	4.481689070	4.481689070	0.0
	0.25	5.754602676	5.754602674	2×10^{-9}
	0.5	7.389056099	7.389056102	3×10^{-9}
	1.0	12.18249396	12.18249396	0.0
2.0	0.0	7.389056099	7.389056099	0.0
	0.25	9.487735836	9.487735837	1×10^{-9}
	0.5	12.18249396	12.18249397	1×10^{-8}
	1.0	20.08553692	20.08553692	0.0

TABLE 2. Comparison of Exact and 8 terms RDTM solution of eq. (2.8).

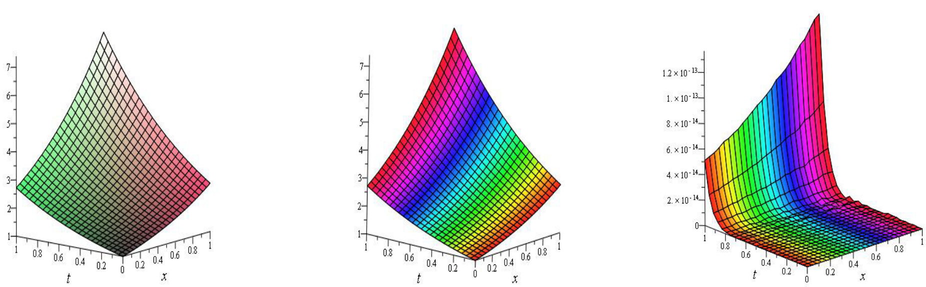

FIGURE 2. **left:** Exact solution, **middle:** RDTM solution and **right:** absolute error for eq. (2.8)

Problem 3: And finally, we handle the nonlinear wave-like equations which is a form of (1.1) with variable coefficients [1, 2]

$$v(x,t)_{tt} = x^2 \frac{\partial}{\partial x}\left(v(x,t)_x v(x,t)_{xx}\right) - x^2\left(\frac{\partial^2}{\partial x^2} v(x,t)\right)^2 - v(x,t) \tag{2.15}$$

with initial conditions

$$v(x,0) = 0 \quad , \quad v(x,0)_t = x^2. \tag{2.16}$$

We rewrite the equation (2.15) same manner as the PROBLEM 1 and PROBLEM 2 to as

$$v_{tt} = x^2\left((v_{xx})^2 + v_x v_{xxx}\right) - x^2 (v_{xx})^2 - v. \tag{2.17}$$

We recall that the processing steps of (2.4),(2.5), (2.11),(2.12) and also apply the reduced differential transform to equation (2.17), transformed form is obtained easily

$$(k+1)(k+2)V_{k+2}(x) = x^2\begin{pmatrix} \sum_{r=0}^{k} \frac{d^2}{dx^2} V_r(x) \frac{d^2}{dx^2} V_{k-r}(x) \\ +\sum_{r=0}^{k} \frac{d}{dx} V_r(x) \frac{d^3}{dx^3} V_{k-r}(x) \end{pmatrix} - x^2 \sum_{r=0}^{k} \frac{d^2}{dx^2} V_r(x) \frac{d^2}{dx^2} V_{k-r}(x) - V_k(x) \tag{2.18}$$

and initial conditions transform

$$V_0(x) = 0 \quad \text{and} \quad V_1(x) = x^2. \tag{2.19}$$

By applying reduced differential inverse transform process of (1.5), (1.6), $V_k(x)$ values are obtained as below

$$V_2(x) = 0,\ V_3(x) = -\frac{1}{6}x^2,\ V_4(x) = 0, \tag{2.20}$$
$$V_5(x) = \frac{1}{120}x^2, V_6(x) = 0, V_7(x) = -\frac{1}{5040}x^2, \ldots$$

So from (1.5)-(1.6), we obtain the RDTM solution of (2.15) following

$$v(x,t) = \sum_{k=0}^{\infty} V_k(x) t^k = x^2\begin{pmatrix} t - \frac{1}{6}t^3 + \frac{1}{120}t^5 \\ -\frac{1}{5040}t^7 \pm \cdots \end{pmatrix} \approx x^2 \sin(t). \tag{2.21}$$

Hence, the approximate solution (2.21) converges rapidly to exact solution of (2.15) and has higher accuracy as shown Table 3, Figure 3.

t	x	Exact Soln	RDTM Soln	Absolute errors
0.5	0.0	0.0	0.0	0.0
	0.25	0.02996409616	0.02996409617	1×10^{-11}
	0.5	0.1198563846	0.1198563847	1×10^{-10}
	1.0	0.4794255386	0.4794255387	1×10^{-10}
1.0	0.0	0.0	0.0	0.0
	0.25	0.05259193655	0.05259193654	1×10^{-11}
	0.5	0.2103677462	0.2103677460	2×10^{-10}
	1.0	0.8414709848	0.8414709847	1×10^{-10}
1.5	0.0	0.0	0.0	0.0
	0.25	0.06234343666	0.06234343665	1×10^{-11}
	0.5	0.2493737466	0.2493737467	1×10^{-10}
	1.0	0.9974949866	0.9974949867	1×10^{-10}
2.0	0.0	0.0	0.0	0.0
	0.25	0.05683108918	0.05683108914	4×10^{-11}
	0.5	0.2273243567	0.2273243567	0.0
	1.0	0.9092974268	0.9092974262	6×10^{-10}

TABLE 3. Comparison of Exact and 12 terms RDTM solution of eq. 2.15.

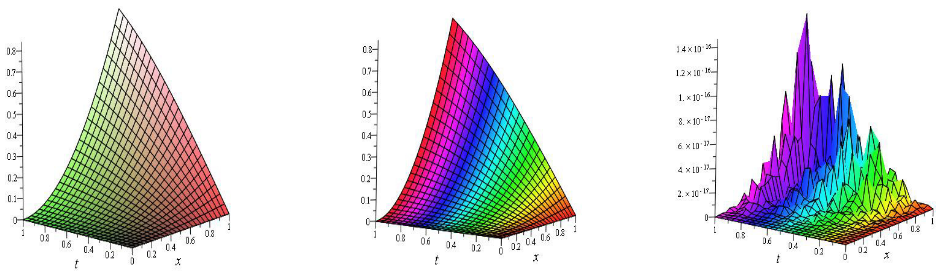

FIGURE 3. **left:** Exact solution, **middle:** RDTM solution and **right:** absolute error for eq. (2.15)

Iterations	CPU times of HPTM	CPU times of ADM	CPU times of RDTM
10	0.353	0.204	0.279
20	0.839	0.638	0.915
30	4.761	4.027	2.186
40	48.545	27.002	3.787
50	499.921	620.988	5.612

TABLE 4. The comparison of computation times which computed with Intel(R) Core (TM) i5-3230M CPU for 2.60 GHz between HPTM, ADM and RDTM for equation (2.15) with initial condition (2.16).

t	x	ADM[4]	RDTM
	0.1	0.0	0.0
0.1	0.3	0.0	0.0
	0.5	0.0	0.0
	0.7	0.0	$1.0e-15$
	0.1	0.0	0.0
0.3	0.3	0.0	0.0
	0.5	0.0	0.0
	0.7	0.0	0.0
	0.1	$1.96024e-16$	$1.0e-16$
0.5	0.3	$1.76248e-15$	0.0
	0.5	$4.89886e-15$	0.0
	0.7	$9.60343e-15$	$1.0e-14$
	0.1	$1.55232e-14$	0.0
0.7	0.3	$1.39708e-13$	0.0
	0.5	$3.88078e-13$	0.0
	0.7	$7.60614e-13$	$2.0e-14$
	0.1	$4.06630e-13$	0.0
0.9	0.3	$3.65968e-12$	$1.0e-15$
	0.5	$1.01658e-11$	0.0
	0.7	$1.99248e-11$	$1.0e-14$

TABLE 5. Comparison of the absolute errors of ADM-RDTM with six terms solution for eq. 2.15.

t	x	HPTM[5]	RDTM
	0.2	0.0	$2.0e-23$
0.2	0.4	0.0	$2.0e-22$
	0.6	0.0	$2.0e-22$
	0.8	0.0	0.0
	0.2	0.0	0.0
0.4	0.4	0.0	$1.0e-22$
	0.6	0.0	0.0
	0.8	0.0	$1.0e-21$
	0.2	0.0	0.0
0.6	0.4	0.0	$1.0e-22$
	0.6	0.0	0.0
	0.8	0.0	0.0
	0.2	$2.86954e-17$	0.0
0.8	0.4	$8.69872e-17$	$1.0e-21$
	0.6	$5.92313e-17$	$2.0e-21$
	0.8	$8.50096e-17$	$1.0e-21$
	0.2	$1.89658e-15$	0.0
1.0	0.4	$2.56856e-15$	$1.0e-21$
	0.6	$5.96845e-15$	$1.0e-21$
	0.8	$1.00236e-17$	0.0

TABLE 6. Comparison of the absolute errors of HPTM-RDTM with ten terms solution for eq. (2.15).

3. Conclusions

In this article, we applied the reduced differential transform method (RDTM), which has an advantage to provide an analytical approximation to the solution, usually an exact solution, in a rapidly convergent sequence, for nonlinear wave-like equations with variable coefficients. RDTM can be performed very easily and it is more effective and reliable, when it is compared with most famous techniques (Adomian Decomposition as in [1] and Homotopy Perturbation Transform as in [2]) shown in Table 5, Table 6. Additionally, RDTM is faster than ADM-HPTM to solve this type of equations as shown in Table 4.

So, our results show that the presented method is powerful technique and provides high accuracy for solving wave-like equations.

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