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NEW RESULTS ON DELAY-INTERVAL-DEPENDENT ROBUST EXPONENTIAL STABILITY FOR UNCERTAIN NEUTRAL-TYPE SYSTEMS WITH MIXED TIME-VARYING DELAYS AND NONLINEAR PERTURBATIONS

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Abstract The design problem of delay-interval-dependent robust exponential stability for uncertain neutral-type system with distributed and discrete time-varying delays, and nonlinear perturbations was studied. We concentrated on norm-bounded uncertainties and nonlinear time-varying parameter perturbations. By using mixed model transformation, Peng-Park's integral inequality, Wirtinger-based integral inequality, and proper Lyapunov-Krasovskii functional, new delay-inteval-dependent robust exponential stability criterion was received and formulated in the form of linear matrix inequalities (LMIs). Moreover, exponential stability criterion was also suggested for a neutral-type system with distributed and discrete time-varying delays, and nonlinear perturbations. Finally, numerical examples showed that the recommended approach achieves the expected results and the predominance of our results to those in the literature.

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1. INTRODUCTION

Nowadays, neutral-type system is popularly in discussed because it can be applied in many fields which composed delays both in its derivatives and state variables [1, 2]. In practical applications, this delays can be noticed in various fields such as mechanics,

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vibrating masses attached to population ecology, distributed networks, heat exchangers, robots in contact with rigid environments, automatic control, [3, 4] and so on. Besides, in terms of applications is able to modelled by systems with distributed delay which appeared in [5, 6].

Recently, the issue of delay-dependent stability on uncertain neutral-type systems with time-varying delays has been studied in [7–9], and the considered system has interval timevarying delays and uncertainties in [10, 11]. In addition, several authors have designed the topic of stability for systems with time-varying delays, and nonlinear perturbations such as [12–14], and have presented some stability conditions for uncertain neutral-type systems with interval time-varying delays, and nonlinear perturbations which appeared in [15-17]. In [14], Cheng et al. have expanded the novel criteria on uncertain delay-differential systems for neutral-type and nonlinear uncertainties, which the variation interval of time delay was divided into two subintervals by introducing the central point. Mohajerpoor, et al. [16] have taken advantage of descriptor transformation and utilizing triple integral terms, Which improved above system. Furthermore, the stability condition for neutraltype systems with mixed time delays and distributed delay have been studied in [18-21]. Pinjai and Mukdasai [18] have considered the issue of a class of delayed neutral-type systems with mixed time-varying delays, and nonlinear uncertain by using decomposition technique of coefficient matrix and the combination of descriptor model transfomation. The relationships between discrete delay, neutral delay and distributed delay for uncertain nutral-type systems has been studied in [20].

On the other hand, the exponential stability of various systems has also been received a lot of attention from researcher as well (for examples, see [18, 22–26]). Ali.[23] has used generalized eigenvalue problem approach for presented a novel exponential stability criterion for the neutral-type differential system with nonlinear uncertainties. Maharajan et al. have discussed the problem of exponential stability for BAM-type neural networks with non-fragile state estimator by fabricating a suitable LyapunovKrasovskii functional and enrolling some analysis techniques in [24].

As far as we can tell, there have proposed few results in the literature interesting the problem of delay-interval-dependent robust exponential stability of the uncertain neutral-type systems with time-varying delays, and nonlinear uncertainties. The exponential stability is important toward an analyzing stability because it can identify the rates convergence of system states to equilibrium point, so we have established the robustness of the exponential stability in Euclidean spaces. Besides, the characteristic of interval time-varying delay shows the ability of time delay on varying in an interval in which the lower bound of delay is not limited to zero.

In this paper, the delay-interval-dependent robust exponential stability criterion was designed for uncertain neutral-type system with distributed and discrete time-varying delays, and nonlinear perturbations. we concentrated on norm-bounded uncertainties and nonlinear time-varying parameter perturbations. First, the problem of delay-intervaldependent exponential stability criterion was desined for neutral-type system with distributed and discrete time-varying delays, and nonlinear perturbations. By using the Leibniz-Newton formula, utilization of zero equation, mixed model transformation, Peng-Park's integral inequality, Wirtinger-based integral inequality, and proper Lyapunov-Krasovskii functional, new delay-interval-dependent robust exponential stability criterion was received and formulated in the form of LMIs. Then, the problem of delay-intevaldependent robust exponential stability criterion was suggested for uncertain neutral system with mixed time-varying delays, and nonlinear perturbations. Finally, we represented the numerical examples to indicate the advantage of the new results, which are superior to the containing results.

2. Preliminaries

Notations: R^+ denotes the set of all real non-negative numbers; R^n and $R^{n \times r}$ denotes the *n*-dimensional Euclidean space and the set of all $n \times r$ real matrices, respectively; A^T denotes the transpose of the matrix A; A is symmetric if $A = A^T$; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\operatorname{Re} \lambda : \lambda \in \lambda(A)\}; \lambda_{\min}(A) = \min\{\operatorname{Re} \lambda : \lambda \in \lambda(A)\};$ $C([-\bar{\delta}, 0], R^n)$ denotes the space of all continuous vector functions mapping $[-\bar{\delta}, 0]$ into R^n where $\bar{\delta} = \max\{\eta_U, \delta_U, g_U\}, \eta_U, \delta_U, g_U \in R^+; x_t = x(t+s), s \in [-\bar{\delta}, 0]; \delta_{UL} = \delta_U - \delta_U;$ $\eta_{UL} = \eta_U - \eta_U; g_{UL} = g_U - g_U; *$ represents the elements below the main diagonal of a symmetric matrix.

We consider uncertain neutral-type delayed systems and nonlinear perturbations

$$\dot{z}(t) = A(t)z(t) + B(t)z(t - \delta(t)) + C(t)\dot{z}(t - \eta(t)) + D(t) \int_{t-g(t)}^{t} z(s)ds
+ g_a(t, z(t)) + g_b(t, z(t - \delta(t))) + g_c(t, \dot{z}(t - \eta(t))), \quad t \ge 0,$$

$$z(t) = \Phi(t), \quad \dot{z}(t) = \Psi(t), \quad \forall t \in [-\bar{\delta}, 0],$$
(2.1)

where $z(t) \in \mathbb{R}^n$ is the state variable. $\delta(t)$, $\eta(t)$ and g(t) are discrete, neutral and distributed interval time-varying delays, respectively, satisfying

$$0 \le \delta_L \le \delta(t) \le \delta_U, \quad \dot{\delta}(t) \le \delta_d, \tag{2.2}$$

$$0 \le \eta_L \le \eta(t) \le \eta_U, \quad \dot{\eta}(t) \le \eta_d, \tag{2.3}$$

$$0 \le g_L \le g(t) \le g_L,\tag{2.4}$$

where δ_L , δ_U , δ_d , η_L , η_U , η_d , g_L and g_U are given nonnegative real constants. $\Phi(t)$ and $\Psi(t)$ are the initial functions that are continuously differentiable on $C([-\bar{\delta}, 0], R^n)$ with the norm $\|\Phi\| = \sup_{s \in [-\bar{\delta}, 0]} \|\Phi(s)\|$, $\|\Psi\| = \sup_{s \in [-\bar{\delta}, 0]} \|\Psi(s)\|$. The uncertainties $g_i(\cdot)$, i = a, b, c, satisfying $g_i(0, \cdot) = 0$, and

$$g_a^T(t, z(t))g_a(t, z(t)) \leq \alpha_a^2 z^T(t)z(t), \qquad (2.5)$$

$$g_b^T(t, z(t - \delta(t)))g_b(t, z(t - \delta(t))) \leq \alpha_b^2 z^T(t - \delta(t))z(t - \delta(t)),$$
(2.6)

$$g_c^T(t, \dot{z}(t-\eta(t)))g_c(t, \dot{z}(t-\eta(t))) \leq \alpha_c^2 \dot{z}^T(t-\eta(t))\dot{z}(t-\eta(t)), \qquad (2.7)$$

where α_a , α_b and α_c are nonnegative real constants. $A(t) = A + \Delta A(t)$, $B(t) = B + \Delta B(t)$, $C(t) = C + \Delta C(t)$, $D(t) = D + \Delta D(t)$, where $A, B, C, D \in \mathbb{R}^{n \times n}$ are real constant matrices, and $\Delta A(t)$, $\Delta B(t)$, $\Delta C(t)$, $\Delta D(t)$ are uncertainties matrices, which the form is according to

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) & \Delta C(t) & \Delta D(t) \end{bmatrix} = L \Delta(t) \begin{bmatrix} G_a & G_b & G_c & G_d \end{bmatrix},$$

where L, G_a, G_b, G_c and G_d are real constant matrices with appropriate dimensions. The uncertainty matrix $\Delta(t)$ is satisfying

$$\Delta(t) = F(t)[I - JF(t)]^{-1}, \qquad (2.8)$$

is said to be admissible where J is an unknown matrix satisfying

$$I - JJ^T > 0. (2.9)$$

The uncertainty matrix F(t) is satisfying

$$F(t)^T F(t) \ge 0.$$
 (2.10)

Definition 2.1. The system (2.1) is robustly exponentially stable if there exist real constants $\alpha > 0$, k > 0 such that, the solution $z(t, \Phi, \Psi)$ of the system (2.1) satisfies

$$||z(t, \Phi, \Psi)|| \le k \max\{||\Phi||, ||\Psi||\} e^{-\alpha t}, \quad t \ge 0.$$

Lemma 2.2. (Jensen's inequality) Let $Q \in \mathbb{R}^{n \times n}$, $Q = Q^T > 0$ be any constant matrix, δ_U be positive real constant and $\dot{z} : [-\delta_U, 0] \to \mathbb{R}^n$ be vector-valued function. Then,

$$-\delta_U \int_{t-\delta_U}^t \dot{z}^T(s) Q \dot{z}(s) ds \le -\left(\int_{t-\delta_U}^t \dot{z}(s) ds\right)^T Q\left(\int_{t-\delta_U}^t \dot{z}(s) ds\right)$$

Lemma 2.3. [27](Sun et al.) Let $Q \in \mathbb{R}^{n \times n}$, $Q = Q^T > 0$ be any constant matrix, δ_L and δ_U be positive real constants. Then,

$$\begin{split} -\delta_U \int_{t-\delta_U}^t z^T(s)Qz(s)ds &\leq -\Big(\int_{t-\delta_U}^t z(s)ds\Big)^T Q\Big(\int_{t-\delta_U}^t z(s)ds\Big),\\ -\frac{(\delta_U^2 - \delta_L^2)}{2} \int_{-\delta_U}^{-\delta_L} \int_{t+s}^t z^T(u)Qz(u)duds \\ &\leq -\Big(\int_{-\delta_U}^{-\delta_L} \int_{t+s}^t z(u)duds\Big)^T Q\Big(\int_{-\delta_U}^{-\delta_L} \int_{t+s}^t z(u)duds\Big). \end{split}$$

Lemma 2.4. [17] Let $Q \in \mathbb{R}^{n \times n}$, $Q = Q^T > 0$ be any positive constant matrix, $\delta(t)$ be discrete time-varying delays with (2.2), $z : [-\delta_U, 0] \to \mathbb{R}^n$ be a vector function. Then,

$$-[\delta_{UL}] \int_{t-\delta_U}^{t-\delta_L} z^T(s) Qz(s) ds \leq -\int_{t-\delta(t)}^{t-\delta_L} z^T(s) ds Q \int_{t-\delta(t)}^{t-\delta_L} z(s) ds \\ -\int_{t-\delta_U}^{t-\delta(t)} z^T(s) ds Q \int_{t-\delta_U}^{t-\delta(t)} z(s) ds$$

Lemma 2.5. [17] Let $Q_1, Q_2, Q_3 \in \mathbb{R}^{n \times n}$ be any costant matrices which $Q_1 \ge 0, Q_3 > 0$, $\begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \ge 0$, $\delta(t)$ be time-varying delays with (2.2), $\dot{z} : [-\delta_U, 0] \to \mathbb{R}^n$ be vector function. Then,

$$-[\delta_{UL}] \int_{t-\delta_U}^{t-\delta_L} \begin{bmatrix} z(s) \\ \dot{z}(s) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} z(s) \\ \dot{z}(s) \end{bmatrix} ds$$

$$\leq \omega_1^T \begin{bmatrix} -Q_3 & Q_3 & 0 & -Q_2^T & 0 \\ * & -Q_3 - Q_3^T & Q_3 & Q_2^T & -Q_2^T \\ * & * & -Q_3 & 0 & Q_2^T \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_1 \end{bmatrix} \omega_1,$$

$$= \begin{bmatrix} z(t-\delta_L), \ z(t-\delta(t)), \ z(t-\delta_U), \ \int_{t-\delta(t)}^{t-\delta_L} z(s) ds, \ \int_{t-\delta_U}^{t-\delta(t)} z(s) ds \end{bmatrix}.$$

Lemma 2.6. [17] Let χ , $M_i \in \mathbb{R}^{n \times n}$, i = 1, 2, ..., 5 be any constant matrices and $\delta(t)$ be time-varying delays with (2.2), $z(t) \in \mathbb{R}^n$ be a vector-valued function, if $\begin{bmatrix} \chi & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \ge$

where ω_1^T

$$\begin{array}{l} 0 \ and \ \omega_2^T = \left[z(t - \delta_L), \ z(t - \delta(t)), \ z(t - \delta_U) \right], \ then \\ & - \int_{t - \delta_U}^{t - \delta_L} \dot{z}^T(s) \chi \dot{z}(s) ds \\ & \leq \omega_2^T \begin{bmatrix} M_1 + M_1^T & -M_1^T + M_2 & 0 \\ * & M_1 + M_1^T - M_2 - M_2^T & -M_1^T + M_2 \\ * & * & -M_2 - M_2^T \end{bmatrix} \omega_2 \\ & + (\delta_{UL}) \omega_2^T \begin{bmatrix} M_3 & M_4 & 0 \\ * & M_3 + M_5 & M_4 \\ * & * & M_5 \end{bmatrix} \omega_2. \end{array}$$

Lemma 2.7. [28] (Wirtinger – based integral inequality) Let $Q \in \mathbb{R}^{n \times n}$, $Q = Q^T > 0$ be any constant matrix, δ_L, δ_U be nonnegative real constants and $\dot{z} : [-\delta_U, -\delta_L] \to \mathbb{R}^n$ be a vector-valued function. Then,

$$-(\delta_{UL})\int_{t-\delta_U}^{t-\delta_L} \dot{z}^T(s)Q\dot{z}(s)ds \le \omega_3^T \begin{bmatrix} -4Q & -2Q & 6Q\\ * & -4Q & 6Q\\ * & * & -12Q \end{bmatrix} \omega_3,$$

where $\omega_3^T = \left[z(t - \delta_L), z(t - \delta_U), \frac{1}{\delta_{UL}} \int_{t - \delta_U}^{t - \delta_L} z(s) ds \right].$

Lemma 2.8. [29, 30] (Peng – Park's integral inequality) Let $Q, S \in \mathbb{R}^{n \times n}$ be any costant matrices which $Q \ge 0$, $\begin{bmatrix} Q & S \\ * & Q \end{bmatrix} \ge 0$, $\delta(t)$ be time-varying delay with $0 \le \delta(t) \le \delta_U$, $\dot{z}: [-\delta_U, 0] \to \mathbb{R}^n$ be a vector-valued function. Then,

$$-\delta_U \int_{t-\delta_U}^t \dot{z}^T(s) Q \dot{z}(s) ds \le \omega_4^T \begin{bmatrix} -Q & Q-S & S \\ * & -2Q+S+S^T & Q-S \\ * & * & -Q \end{bmatrix} \omega_4$$

where $\omega_4^T = [z(t), z(t - \delta(t)), z(t - \delta_U)].$

Lemma 2.9. [31] For any real constant matrices of appropriate dimensions M, S and N with $M = M^T$, and $\Delta(t)$ is given constant by (2.8)-(2.10), then

$$M + S\Delta(t)N + N^T\Delta(t)S^T < 0,$$

holds if and only if

$$\begin{bmatrix} M & S & \beta N^T \\ * & -\beta I & \beta J^T \\ * & * & -\beta I \end{bmatrix} < 0,$$

where any positive real constant β .

3. Main Results

First, the exponential stability criterion will be offered for the following system

$$\dot{z}(t) = Az(t) + Bz(t - \eta(t)) + C\dot{z}(t - \eta(t)) + D \int_{t-g(t)}^{t} z(s)ds + g_a(t, z(t))
+ g_b(t, z(t - \eta(t))) + g_c(t, \dot{z}(t - \eta(t))), t \ge 0,$$

$$z(t) = \phi(t), \quad \dot{z}(t) = \varphi(t), \quad \forall t \in [-\bar{\delta}, 0].$$
(3.1)

Then, the new criterion of system (3.1) will be introduced via LMIs approach, which we present the following notations for later use.

$$\Lambda = \left[\Lambda_{i,j}\right]_{21 \times 21},\tag{3.2}$$

for $\Lambda_{i,j} = \Lambda_{j,i}^T$, where

$$\begin{split} &\Lambda_{1,1} = P_1 A_1 + A_1^T P_1 + 2Q_1^T + 2Q_5^T + Q_9^T A_1 + A_1^T Q_9 + 2\alpha (P_1 + P_2) + Q_{13}^T A_1 \\ &+ A_1^T Q_{13} + P_3 + P_4 + \delta_U^2 (P_5 + P_6) + (\delta_{UL})^2 (W_2 + W_3) + e^{-2\alpha\delta_U} (M_1 + M_1^T) \\ &+ \delta_U^2 e^{-2\alpha\delta_U} M_3 - 4e^{-2\alpha\delta_U} P_8 - e^{-2\alpha\delta_U} P_9 + \delta_U^2 R_1 - e^{-2\alpha\delta_U} R_3 + (\delta_{UL})^2 R_4 \\ &- \delta_U^2 e^{-4\alpha\delta_U} P_{10} - (\delta_{UL})^2 e^{-4\alpha\delta_U} W_7 + g_U^2 P_{12} + (g_{UL})^2 W_{10} + \nu_a \alpha_A^2 I + N_1^T A_1 \\ &+ A_1^T N_1, \ \Lambda_{1,2} = P_1 (B + A_2) - Q_1^T + Q_2 - Q_5^T + Q_6 + Q_9^T (B + A_2) + A_1^T Q_{10} \\ &+ Q_{13}^T (B + A_2) + \delta_U e^{-2\alpha\delta_U} (-M_1^T + M_2) + \delta_U^2 e^{-2\alpha\delta_U} M_4 + e^{-2\alpha\delta_U} R_9 - S) \\ &+ e^{-2\alpha\delta_U} R_3 + N_1^T (B + A_2) + A_1^T N_2, \ \Lambda_{1,3} = -2e^{-2\alpha\delta_U} P_8 + e^{-2\alpha\delta_U} S, \\ &\Lambda_{1,4} = Q_4 + Q_8 - Q_9^T + A_1^T Q_{12} + \delta_U^2 R_2 + (\delta_{UL}) R_5 - N_1^T + A_1^T N_3, \\ &\Lambda_{1,5} = (P_1 + Q_9^T + Q_{13}^T) C + N_1^T C + A_1^T N_4, \ \Lambda_{1,6} = -e^{-2\alpha\delta_U} R_2^T + \delta_U e^{-4\alpha\delta_U} P_{10}, \\ &\Lambda_{1,7} = \delta_U e^{-4\alpha\delta_U} P_{10} + (\delta_{UL}) e^{-4\alpha\delta_U} W_7, \ \Lambda_{1,8} = 6e^{-2\alpha\delta_U} P_8, \ \Lambda_{1,9} = -Q_1^T + Q_3 \\ &- Q_5^T + Q_7 + A_1^T Q_{11} + (P_1 + Q_9^T + Q_{13}^T) A_2, \ \Lambda_{1,10} = P_1 + Q_9^T + Q_{13}^T + N_1^T + A_1^T N_7, \\ &\Lambda_{1,14} = (\delta_{UL}) e^{-4\alpha\delta_U} W_7, \ \Lambda_{1,17} = P_2 + A_1^T Q_{14} - Q_{13}^T, \ \Lambda_{1,20} = (P_1 + Q_9^T + Q_{13}^T) D \\ &+ N_1^T D + A_1^T N_8, \ \Lambda_{2,2} = -2Q_2^T - 2Q_6^T + Q_{10}^T (B + A_2) + (B^T + A_2^T) Q_{10} \\ &- e^{-2\alpha\delta_U} P_4 + \delta_d P_4 + \delta_U e^{-2\alpha\delta_U} (M_1 + M_1^T - M_2 - M_2^T) + \delta_U^2 e^{-2\alpha\delta_U} (M_3 + M_5) \\ &- 2e^{-2\alpha\delta_U} (P_9 - S - S^T) + (\delta_{UL}) e^{-2\alpha\delta_U} (R_6 + R_6^T) + \nu_b \alpha_b^2 I + N_2^T (B + A_2) \\ &+ (B^T + A_2^T) N_2, \ \Lambda_{2,3} = -\delta_U e^{-2\alpha\delta_U} (M_1^T - M_2) + \delta_U^2 e^{-2\alpha\delta_U} M_4 + e^{-2\alpha\delta_U} (P_9 - S) \\ &- (\delta_{UL}) e^{-2\alpha\delta_U} (M_6^T - M_7) + (\delta_{UL})^2 e^{-2\alpha\delta_U} M_9 + e^{-2\alpha\delta_U} R_3 + e^{-2\alpha\delta_U} R_6, \\ &\Lambda_{2,4} = -Q_4 - Q_8 - Q_{10}^T + (B^T + A_2^T) N_2, \ \Lambda_{2,14} = e^{-2\alpha\delta_U} R_5, \ \Lambda_{2,15} = Q_{10}^T C + N_2^T + (B^T + A_2^T) N_5, \ \Lambda_{2,11} = Q_{10} + N_2^T + (B^T + A_2^T) N_3, \\ &\Lambda_{2,5} = Q_{10}^T C + N_2^T$$

$$\begin{split} \Lambda_{3,13} &= -2e^{-2\alpha\delta_U}W_5, \ \Lambda_{3,15} = 6e^{-2\alpha\delta_U}W_5, \ \Lambda_{4,4} = -2Q_{12}^T + \delta_U^2(P_7 + P_8 \\ &+ P_9) + (\delta_{UL})^2e^{-2\alpha\delta_U}(W_4 + W_5 + W_6) + \delta_U^2R_3 + (\delta_{UL})^2R_6 + \frac{\delta_U^4}{4}P_{10} \\ &+ \frac{(\delta_U^2 - \delta_L^2)^2}{4}W_7 + P_{11} + (\delta_{UL})^2W_8 + W_9 - 2N_3^T + 2N_{10}^T, \ \Lambda_{4,5} = Q_{12}^TC \\ &+ N_3^TC - N_4, \ \Lambda_{4,9} = -Q_4^T - Q_8^T - Q_{11} + Q_{12}^TA_2 + N_3^TA_2 - N_9, \\ \Lambda_{4,10} = Q_{12}^T + N_3^T - N_5, \ \Lambda_{4,11} = Q_{12}^TD + N_3^T - N_6, \ \Lambda_{4,12} = Q_{12}^T + N_3^T - N_7, \\ \Lambda_{4,17} = -N_{10}^T + N_{11}, \ \Lambda_{4,20} = Q_{12}^TD + N_3^TD - N_8, \ \Lambda_{5,5} = -e^{-2\alpha\eta_U}P_{11} \\ &+ \eta_dP_{11} + \nu_e\alpha_c^2I + N_4^TC + C^TN_4, \ \Lambda_{5,9} = C^TQ_{11} + N_4^TA_2 + C^TN_9, \\ \Lambda_{5,10} = N_4^T + C^TN_5, \ \Lambda_{5,11} = N_4^T + C^TN_6, \ \Lambda_{5,12} = N_4^T + C^TN_7, \\ \Lambda_{5,17} = C^TQ_{14}^T, \ \Lambda_{5,20} = N_4^TD + C^TN_8, \ \Lambda_{6,6} = -e^{-2\alpha\delta_U}P_6 - e^{-2\alpha\delta_U}R_1 \\ &- e^{-4\alpha\delta_U}P_{10}, \ \Lambda_{6,7} = -e^{-4\alpha\delta_U}P_{10}, \ \Lambda_{7,7} = -e^{-2\alpha\delta_U}P_6 - e^{-2\alpha\delta_U}R_3 \\ &- e^{-2\alpha\delta_U}R_4 - e^{-4\alpha\delta_U}P_{10} - e^{-4\alpha\delta_U}W_7, \ \Lambda_{7,14} = e^{-4\alpha\delta_U}W_7, \ \Lambda_{8,8} = -\delta_U^2e^{-2\alpha\delta_U} \\ \times P_5 - 12e^{-2\alpha\delta_U}P_8, \ \Lambda_{9,9} = -2Q_3^T - 2Q_7^T + N_1^TA_2 + A_2^TN_9, \ \Lambda_{9,10} = Q_{11}^T \\ &+ A_2^TN_5^T + N_9^T, \ \Lambda_{9,11} = Q_{11}^T + A_2^TN_6^T + N_9^T, \ \Lambda_{9,12} = Q_{11}^T + A_2^TN_7^T + N_9^T, \\ \Lambda_{9,20} = N_9^TD + Q_{11}^TD + A_2^TN_8, \ \Lambda_{10,10} = -\nu_a I + 2N_5^T, \ \Lambda_{10,11} = N_5^T + N_6, \\ \Lambda_{10,12} = N_5^T + N_7, \ \Lambda_{10,17} = Q_{14}, \ \Lambda_{10,20} = N_5^TD + N_8, \ \Lambda_{11,11} = -\nu_b \\ &+ 2N_6^T, \ \Lambda_{11,12} = N_6^T + N_7, \ \Lambda_{11,17} = Q_{14}, \ \Lambda_{12,20} = N_7^TD + N_8, \ \Lambda_{13,13} = e^{-2\alpha\delta_U} \\ \times W_1 + (\delta_{UL})e^{-2\alpha\delta_U}M_5^T, \ \Lambda_{13,15} = -(\delta_{UL})^2e^{-2\alpha\delta_U}W_5, \ \Lambda_{14,14} = -e^{-2\alpha\delta_U}W_5, \\ \Lambda_{16,16} = -e^{-2\alpha\delta_U}W_7, \ \Lambda_{15,15} = -(\delta_{UL})^2e^{-2\alpha\delta_U}W_2 - 12e^{-2\alpha\delta_U}W_5, \\ \Lambda_{16,16} = -e^{-2\alpha\delta_U}W_6, \ \Lambda_{17,17} = -2Q_{14}^T - 2N_{11}^T, \ \Lambda_{17,20} = Q_{14}^TD, \\ \Lambda_{18,18} = (\eta_U - \eta_L)e^{-2\alpha\eta_U}W_8 - e^{-2\alpha\eta_U}W_9, \ \Lambda_{19,19} = -(\eta_{UL})e^{-2\alpha\eta_U}W_8, \\ \Lambda_{20,20} = -e^{-2\alpha\delta_U}W_6, \ \Lambda_{17,17} = -2\alpha$$

Theorem 3.1. For $||C|| + \alpha_c < 1$, $\alpha > 0$, if there exist positive definite symmetric matrices P_i , W_j , i = 1, 2, ..., 12, j = 1, 2, ..., 10, any appropriate dimensional matrices S, Q_k , M_l , N_m , R_n , k = 1, 2, ..., 14, l = 1, 2, ..., 10, m = 1, 2, ..., 11, n = 1, 2, ..., 6, and positive real constants α_s , ν_s , s = a, b, c, satisfying the following LMIs

$$\begin{bmatrix} P_7 & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \ge 0,$$
(3.3)

$$\begin{bmatrix} W_4 & M_6 & M_7 \\ * & M_8 & M_9 \\ * & * & M_{10} \end{bmatrix} \ge 0,$$
(3.4)

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \geq 0,$$

$$(3.5)$$

$$\begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} \ge 0,$$

$$(3.6)$$

$$\begin{bmatrix} P_9 & S\\ * & P_9 \end{bmatrix} \ge 0, \tag{3.7}$$

$$\Lambda < 0, \tag{3.8}$$

then the system (3.1) is exponentially stable.

Proof. Firstly, we improve the bound of interval time-varying delays by using the decomposition technique. Let constant matrix A as

$$A = A_1 + A_2, (3.9)$$

where $A_1, A_2 \in \mathbb{R}^{n \times n}$ are constant matrices. Ensure the exponential stability of the system (3.1) by choosing to take advantage of the zero equation as follows

$$0 = z(t) - z(t - \delta(t)) - \int_{t - \delta(t)}^{t} \dot{z}(s) ds.$$
(3.10)

By (3.9) and (3.10), the system (3.1) can be represented in the form of the descriptor system

$$\dot{z}(t) = A_1 z(t) + (A_2 + B) z(t - \delta(t)) + A_2 \int_{t - \delta(t)}^{t} \dot{z}(s) ds + C \dot{z}(t - \eta(t)) + D \int_{t - g(t)}^{t} z(s) ds + g_a(t, z(t)) + g_b(t, z(t - \delta(t))) + g_c(t, \dot{z}(t - \eta(t))).$$
(3.11)

Modify the system (3.11) in term of descriptor systems, which is the form as follows

$$\dot{z}(t) = w(t), \qquad (3.12)$$

$$0 = -w(t) + A_1 z(t) + (A_2 + B) z(t - \delta(t)) + A_2 \int_{t-\delta(t)}^t \dot{z}(s) ds + C\dot{z}(t - \eta(t)) + D \int_{t-g(t)}^t z(s) ds + g_a(t, z(t)) + g_b(t, z(t - \delta(t))) + g_c(t, \dot{z}(t - \eta(t))). \qquad (3.13)$$

Next, we consider Lyapunov-Krasovskii functional for a class of neutral-type delayed systems (3.11), (3.12) and (3.13) as

$$V(t) = \sum_{i=1}^{9} V_i(t), \qquad (3.14)$$

where

$$\begin{split} & \times \begin{bmatrix} P_1 & 0 & 0 & 0 \\ Q_1 & Q_2 & Q_3 & Q_4 \\ Q_5 & Q_6 & Q_7 & Q_8 \\ Q_9 & Q_{10} & Q_{11} & Q_{12} \end{bmatrix} \begin{bmatrix} z(t) \\ f_{t-\delta(t)}^{-\delta(t)} \dot{z}(s)ds \end{bmatrix}, \\ & V_2(t) &= z^T(t)P_2z(t) = \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_2 & 0 \\ Q_{13} & Q_{14} \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}, \\ & V_3(t) &= \int_{t-\delta_U}^t e^{2\alpha(s-t)} z^T(s)P_3z(s)ds + \int_{t-\delta(t)}^t e^{2\alpha(s-t)} z^T(s)P_4z(s)ds \\ &\quad + \int_{t-\delta_U}^{t-\delta_U} e^{2\alpha(s-t)} z^T(s)W_1z(s)ds, \\ & V_4(t) &= \delta_U \int_{-\delta_U}^0 \int_{t+s}^t e^{2\alpha(\theta-t)} z^T(\theta) \begin{bmatrix} P_5 + P_6 \end{bmatrix} z(\theta)d\theta ds \\ &\quad + (\delta_{UL}) \int_{-\delta_U}^{-\delta_L} \int_{t+s}^t e^{2\alpha(\theta-t)} z^T(\theta) \begin{bmatrix} P_7 + P_8 + P_9 \end{bmatrix} \dot{z}(\theta)d\theta ds \\ &\quad + (\delta_{UL}) \int_{-\delta_U}^{-\delta_L} \int_{t+s}^t e^{2\alpha(\theta-t)} \dot{z}^T(\theta) \begin{bmatrix} W_4 + W_5 + W_6 \end{bmatrix} \dot{z}(\theta)d\theta ds, \\ &\quad + (\delta_{UL}) \int_{-\delta_U}^{-\delta_L} \int_{t+s}^t e^{2\alpha(\theta-t)} \dot{z}^T(\theta) \begin{bmatrix} W_4 + W_5 + W_6 \end{bmatrix} \dot{z}(\theta)d\theta ds, \\ &\quad + (\delta_{UL}) \int_{-\delta_U}^{-\delta_L} \int_{t+s}^t e^{2\alpha(\theta-t)} \left[z(\theta) \\ \dot{z}(\theta) \end{bmatrix}^T \begin{bmatrix} R_4 & R_3 \\ R_4 & R_5 \end{bmatrix} \begin{bmatrix} z(\theta) \\ \dot{z}(\theta) \end{bmatrix} d\theta ds \\ &\quad + (\delta_UL) \int_{-\delta_U}^{-\delta_L} \int_{t+\theta}^t e^{2\alpha(u+\theta-t)} \dot{z}^T(u)P_{10}\dot{z}(u)du d\theta ds \\ &\quad + (\frac{\delta_U^2 - \delta_L^2}{2}) \int_{-\delta_U}^{-\delta_L} \int_{s}^0 \int_{t+\theta}^t e^{2\alpha(u+\theta-t)} \dot{z}^T(u)W_7\dot{z}(u)du d\theta ds \\ &\quad + (\frac{\delta_U^2 - \delta_L^2}{2}) \int_{-\delta_U}^{-\delta_L} \int_{s}^0 \int_{t+\theta}^t e^{2\alpha(u+\theta-t)} \dot{z}^T(\theta)W_{10}\dot{z}(\theta) d\theta ds \\ &\quad + \int_{t-\eta_L}^{t-\eta_L} e^{2\alpha(s-t)} \dot{z}^T(s)W_9\dot{z}(s) ds, \\ V_9(t) &= g_U \int_{-g_U}^0 \int_{t+s}^t e^{2\alpha(\theta-t)} z^T(\theta)P_{12}z(\theta) d\theta ds \\ &\quad + (g_UL) \int_{-g_U}^{-g_L} \int_{t+s}^t e^{2\alpha(\theta-t)} z^T(\theta)W_{10}z(\theta) d\theta ds. \end{aligned}$$

The differential of $V_1(t)$ along the trajectory of system (3.11), we obtian

$$\dot{V}_1(t) = 2z^T(t)P_1\dot{z}(t)$$

$$= 2z^{T}(t)P_{1}\left[A_{1}z(t) + (A_{2} + B)z(t - \delta(t)) + A_{2}\int_{t-\delta(t)}^{t} \dot{z}(s)ds + C\dot{z}(t - \eta(t)) + D\int_{t-g(t)}^{t} z(s)ds + g_{a}(t, z(t)) + g_{b}(t, z(t - \delta(t))) + g_{c}(t, \dot{z}(t - \eta(t)))\right] + 2\left[z^{T}(t)Q_{1}^{T} + z^{T}(t - \delta(t))Q_{2}^{T} + \int_{t-\delta(t)}^{t} \dot{z}^{T}(s) \times Q_{3}^{T}ds + \dot{z}(t)Q_{4}^{T}\right]\left[z(t) - z(t - \delta(t)) - \int_{t-\delta(t)}^{t} \dot{z}(s)ds\right] + 2\left[z^{T}(t) \times Q_{5}^{T} + z^{T}(t - \delta(t))Q_{6}^{T} + \int_{t-\delta(t)}^{t} \dot{z}^{T}(s)Q_{7}^{T}ds + \dot{z}(t)Q_{8}^{T}\right]\left[z(t) - z(t - \delta(t)) - \int_{t-\delta(t)}^{t} \dot{z}(s)ds\right] + 2\left[z^{T}(t) - z(t - \delta(t))Q_{10}^{T} + \int_{t-\delta(t)}^{t} \dot{z}(s)ds\right] + 2\left[z^{T}(t)Q_{9}^{T} + z^{T}(t - \delta(t))Q_{10}^{T} + \int_{t-\delta(t)}^{t} \dot{z}(s)ds\right] + 2\left[z^{T}(t)Q_{9}^{T} + z^{T}(t - \delta(t))Q_{10}^{T} + \int_{t-\delta(t)}^{t} \dot{z}(s)ds + z(t)Q_{12}^{T}\right]\left[-\dot{z}(t) + A_{1}z(t) + (A_{2} + B) \times z(t - \delta(t)) + A_{2}\int_{t-\delta(t)}^{t} \dot{z}(s)ds + C\dot{z}(t - \eta(t)) + D\int_{t-g(t)}^{t} z(s)ds + g_{a}(t, z(t)) + g_{b}(t, z(t - \delta(t))) + g_{c}(t, \dot{z}(t - \eta(t)))\right] + 2\alpha z^{T}(t)P_{1}z(t) - 2\alpha V_{1}(t).$$

Calculating $\dot{V}_2(t)$ in accordance with the solutions of the systems (3.12) and (3.13), we get

$$\begin{split} \dot{V}_{2}(t) &= 2z^{T}(t)P_{2}\dot{z}(t) = 2\begin{bmatrix} z(t)\\ w(t) \end{bmatrix}^{T} \begin{bmatrix} P_{2} & Q_{13}^{T}\\ 0 & Q_{14}^{T} \end{bmatrix} \begin{bmatrix} \dot{z}(t)\\ 0 \end{bmatrix} \\ &= 2z^{T}(t)P_{2}w(t) + 2z^{T}(t)Q_{13}^{T} \begin{bmatrix} -w(t) + A_{1}z(t) + (A_{2} + B)z(t - \delta(t)) \\ &+ A_{2}\int_{t-\delta(t)}^{t} \dot{z}(s)ds + C\dot{z}(t - \eta(t)) + D\int_{t-g(t)}^{t} z(s)ds + g_{a}(t, z(t)) \\ &+ g_{b}(t, z(t - \delta(t))) + g_{c}(t, \dot{z}(t - \eta(t))) \end{bmatrix} + 2w^{T}(t)Q_{14}^{T} \begin{bmatrix} -w(t) + A_{1}z(t) \\ &+ (A_{2} + B)z(t - \delta(t)) + A_{2}\int_{t-\delta(t)}^{t} \dot{z}(s)ds + C\dot{z}(t - \eta(t)) \\ &+ D\int_{t-g(t)}^{t} z(s)ds + g_{a}(t, z(t)) + g_{b}(t, z(t - \delta(t))) + g_{c}(t, \dot{z}(t - \eta(t))) \end{bmatrix} \\ &+ 2\alpha z^{T}(t)P_{2}z(t) - 2\alpha V_{2}(t). \end{split}$$

The time derivative of $V_3(t)$ is calculated as

$$\dot{V}_3(t) \leq z^T(t)[P_3 + P_4]z(t) - e^{-2\alpha\delta_U}z^T(t - \delta_U)P_3z(t - \delta_U)$$

$$-e^{-2\alpha\delta_U}z^T(t-\delta(t))P_4z(t-\delta(t))+\delta_dz^T(t-\delta(t))P_4z(t-\delta(t))$$

+ $e^{-2\alpha\delta_L}z^T(t-\delta_L)W_1z(t-\delta_L)-e^{-2\alpha\delta_U}z^T(t-\delta_U)W_1z(t-\delta_U)$
- $2\alpha V_3(t).$

For any scalar $s \in [t - \delta_U, t]$, we obviously $e^{-2\alpha\delta_U} \leq e^{-2\alpha\delta_L} \leq e^{2\alpha(s-t)} \leq 1$. Combine with Lemma 2.2 and Lemma 2.4, we obtain $\dot{V}_4(t)$ as the follow

$$\begin{aligned} \dot{V}_{4}(t) &\leq (\delta_{U})^{2} z^{T}(t) [P_{5} + P_{6}] z(t) - e^{-2\alpha\delta_{U}} \left(\frac{1}{\delta_{U}} \int_{t-\delta_{U}}^{t} z^{T}(s) ds \right) (\delta_{U})^{2} P_{5} \\ &\times \left(\frac{1}{\delta_{U}} \int_{t-\delta_{U}}^{t} z(s) ds \right) - e^{-2\alpha\delta_{U}} \left[\left(\int_{t-\delta(t)}^{t} z^{T}(s) ds \right) P_{6} \\ &\left(\int_{t-\delta(t)}^{t} z(s) ds \right) + \left(\int_{t-\delta_{U}}^{t-\delta(t)} z^{T}(s) ds \right) P_{6} \times \left(\int_{t-\delta_{U}}^{t-\delta(t)} z(s) ds \right) \right] \\ &+ (\delta_{UL})^{2} z^{T}(t) [W_{2} + W_{3}] z(t) - e^{-2\alpha\delta_{U}} \left(\frac{1}{\delta_{UL}} \int_{t-\delta_{U}}^{t-\delta_{L}} z^{T}(s) ds \right) \\ &\times (\delta_{UL})^{2} W_{2} \left(\frac{1}{\delta_{UL}} \int_{t-\delta_{U}}^{t-\delta_{L}} z(s) ds \right) - e^{-2\alpha\delta_{U}} \left[\left(\int_{t-\delta(t)}^{t-\delta_{L}} z^{T}(s) ds \right) \\ &\times W_{3} \left(\int_{t-\delta(t)}^{t-\delta_{L}} z(s) ds \right) + \left(\int_{t-\delta_{U}}^{t-\delta(t)} z^{T}(s) ds \right) W_{3} \left(\int_{t-\delta_{U}}^{t-\delta(t)} z(s) ds \right) \right] \\ &- 2\alpha V_{4}(t). \end{aligned}$$

By taking advantage of the Lemma 2.6 - Lemma 2.8, the differential of $V_5(t)$ is calculated as

$$\begin{split} \dot{V}_{5}(t) &\leq \delta_{U}^{2} \dot{z}^{T}(t) [P_{7} + P_{8} + P_{9}] \dot{z}(t) + \delta_{U} e^{-2\alpha\delta_{U}} \left(\begin{bmatrix} z(t) \\ z(t - \delta(t)) \\ z(t - \delta(t)) \\ z(t - \delta_{U}) \end{bmatrix}^{T} \\ &\times \begin{bmatrix} M_{1} + M_{1}^{T} & -M_{1}^{T} + M_{2} & 0 \\ * & M_{1} + M_{1}^{T} - M_{2} - M_{2}^{T} & -M_{1}^{T} + M_{2} \\ * & & -M_{2} - M_{2}^{T} \end{bmatrix} \begin{bmatrix} z(t) \\ z(t - \delta(t)) \\ z(t - \delta_{U}) \end{bmatrix} \\ &+ \delta_{U} \begin{bmatrix} z(t) \\ z(t - \delta(t)) \\ z(t - \delta_{U}) \\ z(t - \delta_{U}) \end{bmatrix}^{T} \begin{bmatrix} M_{3} & M_{4} & 0 \\ * & M_{3} + M_{5} & M_{4} \\ * & * & M_{5} \end{bmatrix} \begin{bmatrix} z(t) \\ z(t - \delta(t)) \\ z(t - \delta_{U}) \end{bmatrix} \\ &- e^{-2\alpha\delta_{U}} \begin{bmatrix} z(t) \\ z(t - \delta_{U}) \\ \frac{1}{\delta_{U}} \int_{t - \delta_{U}}^{t} z(s) ds \end{bmatrix}^{T} \begin{bmatrix} 4P_{8} & 2P_{8} & -6P_{8} \\ * & 4P_{8} & -6P_{8} \\ * & * & 12P_{8} \end{bmatrix} \begin{bmatrix} z(t) \\ z(t - \delta_{U}) \\ \frac{1}{\delta_{U}} \int_{t - \delta_{U}}^{t} z(s) ds \end{bmatrix} \\ &- e^{-2\alpha\delta_{U}} \begin{bmatrix} z(t) \\ z(t - \delta_{U}) \\ \frac{1}{\delta_{U}} \int_{t - \delta_{U}}^{t} z(s) ds \end{bmatrix}^{T} \begin{bmatrix} P_{9} & -P_{9} + S & -S \\ * & 2P_{9} - S - S^{T} & -P_{9} + S \\ * & * & P_{9} \end{bmatrix} \begin{bmatrix} z(t) \\ z(t - \delta(t)) \\ z(t - \delta_{U}) \end{bmatrix}$$

$$\begin{split} &+(\delta_{UL})^{2}\dot{z}^{T}(t)[W_{4}+W_{5}+W_{6}]\dot{z}(t)+(\delta_{UL})e^{-2\alpha\delta_{U}}\left(\begin{bmatrix}z(t-\delta_{L})\\z(t-\delta_{U})\\z(t-\delta_{U})\end{bmatrix}^{T}\right) \\ &\times \begin{bmatrix}M_{6}+M_{6}^{T}&-M_{6}^{T}+M_{7}&0*&M_{6}+M_{6}^{T}-M_{7}-M_{7}^{T}&-M_{6}^{T}+M_{7}*&&-M_{7}-M_{7}^{T}\end{bmatrix}\begin{bmatrix}z(t-\delta_{L})\\z(t-\delta_{U})\\z(t-\delta_{U})\end{bmatrix}^{T}\left[M_{8}&M_{9}&0*&M_{8}+M_{10}&M_{9}*&&*&M_{10}\end{bmatrix}\begin{bmatrix}z(t-\delta_{L})\\z(t-\delta_{U})\\z(t-\delta_{U})\end{bmatrix}\right) \\ &-e^{-2\alpha\delta_{U}}\begin{bmatrix}z(t-\delta_{L})\\z(t-\delta_{U})\\\frac{1}{\delta_{UL}}\int_{t-\delta_{U}}^{t-\delta_{L}}z(s)ds\end{bmatrix}^{T}\begin{bmatrix}4W_{5}&2W_{5}&-6W_{5}*&4W_{5}&-6W_{5}*&&*&12W_{5}\end{bmatrix} \\ &\times \begin{bmatrix}z(t-\delta_{L})\\z(t-\delta_{U})\\\frac{1}{\delta_{UL}}\int_{t-\delta_{U}}^{t-\delta_{L}}z(s)ds\end{bmatrix} - e^{-2\alpha\delta_{U}}\left(\int_{t-\delta_{U}}^{t-\delta_{L}}\dot{z}^{T}(s)ds\right)W_{6}\left(\int_{t-\delta_{U}}^{t-\delta_{L}}\dot{z}(s)ds\right) \\ &-2\alpha V_{5}(t). \end{split}$$

According to Lemma 2.5 and calculating $\dot{V}_6(t)$, we obtain

$$\begin{split} \dot{V}_{6}(t) &\leq \delta_{U}^{2} \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix}^{T} \begin{bmatrix} R_{1} & R_{2} \\ * & R_{3} \end{bmatrix} \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix} + e^{-2\alpha\delta_{U}} \begin{bmatrix} z(t) \\ z(t-\delta(t)) \\ z(t-\delta_{U}) \\ \int_{t-\delta(t)}^{t} z(s)ds \\ \int_{t-\delta(t)}^{t-\delta(t)} z(s)ds \end{bmatrix}^{T} \\ &\times \begin{bmatrix} -R_{3} & R_{3} & 0 & -R_{2}^{T} & 0 \\ * & -R_{3} - R_{3} & R_{3} & R_{2}^{T} & -R_{2}^{T} \\ * & * & -R_{3} & 0 & R_{2}^{T} \\ * & * & * & -R_{1} & 0 \\ * & * & * & * & -R_{1} \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-\delta_{U}) \\ \int_{t-\delta(t)}^{t-\delta(t)} z(s)ds \end{bmatrix} \\ &+ (\delta_{UL})^{2} \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix}^{T} \begin{bmatrix} R_{4} & R_{5} \\ * & R_{6} \end{bmatrix} \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix} + e^{-2\alpha\delta_{U}} \begin{bmatrix} z(t-\delta_{L}) \\ z(t-\delta(t)) \\ z(t-\delta(t)) \\ z(t-\delta(t)) \\ \int_{t-\delta(t)}^{t-\delta(L)} z(s)ds \end{bmatrix}^{T} \\ &\times \begin{bmatrix} -R_{6} & R_{6} & 0 & -R_{5}^{T} & 0 \\ * & -R_{6} - R_{6} & R_{6} & R_{5}^{T} & -R_{5}^{T} \\ * & * & -R_{4} & 0 \\ * & * & * & -R_{4} \end{bmatrix} \begin{bmatrix} z(t-\delta_{L}) \\ z(t-\delta(t)) \\ z$$

Now, applying Lamma 2.3, differenting lead to

$$\begin{split} \dot{V}_{7}(t) &\leq \frac{\delta_{U}^{4}}{4} \dot{z}^{T}(t) P_{10} \dot{z}(t) - e^{-4\alpha\delta_{U}} \left(\delta_{U} z(t) - \int_{t-\delta(t)}^{t} z(s) ds \right. \\ &\left. - \int_{t-\delta_{U}}^{t-\delta(t)} z(s) ds \right)^{T} P_{10} \left(\delta_{U} z(t) - \int_{t-\delta(t)}^{t} z(s) ds - \int_{t-\delta_{U}}^{t-\delta(t)} z(s) ds \right) \\ &\left. + \frac{(\delta_{U}^{2} - \delta_{L}^{2})^{2}}{4} \dot{z}^{T}(t) W_{7} \dot{z}(t) - e^{-4\alpha\delta_{U}} \left((\delta_{UL}) z(t) - \int_{t-\delta(t)}^{t-\delta_{L}} z(s) ds \right) \\ &\left. - \int_{t-\delta_{U}}^{t-\delta(t)} z(s) ds \right)^{T} W_{7} \left((\delta_{UL}) z(t) - \int_{t-\delta(t)}^{t-\delta_{L}} z(s) ds \right) \\ &\left. - \int_{t-\delta_{U}}^{t-\delta(t)} z(s) ds \right) - 2\alpha V_{7}(t). \end{split}$$

Besides, for any scalar $s \in [t - \eta_U, t]$, we obtain $e^{-2\alpha\eta_U} \leq e^{-2\alpha\eta_L} \leq e^{2\alpha(s-t)} \leq 1$. The time derivative of $V_8(t)$ is calculated as

$$\begin{aligned} \dot{V}_8(t) &\leq \dot{z}^T(t)(P_{11} + W_9)\dot{z}(t) - e^{-2\alpha\eta_U}\dot{z}^T(t - \eta(t))P_{11}\dot{z}(t - \eta(t)) \\ &+ \eta_d \dot{z}^T(t - \eta(t))P_{11}\dot{z}(t - \eta(t)) + (\eta_U - \eta_L)e^{-2\alpha\eta_U} \\ &\times \dot{z}^T(t - \eta_L)W_8\dot{z}(t - \eta_L) - (\eta_U - \eta_L)e^{-2\alpha\eta_U}\dot{z}^T(t - \eta_U) \\ &\times W_8\dot{z}(t - \eta_U) - e^{-2\alpha\eta_L}\dot{z}^T(t - \eta_L)W_9\dot{z}(t - \eta_L - 2\alpha V_8(t). \end{aligned}$$

Further, for any scalar $s \in [t - g_U, t]$, we have $e^{-2\alpha g_U} \leq e^{\alpha(s-t)} \leq 1$. Combine with Lamma 2.2, we obtain $\dot{V}_9(t)$ as follows

$$\begin{aligned} \dot{V}_{9}(t) &\leq g_{U}^{2} z^{T}(t) P_{12} z(t) - e^{-2\alpha g_{U}} \int_{t-g(t)}^{t} z^{T}(s) ds P_{12} \int_{t-g(t)}^{t} z(s) ds \\ &+ (g_{UL})^{2} z^{T}(t) W_{10} z(t) - e^{-2\alpha g_{U}} \int_{t-g_{U}}^{t-g_{L}} z^{T}(s) ds W_{10} \\ &\times \int_{t-g_{U}}^{t-g_{L}} z(s) ds - 2\alpha V_{9}(t). \end{aligned}$$

Consider (2.5)-(2.7), we inspected that the following inequalities hold:

$$\nu_a(\alpha_a^2 z^T(t) z(t) - g_a^T(t, z(t)) g_a(t, z(t)))) \ge 0, \qquad (3.15)$$

$$\nu_b(\alpha_b^2 z^T (t - \delta(t)) z(t - \delta(t)) g_b^T (t, z(t - \delta(t)) g_b(t, z(t - \delta(t)))) \ge 0, \qquad (3.16)$$

$$\nu_c(\alpha_c^2 \dot{z}^T (t - \eta(t)) \dot{z} (t - \eta(t)) - g_c^T (t, \dot{z} (t - \eta(t)) g_c (t, \dot{z} (t - \eta(t)))) \ge 0,$$
(3.17)

where ν_a , ν_b , ν_c , are positive real constants.

By the use zero equations, we obtain the following equations

$$2\left[z^{T}(t)N_{1}^{T} + z^{T}(t - \delta(t))N_{2}^{T} + \dot{z}^{T}(t)N_{3}^{T} + \dot{z}^{T}(t - \eta(t)))N_{4}^{T} + g_{a}(t, z(t))N_{5}^{T} + g_{b}(t, z(t - \delta(t)))N_{6}^{T} + g_{c}(t, z(t - \eta(t)))N_{7}^{T} + \int_{t-g(t)}^{t} z^{T}(s)dsN_{8}^{T} + \int_{t-\delta(t)}^{t} \dot{z}^{T}(s)dsN_{9}^{T}\right] \left[-\dot{z}(t) + A_{1}z(t)) + (A_{2} + B)z(t - \delta(t)) + A_{2}\int_{t-\delta(t)}^{t} \dot{z}^{T}(s)ds + C\dot{z}(t - \eta(t)) + D\int_{t-g(t)}^{t} z(s)ds + g_{a}(t, z(t)) + g_{b}(t, z(t - \delta(t))) + g_{c}(t, z(t - \eta(t)))\right] = 0,$$

$$(3.18)$$

$$2\left[\dot{z}^{T}(t)N_{10}^{T} + w^{T}(t)N_{11}^{T}\right]\left[\dot{z}(t) - w(t)\right] = 0, \qquad (3.19)$$

where any real matrices N_m , m = 1, 2, ..., 11 with appropriate dimensions. Due to the use (3.15)-(3.19), it is apparently that

$$\dot{V}(t) + 2\alpha V(t) \leq \xi^{T}(t)\Lambda\xi(t),$$
(3.20)
where $\xi^{T}(t) = [z(t), z(t-\delta(t)), z(t-\delta_{U}), \dot{z}(t), \dot{z}(t-\eta(t)), \int_{t-\delta(t)}^{t} z(s)ds, \int_{t-\delta_{U}}^{t-\delta(t)} z(s)ds, \frac{1}{\delta_{U}} \int_{t-\delta_{U}}^{t} z(s)ds, \int_{t-\delta(t)}^{t} \dot{z}(s)ds, g_{a}(t, z(t)), g_{b}(t, z(t-\delta(t))), g_{c}(t, \dot{z}(t-\eta(t))), z(t-\delta_{L}), \int_{t-\delta(t)}^{t-\delta_{L}} z(s)ds, \frac{1}{(\delta_{UL})} \int_{t-\delta_{U}}^{t-\delta_{L}} z(s)ds, \int_{t-\delta_{U}}^{t-\delta_{L}} \dot{z}(s)ds,$

 $w(t), \dot{z}(t-\eta_L), \dot{z}(t-\eta_U), \int_{t-g(t)}^{t} z(s)ds, \int_{t-g_U}^{t-g_L} z(s)ds$ and Λ is defined in (3.2). By condition (3.8), we obtain

$$\dot{V}(t) + 2\alpha V(t) \le 0, \qquad \forall t \in \mathbb{R}^+,$$
(3.21)

which gives

$$\dot{V}(0) \le V(0)e^{-2\alpha t}, \quad \forall t \in \mathbb{R}^+.$$
 (3.22)

It is readily visible that

$$\lambda_{\min}(P_1) \| z(t) \|^2 \le V(t) \le V(0) e^{-2\alpha t} \le N \max\{\|\Phi\}, \|\Psi\|\}^2 e^{-2\alpha t}, \quad \forall t \in \mathbb{R}^+,$$

and

$$||z(t, \Phi, \Psi)|| \le \sqrt{\frac{N}{\lambda_{\min}(P_1)}} \max\{||\Phi\}, ||\Psi||\} e^{-\alpha t}, \quad \forall t \in \mathbb{R}^+,$$
 (3.23)

where

$$N = \lambda_{\max}(P_1 + P_2) + \delta_U \lambda_{\max}(P_3 + P_4) + (\delta_{UL})\lambda_{\max}(W_1) + \delta_U^3 \lambda_{\max}(P_5 + P_6 + P_7 + P_8 + P_9) + (\delta_{UL})^3 \lambda_{\max}(W_2 + W_3 + W_4 + W_5 + W_6) + \delta_U^3 \lambda_{\max} \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} + (\delta_{UL})^3 \lambda_{\max} \begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} + \frac{\delta_U^5}{2} \lambda_{\max}(P_{10}) + \frac{(\delta_U^2 - \delta_L^2)}{2} (\delta_{UL})^3 \lambda_{\max}(W_7) + \eta_U \lambda_{\max}(P_{11}) + (\eta_{UL})^2 \lambda_{\max}(W_8) + \eta_L \lambda_{\max}(W_9) + g_U^3 \lambda_{\max}(P_{12}) + (g_{UL})^3 \lambda_{\max}(W_{10}).$$

Therefore, if the LMIs conditions (3.3)-(3.8) hold, we conclude that the system (3.1) is exponentially stable. This proof is complete.

Based on Theorem 3.1, we consider the new delay-interval-dependent robust exponential stability for uncertain neutral-type system with distributed and discrete time-varying delays, and nonlinear perturbations for (2.1). Then, the corresponding result is summarized in Theorem 3.2.

Theorem 3.2. For $||C(t)|| + \alpha_c < 1$, $\alpha > 0$, if there exist positive definite symmetric matrices P_i , W_j , i = 1, 2, ..., 12, j = 1, 2, ..., 10, any appropriate dimensional matrices S, Q_k , M_l , N_m , R_n , k = 1, 2, ..., 14, l = 1, 2, ..., 10, m = 1, 2, ..., 11, n = 1, 2, ..., 6, and positive real constants $\beta \alpha_s$, ν_s , s = a, b, c, satisfying the following LMIs (3.3)-(3.7) and

$$\begin{bmatrix} \Lambda & \Gamma_1 & \beta \Gamma_2^T \\ * & -\beta I & \beta J^T \\ * & * & -I \end{bmatrix} < 0,$$
(3.24)

then the system (2.1) is robustly exponentially stable.

Proof. Result of the use the similar method in the proof of Theorem 3.1, and substitution A_1 , B, C and D in LMI (3.8) with $A_1 + L\Delta(t)G_a$, $B + L\Delta(t)G_b$, $C + L\Delta(t)G_c$ and $D + L\Delta(t)G_d$, respectively, we conclude that condition (3.8) for system (2.1) is equivalent to following are required

$$\Lambda + \Gamma_1 \Delta(t) \Gamma_2 + \Gamma_2^T \Delta^T(t) \Gamma_1^T < 0, \qquad (3.25)$$

4. Numerical examples

In this section, we allow the numerical examples to show the performance of the systems (2.1) and (3.1).

Example 4.1. Consider the uncertain neutral-type system with distributed and discrete interval time-varying delays, and nonlinear perturbations (2.1), where

$$A_{1} = \begin{bmatrix} -0.8 & 0\\ 0.1 & -0.8 \end{bmatrix}, A_{2} = \begin{bmatrix} -0.1 & 0.2\\ 0 & -0.1 \end{bmatrix}, B = \begin{bmatrix} -1.1 & -0.2\\ -0.1 & -1.1 \end{bmatrix}, C = \begin{bmatrix} -0.2 & 0\\ 0.2 & -0.1 \end{bmatrix}, D = \begin{bmatrix} -0.12 & -0.12\\ -0.12 & 0.12 \end{bmatrix}, L = I, G_{a} = G_{b} = G_{c} = G_{d} = 0.1I, \alpha_{a} = 0.1, \alpha_{b} = \alpha_{c} = 0.05.$$

We have found that the LMI (3.24) is feasible, which consider for $\eta_L = g_L = 0.1$, $\eta_U = g_U = 0.2$, and $\eta_d = \delta_d$. In Table 1, we show the maximum allowable bound δ_U for ensuring Theorem 3.2 of the system(2.1), which for the exponential convergence rate $\alpha = 0.5$, $\eta_d = \delta_d = 0.1$ and $\delta_L = 0.5$, we obtain $\delta_U = 0.7487$.

$\delta_d = \eta_d$	δ_L	$\alpha = 0.0$	$\alpha = 0.1$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
0.0	0.5	1.2526	1.0711	0.8731	0.7487	0.6250	0.5442
	1.0	1.4990	1.3550	1.2069	1.0560	-	-
0.1	0.5	1.2108	1.4010	0.8510	0.7340	0.6121	0.5346
	1.0	1.4625	1.3352	1.1951	1.0507	-	-

TABLE 1. Upper bounds of time delay δ_U for various values of α and δ_L .

Example 4.2. Consider the uncertain neutral-type system with distributed and discrete interval time-varying delays (2.1), where

$A_1 =$	$\begin{bmatrix} -0.8\\ 0.1 \end{bmatrix}$	$\begin{bmatrix} 0\\ -0.8 \end{bmatrix}$	$, A_2 =$	$\begin{bmatrix} -0.1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix}$, B =	$\begin{bmatrix} -1.1 \\ -0.1 \end{bmatrix}$	$\begin{bmatrix} -0.2 \\ -1.1 \end{bmatrix}$,	C =	$\begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -0.1 \end{bmatrix}$,
D =	$-0.12 \\ -0.12$	-0.12 0.12	, L = I,	$G_a = 0$	$G_b = G$	$G_c = G_d =$	= 0.1I,					
$g_a(t, z)$	$\bar{g}(t)) = g$	$v_b(t, z(t))$	$-\delta(t))) =$	$= g_c(t, z)$	$\dot{z}(t-\eta$	(t))) = 0	•	_	_			

By appying Theorem 3.2, we show the upper bounds on distributed time delay g_U for different α , which apply the conditions in [19], [20] and [21], where $\eta_L = \delta_L = g_L = 0$, $\eta_U = \delta_U = 0.1$ and $\eta_d = \delta_d = 0$. It is clear that our result (Theorem 3.2) are better than those results, which appeared in [19], [20] and [21]. Moreover, we show the ensuring exponential stability of system (2.1).

TABLE 2. Upper bounds of time delay g_U for various values of α .

Method	$\alpha = 0.0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.5$
Chen et al. $[19]$	6.67	-	-	-	-
Chen et al. $[20]$	6.67	-	-	-	-
Zhu et al. $[21]$	6.8925	-	-	-	-
Theorem 3.2	7.2682	4.4142	3.3197	2.6887	1.9472

5. Conclusions

The problem of delay-interval-dependent robust exponential stability criterion for uncertain neutral-type system with distributed and discrete time-varying delays, and nonlinear perturbations was studied. We concentrated on norm-bounded uncertainties and nonlinear time-varying parameter perturbations. New delay-interval-dependent robust exponential stability criterion for uncertain neutral-type system with distributed and discrete interval time-varying delays, and nonlinear perturbations was received and formulated in terms of LMIs by using mixed model transformation, Peng-Park's integral inequality, Wirting-based integral inequality and proper Lyapunov-Krasovskii function. Moreover, Exponential stability criterion for a neutral-type system with distributed and discrete interval time-varying delays, and nonlinear perturbations was presented as well. In the examples, we were presented some results that showed the potential of our results surpass those results were previously seen.

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