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NEW EXPONENTIAL STABILITY CRITERION FOR NEUTRAL SYSTEM WITH INTERVAL TIME-VARYING MIXED DELAYS AND NONLINEAR UNCERTAINTIES

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Abstract The paper investigates the exponential stability analysis of neutral system with interval time-varying discrete, neutral and distributed delays, and nonlinear uncertainties. The uncertainties under consideration are nonlinear time-varying parameter perturbations. Based on Jensen's integral inequality, Wirtinger-base integral inequality, Leibniz-Newton fomula, Peng-Park's integral inequality, mixed model transformation, utilization of zero equation, decomposition matrix technique and the appropriate Lyapunov-Krasovskii functional (LKF), a new delay-range-dependent exponential stability criterion is constructed as linear matrix inequalities (LMIs) for considered system. Furthermore, we derive the improved delay-range-dependent exponential stability criterion of neutral system with discrete and neutral time-varying delays, and nonlinear uncertainties. Numerical examples are proposed to show the usefulness of our method.

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1. Introduction

The research of neutral time-delay systems, which include delays both in the state and in the derivatives of the state, has received considerable attention during the past few decades due to their extensive applications in modeling dynamic behavior of many biological and cognitive activities such as locomotion, mastication, heartbeat, memorization and respiration, see [3, 4, 8]. On the other hand, nonlinear uncertainties are commonly encountered because it is very problematic to derive a certain mathematical model as result of slowly varying parameters, environmental noise, and so on. Therefore,

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special consideration has been given to study the delay-dependent stability criteria for neutral time-delay systems with nonlinear uncertainties in recent years [1, 4, 7, 10, 13–15]. Delay-dependent make use of information on the length of delays. In [6], Lakshmanan investigated the problem for neutral system with time-varying delays by using the new LyapunovKrasovskii functional with triple integral terms, some integral inequalities and convex combination technique. Mohajerpoor [11] studied delay-dependent robust stability problem for neutral system with time-varying delays with the use of the linear matrix inequality technology and Lyapunov functional approach. Rakkiyappan et al. [15] derived the asymptotic stability for neutral systems with interval time-varying delays and nonlinear perturbations by using a new Lyapunov functional and some integral inequalities without introducing any free-weighting matrices.

The results claimed above are only associated with the asymptotic stability. However, the exponential stability problem is also significant because it can set the convergence rate of system states to equilibrium points. Liu et al. [9] investigated the global exponential stability criteria of neutral systems with interval time-varying delays and nonlinear uncertainties with the use of the lower bounds lemma, delay-partitioning technique and LyapunovKrasovskii stability theory. Ali [1] proposed the exponential stability criteria by using the generalized eigenvalue problem approach and the free-weighting matrix method. Liu et al. [10] studied the exponential stability criteria for neutral system with nonlinear uncertainties by using the free-weighting matrices methods within a convex optimization approach.

Motivated by above observations, the problem of exponential stability for neutral system with interval time-varying discrete, neutral and distributed delays, and nonlinear uncertainties is studied. Based on Jensen's integral inequality, Wirtinger-base integral inequality, Leibniz-Newton fomula, Peng-Park's integral inequality, mixed model transformation, utilization of zero equation, decomposition matrix technique and the appropriate Lyapunov-Krasovskii functional (LKF), a new delay-range-dependent exponential stability criterion for the proposed system is constructed in the form of linear matrix inequalities (LMIs). Furthermore, we obtain the improved delay-range-dependent exponential stability criterion of neutral system with interval time-varying discrete and neutral delays, and nonlinear uncertainties. We present the performance of the proposed method by two numerical examples.

2. Preliminaries

We introduce the following neutral system with interval time-varying delays and nonlinear uncertainties of the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bx(t-\lambda(t)) + C\dot{x}(t-\sigma(t)) + f_1(t,x(t)) + f_2(t,x(t-\lambda(t))) \\ &\quad + f_3(t,\dot{x}(t-\rho(t))) + D\int_{t-\rho(t)}^{t} x(s)ds, \quad t \geq 0, \qquad (2.1) \\ x(t) &= \phi(t), \quad \forall t \in [-\max\{\lambda_2,\sigma_2,\rho_2\},0], \end{aligned}$$

where $x(t) \in R^n$ is the state variable, $\phi(t)$ is continuously real-valued function on $[-\max\{\lambda_2,\sigma_2,\rho_2\},0]$ with $\|\phi\| = \sup_{s\in[-\max\{\lambda_2,\sigma_2,\rho_2\},0]} \|\phi(s)\|$. A, B, C, $D \in R^{n\times n}$.

$\lambda(t)$, $\sigma(t)$ and $\rho(t)$ are time-varying discrete, neutral and distributed delays, respectively,

$$0 \le \lambda_1 \le \lambda(t) \le \lambda_2, \quad 0 \le \dot{\lambda}(t) \le \lambda_d, \tag{2.2}$$
$$0 \le \sigma_1 \le \sigma(t) \le \sigma_2, \quad 0 \le \dot{\sigma}(t) \le \sigma_d, \tag{2.3}$$
$$0 \le \rho_1 \le \rho(t) \le \rho_2, \quad 0 \le \dot{\rho}(t) \le \rho_d, \tag{2.4}$$

where σ_1, σ_2, σ_d, λ_1, λ_2, λ_d, ρ_1, ρ_2 and ρ_d are positive real constants. $f_1(t, x(t))$, $f_2(t, x(t-\lambda(t)))$ and $f_3(t, \dot{x}(t-\sigma(t)))$ are nonlinear uncertainties and are assumed to satisfy the following inequalities

$$\delta_1^T(t)\delta_1(t) \le \alpha_1^2 x^T(t)x(t), \tag{2.5}$$
$$\delta_2^T(t)\delta_2(t) \le \alpha_2^2 x^T(t-\lambda(t))x(t-\lambda(t)), \tag{2.6}$$
$$\delta_3^T(t)\delta_3(t) \le \alpha_3^2 \dot{x}^T(t-\sigma(t))\dot{x}(t-\sigma(t)), \tag{2.7}$$

where $\delta_1(t) = f_1(t, x(t))$, $\delta_2(t) = f_2(t, x(t-\lambda(t)))$ and $\delta_3(t) = f_3(t, \dot{x}(t-\sigma(t)))$. α_1, α_2 and α_3 are know positive real constants. The Leibniz-Newton equation is considered

$$0 \;=\; x(t) - x(t-\lambda(t)) - \int_{t-\lambda(t)}^{t} \dot{x}(s)ds. \tag{2.8}$$

For $E \in R^{n\times n}$ will be chosen to assure the exponential stability of system (2.1), we utilize the previous Leibniz-Newton equation

$$0 \;=\; Ex(t) - Ex(t-\lambda(t)) - E\int_{t-\lambda(t)}^{t} \dot{x}(s)ds. \tag{2.9}$$

To improve the discrete delay $\lambda(t)$ in (2.2), we separate constant matrix B as

$$B = B_1 + B_2, \tag{2.10}$$

where B_1, $B_2 \in R^{n\times n}$ are constant matrices. By (2.9) and (2.10), system (2.1) can be represented by the form

$$\begin{aligned}\dot{x}(t) \;=\;& \big[A + B_1 + E\big]x(t) + \big[B_2 - E\big]x(t-\lambda(t)) + C\dot{x}(t-\sigma(t)) + f_1(t, x(t)) \\ &+ f_2(t, x(t-\lambda(t))) + f_3(t, \dot{x}(t-\sigma(t))) - \big[B_1 + E\big]\int_{t-\lambda(t)}^{t} \dot{x}(s)ds \\ &+ D\int_{t-\rho(t)}^{t} x(s)ds. \end{aligned}\tag{2.11}$$

We rewrite system (2.1) by using the model transformation method as follow

$$\dot{x}(t) \;=\; z(t), \tag{2.12}$$
$$\begin{aligned}0 \;=\;& -z(t) + \big[A + B_1\big]x(t) + Bx(t-\lambda(t)) + C\dot{x}(t-\sigma(t)) + f_1(t, x(t)) \\ &+ f_2(t, x(t-\lambda(t))) + f_3(t, \dot{x}(t-\sigma(t))) - B_1\int_{t-\lambda(t)}^{t} \dot{x}(s)ds \\ &+ D\int_{t-\rho(t)}^{t} x(s)ds. \end{aligned}\tag{2.13}$$

Definition 2.1. [5] If there exist scalars $\beta > 0$ and $\alpha > 0$ satisfy:

$$\|x(t, \phi)\| \le \beta\|\phi\|e^{-\alpha t}, \quad \forall t \ge 0,$$

then system (2.1) is **exponentially stable**.

Lemma 2.2. *(Jensen's inequality [18]) For any positive definite symmetric matrix $W \in R^{n\times n}$, $0 < k(t) < k_2$ and vector function $\dot{\omega} : [-k_2, 0] \to R^n$ be well-defined, then*

$$-k_2 \int_{-k_2}^{0} \dot{\omega}^T(s+t) W \dot{\omega}(s+t) ds \leq -\Big(\int_{-k_2}^{0} \dot{\omega}(s+t) ds \Big)^T W \Big(\int_{-k_2}^{0} \dot{\omega}(s+t) ds \Big).$$

Lemma 2.3. [2] *For any positive definite symmetric matrix $W \in R^{n\times n}$, $0 < k(t) < k_2$ and vector function $\dot{\omega} : [-k_2, 0] \to R^n$ be well-defined, then*

$$-\int_{-k_2}^{0} \int_{t+s}^{t} \dot{\omega}^T(u) W \dot{\omega}(u) du ds \leq \psi_1^T(t) \Omega_1 \psi_1(t),$$

where $\psi_1(t) = \begin{bmatrix} \omega(t) \\ \frac{1}{k_2} \int_{t-k_2}^{t} \omega(s) ds \end{bmatrix}$ *and* $\Omega_1 = \begin{bmatrix} -2W & 2W \\ * & -2W \end{bmatrix}$.

Lemma 2.4. *(Sun et al. [19]) For any positive definite symmetric matrix $W \in R^{n\times n}$, $0 < k_1 < k(t) < k_2$ and vector function $\dot{\omega} : [-k_2, -k_1] \to R^n$ be well-defined, then*

$$-k_2 \int_{t-k_2}^{t} \dot{\omega}^T(s) W \dot{\omega}(s) ds \leq -\psi_2^T(t) W \psi_2(t),$$
$$-\frac{(k_2^2 - k_1^2)}{2} \int_{-k_2}^{-k_1} \int_{t+s}^{t} \omega^T(u) W \omega(u) du ds \leq -\psi_3^T(t) W \psi_3(t),$$

where $\psi_2(t) = \Big(\int_{t-k_2}^{t} \dot{\omega}(s) ds \Big)$ *and* $\psi_3(t) = \Big(\int_{-k_2}^{-k_1} \int_{t+s}^{t} \omega(u) du ds \Big)$.

Lemma 2.5. [18] *For any positive definite symmetric matrix $W \in R^{n\times n}$, $0 < k_1 < k(t) < k_2$ and vector function $\omega : [-k_2, -k_1] \to R^n$ be well-defined, then*

$$-[k_2 - k_1] \int_{t-k_2}^{t-k_1} \omega^T(s) W \omega(s) ds \leq -\psi_4^T(t) W \psi_4(t) - \psi_5^T(t) W \psi_5(t),$$

where $\psi_4(t) = \int_{t-k(t)}^{t-k_1} \omega(s) ds$ *and* $\psi_5(t) = \int_{t-k_2}^{t-k(t)} \omega(s) ds$.

Lemma 2.6. [18] *For any constant matrices $Y_1, Y_2, Y_3 \in R^{n\times n}$, $Y_1 \geq 0, Y_3 > 0$, $\begin{bmatrix} Y_1 & Y_2 \\ * & Y_3 \end{bmatrix} \geq 0$, $k(t)$ is time-varying delay with $0 \leq k_1 \leq k(t) \leq k_2$, $k_1, k_2 \in R$ and vector functions $\dot{\omega} : [-k_2, -k_1] \to R^n$ be well-defined, then*

$$-(k_2 - k_1) \int_{t-k_2}^{t-k_1} \begin{bmatrix} \omega(s) \\ \dot{\omega}(s) \end{bmatrix}^T \begin{bmatrix} Y_1 & Y_2 \\ * & Y_3 \end{bmatrix} \begin{bmatrix} \omega(s) \\ \dot{\omega}(s) \end{bmatrix} ds \leq \psi_6^T(t) \Omega_2 \psi_6(t),$$

where $\psi_6(t) = \begin{bmatrix} \omega(t-k_1) \\ \omega(t-k(t)) \\ \omega(t-k_2) \\ \int_{t-k(t)}^{t-k_1} \omega(s) ds \\ \int_{t-k_2}^{t-k(t)} \omega(s) ds \end{bmatrix}$ *and* $\Omega_2 = \begin{bmatrix} -Y_3 & Y_3 & 0 & -Y_2^T & 0 \\ * & -Y_3 - Y_3^T & Y_3 & Y_2^T & -Y_2^T \\ * & * & -Y_3 & 0 & Y_2^T \\ * & * & * & -Y_1 & 0 \\ * & * & * & * & -Y_1 \end{bmatrix}$.

Lemma 2.7. [18] *For any constant matrices $W, Y_i \in R^{n\times n}$, $i = 4, 5, ..., 8$, $0 \leq k_1 \leq k(t) \leq k_2$ and vector function $\dot{\omega} : [-k_2, -k_1] \to R^n$ be well-defined, then*

$$-\int_{t-k_2}^{t-k_1} \dot{\omega}^T(s) W \dot{\omega}(s) ds \quad \leq \quad \psi_7^T(t) \Omega_3 \psi_7(t) + (k_2 - k_1) \psi_7^T(t) \Omega_4 \psi_7(t),$$

where $\psi_7(t) = \begin{bmatrix} \omega(t-k_1) \\ \omega(t-k(t)) \\ \omega(t-k_2) \end{bmatrix}$, $\Omega_3 = \begin{bmatrix} Y_4+Y_4^T & -Y_4^T+Y_5 & 0 \\ * & Y_4+Y_4^T-Y_5-Y_5^T & -Y_4^T+Y_5 \\ * & * & -Y_5-Y_5^T \end{bmatrix}$,

$\Omega_4 = \begin{bmatrix} Y_6 & Y_7 & 0 \\ * & Y_6+Y_8 & Y_7 \\ * & * & Y_8 \end{bmatrix}$ *and* $\begin{bmatrix} W & Y_4 & Y_5 \\ * & Y_6 & Y_7 \\ * & * & Y_8 \end{bmatrix} \geq 0.$

Lemma 2.8. ($Wirtinger - based\ integral\ inequality$ [17]) *For any positive definite symmetric matrix* $W \in R^{n\times n}$, $0 < k_1 < k(t) < k_2$ *and vector function* $\dot{\omega} : [-k_2, -k_1] \to R^n$ *be well-defined, then*

$$-(k_2-k_1)\int_{t-k_2}^{t-k_1} \dot{\omega}^T(s)W\dot{\omega}(s)ds \leq \psi_8^T(t)\Omega_5\psi_8(t),$$

where $\psi_8(t) = \left[\omega^T(t-k_1),\ \omega^T(t-k_2),\ \frac{1}{k_2-k_1}\int_{t-k_2}^{t-k_1}\omega^T(s)ds\right]^T$
and $\Omega_5 = \begin{bmatrix} -4W & -2W & 6W \\ * & -4W & 6W \\ * & * & -12W \end{bmatrix}$.

Lemma 2.9. ($Peng - Park's\ integral\ inequality$ [12]) *Let* W *and* S *be real constant matrices such that* $\begin{bmatrix} W & S \\ * & W \end{bmatrix} \geq 0$, k_2 *and* $k(t)$ *be positive scalars satisfying* $0 < k(t) < k_2$, *vector function* $\dot{\omega} : [-k_2, 0] \to R^n$ *be well-defined, then*

$$-k_2\int_{t-k_2}^{t} \dot{\omega}^T(s)W\dot{\omega}(s)ds \leq \psi_9^T(t)\Omega_6\psi_9(t),$$

where $\psi_9(t) = \begin{bmatrix} \omega(t) \\ \omega(t-k(t)) \\ \omega(t-k_2) \end{bmatrix}$ *and* $\Omega_6 = \begin{bmatrix} -W & W-S & S \\ * & -2W+S+S^T & W-S \\ * & * & -W \end{bmatrix}$.

Lemma 2.10. *(An extended Wirtinger's integral inequality* [20]*) For any positive definite symmetric matrix* $W \in R^{n\times n}$, *scalars* k_1, k_2 *and vector function* $\omega : [k_1, k_2] \to R^n$ *be well-defined, then*

$$(k_2-k_1)\int_{k_1}^{k_2} \omega^T(u)W\omega(u)du \geq {\Omega_7}^T W\Omega_7 + 3{\Omega_8}^T W\Omega_8 + 5{\Omega_9}^T W\Omega_9, \tag{2.14}$$

where $\Omega_7 = \int_{k_1}^{k_2}\omega(u)du$, $\Omega_8 = \int_{k_1}^{k_2}\omega(u)du - \frac{2}{k_2-k_1}\int_{k_1}^{k_2}du\int_{k_1}^{u}\omega(r)dr$
and $\Omega_9 = \int_{k_1}^{k_2}\omega(u)du - \frac{6}{k_2-k_1}\int_{k_1}^{k_2}du\int_{k_1}^{u}\omega(r)dr + \frac{12}{(k_2-k_1)^2}\int_{k_1}^{k_2}du\int_{k_1}^{u}ds\int_{k_1}^{s}\omega(r)dr.$

Lemma 2.11. [21] *For any positive definite symmetric matrix* $W \in R^{n\times n}$, *scalars* k_1, k_2 *and vector function* $\dot{\omega} : [k_1, k_2] \to R^n$ *be well-defined, then*

$$\int_{k_1}^{k_2}\int_{u}^{k_2} \dot{\omega}^T(s)W\dot{\omega}(s)dsdu \geq 2\Omega_{10}^T W\Omega_{10} + 4\Omega_{11}^T W\Omega_{11} + 6\Omega_{12}^T W\Omega_{12}, \tag{2.15}$$

where $\Omega_{10} = \omega(k_2) - \frac{1}{k_2-k_1}\int_{k_1}^{k_2}\omega(s)ds$,
$\Omega_{11} = \omega(k_2) + \frac{2}{k_2-k_1}\int_{k_1}^{k_2}\omega(s)ds - \frac{6}{(k_2-k_1)^2}\int_{k_1}^{k_2}\int_{u}^{k_2}\omega(s)dsdu$
and $\Omega_{12} = \omega(k_2) - \frac{3}{k_2-k_1}\int_{k_1}^{k_2}\omega(s)ds + \frac{24}{(k_2-k_1)^2}\int_{k_1}^{k_2}\int_{u}^{k_2}\omega(s)dsdu$
$-\frac{60}{(k_2-k_1)^3}\int_{k_1}^{k_2}\int_{u}^{k_2}\int_{s}^{k_2}\omega(r)drdsdu.$

3. Main Results

The purpose of this section is to show our main results. We define a new parameter

$$\Xi = \left[\Xi_{(i,j)}\right]_{27\times 27}, \tag{3.1}$$

where $\Xi_{(i,j)} = \Xi^T_{(j,i)}$,
$\Xi_{1,1} = Z_1A + Z_1B_1 + W + A^TZ_1^T + B_1^TZ_1^T + x^T + Q_1^T + Q_5^T + Q_1 + Q_5 + Q_9^TA + Q_9^TB_1 + A^TQ_9 + B_1^TQ_9 + Z_3 + \lambda_2 Z_{26} + \lambda_2^2 Z_{11} + \lambda_2^2 Z_{12} + \lambda_2^2 G_1 - e^{-2\alpha\lambda_2}G_3 - e^{-4\alpha\lambda_2}\lambda_2^2 Z_{20} - 2e^{-4\alpha\lambda_2}\lambda_2^2 Z_{21} + e^{-2\alpha\lambda_2}[F_1 + F_1^T - 4Z_{16} - Z_{17} + \lambda_2 F_3] + \varepsilon_1\eta^2 I + 2\alpha Z_1 + \lambda_2^2 Z_{18} - 12e^{-2\alpha\lambda_2}Z_{22} + 2\alpha Z_2 + Z_4 - \lambda_1 Z_{26} + \lambda_1^2 Z_{13} + \lambda_2^2 G_4 - 2\lambda_1\lambda_2 G_4 + \lambda_1^2 G_4 + \frac{\lambda_1^2}{2} Z_{23} - e^{-4\alpha\lambda_2}(\lambda_2^2 - 2\lambda_1\lambda_2 + \lambda_1^2)Z_{24} - 2e^{-4\alpha\lambda_2}Z_{23} + \lambda_2^2 Z_{14} - \lambda_1^2 Z_{14}$,
$\Xi_{1,2} = Z_1B_2 - W - Q_1^T - Q_5^T + Q_2 + Q_6 + Q_9^TB_2 + A^TQ_{10} + B_1^TQ_{10}$,
$\Xi_{1,3} = -2e^{-2\alpha\lambda_2}Z_{16} + e^{-2\alpha\lambda_2}S$, $\Xi_{1,4} = \Xi_{1,5} = \Xi_{1,6} = Z_1 + Q_9$,
$\Xi_{1,7} = Z_1C + Q_9C$, $\Xi_{1,8} = -Z_1B_1 - WQ_1^T - Q_5^T + Q_3 + Q_7 - Q_9^TB_1 + A^TQ_{11} + B_1Q_{11}$, $\Xi_{1,9} = -e^{-2\alpha\lambda_2}G_2^T$, $\Xi_{1,11} = \lambda_2^2 e^{-4\alpha\lambda_2}Z_{20} + \lambda_2^2 e^{-4\alpha\lambda_2}Z_{21} + 6e^{-2\alpha\lambda_2}Z_{16} + 12e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{1,12} = Q_4 + Q_8 - Q_9^T + A^TQ_{12} + B_1^TQ_{12} + \lambda_2 G_2 + Q_{13}^T + \lambda_2^2 G_5 - 2\lambda_1\lambda_2 G_5 + \lambda_1^2 G_5$, $\Xi_{1,14} = -120e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{1,16} = 360e^{-2\alpha\lambda_2}Z_{22}$,
$\Xi_{1,17} = A^TQ^{14} + B_1^TQ_{14} + Z_2 - Q_{13}$, $\Xi_{1,18} = e^{-4\alpha\lambda_2}\lambda_2 Z_{24} - e^{-4\alpha\lambda_2}\lambda_1 Z_{24}$,
$\Xi_{1,19} = 2e^{-4\alpha\lambda_2}Z_{23}$, $\Xi_{1,23} = Z_1^TD + Q_9^TD$, $\Xi_{2,1} = B_2^TZ_1^T - x^T - Q_1 - Q_5 + Q_2^T + Q_6^T + B_2^TQ_9 + Q_{10}^TA + Q_{10}^TB_1 - e^{-2\alpha\lambda_2}F_1 + e^{-2\alpha\lambda_2}F_2^T + \lambda_2 e^{-2\alpha\lambda_2}F_4^T + e^{-2\alpha\lambda_2}Z_{17}^T - e^{-2\alpha\lambda_2}S + e^{-2\alpha\lambda_2}G_3$, $\Xi_{2,2} = -Q_2^T - Q_2 - Q_6^T - Q_6 + Q_{10}B_2 + B_2^TQ_{10} - \lambda_2 e^{-2\alpha\lambda_2}Z_{26} + \lambda_2\lambda_d e^{-2\alpha\lambda_2}Z_{26} + e^{-2\alpha\lambda_2}F_1 + e^{-2\alpha\lambda_2}F_1^T - e^{-2\alpha\lambda_2}F_2 - e^{-2\alpha\lambda_2}F_2^T + \lambda_2 e^{-2\alpha\lambda_2}F_3 + \lambda_2 e^{2\alpha\lambda_2}F_5 - 2e^{-2\alpha\lambda_2}Z_{17} + e^{-2\alpha\lambda_2}S + e^{-2\alpha\lambda_2}S^T - e^{-2\alpha\lambda_2}G_3 - e^{-2\alpha\lambda_2}G_3^T + \varepsilon_2 x^2 I - \lambda_1 e^{-2\alpha\lambda_2}Z_{26} + \lambda_1\lambda_d e^{-2\alpha\lambda_2}Z_{26} - e^{-2\alpha\lambda_2}G_6 - e^{-2\alpha\lambda_2}G_6^T + e^{-2\alpha\lambda_2}F_6 + e^{-2\alpha\lambda_2}F_6^T - e^{-2\alpha\lambda_2}F_7 - e^{-2\alpha\lambda_2}F_7^T + \lambda_2 F_8 - \lambda_1 F_8 + F_{10}$,
$\Xi_{2,3} = \lambda_2 e^{-2\alpha\lambda_2}F_4 + e^{-2\alpha\lambda_2}Z_{17} - e^{-2\alpha\lambda_2}S - e^{-2\alpha\lambda_2}F_1^T + e^{-2\alpha\lambda_2}F_2 + e^{-2\alpha\lambda_2}G_3 + e^{-2\alpha\lambda_2}G_6 - e^{-2\alpha\lambda_2}F_6^T + e^{-2\alpha\lambda_2}F_7 + \lambda_2 F_9 - \lambda_1 F_9$, $\Xi_{2,4} = \Xi_{2,5} = \Xi_{2,6} = Q_{10}^T$,
$\Xi_{2,7} = Q_{10}^TC$, $\Xi_{2,8} = -Q_2^T - Q_6^T - Q_3 - Q_7 - Q_{10}^TB_1 + B_2^TQ_{11}$, $\Xi_{2,9} = e^{-2\alpha\lambda_2}G_2^T$,
$\Xi_{2,10} = -e^{-2\alpha\lambda_2}(G_2^T - G_5^T)$, $\Xi_{2,12} = -Q_{10}^T - Q_4 - Q_8 + B_2^TQ_{12}$, $\Xi_{2,17} = B_2^TQ_{14}$,
$\Xi_{2,18} = e^{-2\alpha\lambda_2}G_6^T - e^{-2\alpha\lambda_2}F_6 + e^{-2\alpha\lambda_2}F_7^T + \lambda_2 F_9^T - \lambda_1 F_9^T$, $\Xi_{2,20} = e^{-2\alpha\lambda_2}G_5^T$,
$\Xi_{2,23} = Q_{10}^TD$, $\Xi_{3,1} = -2e^{-2\alpha\lambda_2}Z_{16}^T + e^{-2\alpha\lambda_2}S^T, \Xi_{3,2} = -e^{-2\alpha\lambda_2}F_1 + e^{-2\alpha\lambda_2}F_2^T + \lambda_2 e^{-2\alpha\lambda_2}F_4^T + e^{-2\alpha\lambda_2}Z_{17}^T - e^{-2\alpha\lambda_2}S^T + e^{-2\alpha\lambda_2}G_3^T + e^{-2\alpha\lambda_2}G_6^T - e^{-2\alpha\lambda_2}F_6 + e^{-2\alpha\lambda_2}F_7^T + \lambda_2 F_9^T - \lambda_1 F_9^T$, $\Xi_{3,3} = -e^{-2\alpha\lambda_2}Z_3 - e^{-2\alpha\lambda_2}F_2 - e^{-2\alpha\lambda_2}F_2^T + \lambda_2 e^{-2\alpha\lambda_2}F_5 - 4e^{-2\alpha\lambda_2}Z_{16} - e^{-2\alpha\lambda_2}Z_{17} - e^{-2\alpha\lambda_2}G_3 - e^{-2\alpha\lambda_2}G_6 + e^{-2\alpha\lambda_2}F_7 - e^{-2\alpha\lambda_2}F_7^T + \lambda_2 F_{10} - \lambda_1 F_{10} - e^{-2\alpha\lambda_2}Z_5$, $\Xi_{3,10} = e^{-2\alpha\lambda_2}G_3^T + e^{-2\alpha\lambda_2}G_5^T$,
$\Xi_{3,11} = 6e^{-2\alpha\lambda_2}G_6$, $\Xi_{4,1} = Z_1^T + Q_9$, $\Xi_{4,2} = Q_{10}$, $\Xi_{4,4} = -\varepsilon_1 I$, $\Xi_{4,8} = Q_{11}$,
$\Xi_{4,12} = Q_{12}$, $\Xi_{4,17} = Q_{14}$, $\Xi_{5,1} = Z_1^T + Q_9$, $\Xi_{5,2} = Q_{10}$, $\Xi_{5,5} = -\varepsilon_2 I$,
$\Xi_{5,8} = Q_{11}$, $\Xi_{5,12} = Q_{12}$, $\Xi_{5,17} = Q_{14}$, $\Xi_{6,1} = Z_1 + Q_9$, $\Xi_{6,2} = Q_{10}$,
$\Xi_{6,6} = -\varepsilon_3 I$, $\Xi_{6,8} = Q_{11}$, $\Xi_{6,12} = Q_{12}$, $\Xi_{6,17} = Q_{14}$, $\Xi_{7,1} = C^TZ_1^T + C^TQ_9$,
$\Xi_{7,2} = C^TQ_{10}$, $\Xi_{7,7} = -r^2 e^{-2\alpha\sigma_2}Z_{25} + \sigma_d e^{-2\alpha\sigma_2}Z_{25} + z^2\varepsilon I - \sigma_1 e^{-2\alpha\sigma_2}Z_{25} - \sigma_1\sigma_d Z_{25}e^{-2\alpha\sigma_2}$, $\Xi_{7,8} = C^TQ_{11}$, $\Xi_{7,12} = C^TQ_{12}$, $\Xi_{7,17} = C^TQ_{14}$,
$\Xi_{8,1} = -x^T - B_1^TZ_1^T - Q_1 - Q_5 + Q_3^T + Q_7^T - B_1^TQ_9 + Q_{11}^TA + Q_{11}^TB_1$,
$\Xi_{8,2} = -Q_2 - Q_6 - Q_3^T - Q_7^T - B_1^TQ_{10} + Q_{11}B_2$, $\Xi_{8,4} = \Xi_{8,5} = \Xi_{8,6} = Q_{11}^T$,
$\Xi_{8,7} = Q_{11}^TC$, $\Xi_{8,8} = -Q_3^T - Q_7^T - Q_3 - Q_7 - Q_{11}^TB_1 - B_1^TQ_{11}$, $\Xi_{8,12} = -Q_4 - Q_8 - Q_{11}^T - B_1^TQ_{12}$, $\Xi_{8,17} = -B_1^TQ_{14}$, $\Xi_{8,23} = Q_{11}^TD$, $\Xi_{9,1} = -e^{-2\alpha\lambda_2}G_2$,
$\Xi_{9,2} = e^{-2\alpha\lambda_2}G_2$, $\Xi_{9,9} = -e^{-2\alpha\lambda_2}Z_{12} - e^{-2\alpha\lambda_2}G_1$, $\Xi_{10,2} = -e^{-2\alpha\lambda_2}G_2 - e^{-2\alpha\lambda_2}G_5$,

$\Xi_{10,3} = e^{-2\alpha\lambda_2}G_2 + e^{-2\alpha\lambda_2}G_5$, $\Xi_{10,10} = -e^{-2\alpha\lambda_2}Z_{12} - e^{-2\alpha\lambda_2}G_1 - e^{-2\alpha\lambda_2}G_4 - e^{-2\alpha\lambda_2}Z_{14}$, $\Xi_{11,1} = \lambda_2^2 e^{-4\alpha\lambda_2}Z_{20} + 2\lambda_2^2 e^{-4\alpha\lambda_2}Z_{21} + 6e^{-2\alpha\lambda_2}Z_{16}^T + 12e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{11,3} = 6e^{-2\alpha\lambda_2}Z_{16}^T$, $\Xi_{11,11} = -\lambda_2^2 e^{-2\alpha\lambda_2}Z_{11} - \lambda_2^2 e^{-4\alpha\lambda_2}Z_{20} - 2\lambda_2^2 e^{-4\alpha\lambda_2}Z_{21} - 12e^{-2\alpha\lambda_2}Z_{16} - 9\lambda_2^2 e^{-2\alpha\lambda_2}Z_{18} - 72e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{11,13} = 36\lambda_2 e^{-2\alpha\lambda_2}Z_{18}$, $\Xi_{11,14} = 480e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{11,15} = -60\lambda_2 e^{-2\alpha\lambda_2}Z_{18}$, $\Xi_{11,16} = -1080e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{12,1} = A_4^T + Q_8^T - Q_9 + Q_{12}^T A + Q_{12}^T B_1 + \lambda_2^2 G_2^T + Q_{13} + \lambda_2^2 G_5^T - 2\lambda_1\lambda_2 G_5^T + \lambda_1^2 G_5^T$, $\Xi_{12,2} = -Q_4^T - Q_8^T - Q_{10} + Q_{12}^T B_2$, $\Xi_{12,4} = \Xi_{12,5} = \Xi_{12,6} = Q_{12}^T$, $\Xi_{12,7} = Q_{12}^T C$, $\Xi_{12,8} = -Q_4^T - Q_8^T - Q_{11} - Q_{12}^T B_1$, $\Xi_{12,12} = -Q_{12}^T - Q_{12} + \frac{1}{4}\lambda_2^2 Z_{20} + \frac{1}{2}\lambda_2^4 Z_{21} + \lambda_2^2 Z_{16} + \lambda_2^2 Z_{17} + \lambda_2 Z_{15} + \lambda_2^2 G_3 + G_2 Z_{25} + \frac{1}{2}\lambda_2^2 Z_{22} + 2T_1 - \sigma_1 Z_{25} + \lambda_2^2 G_6 - 2\lambda_1\lambda_2 G_6 + \lambda_1^2 G_6 + \lambda_2 Z_{19} - \lambda_1 Z_{19} + \frac{\lambda_2^4}{4}Z_{24} - \frac{\lambda_2^2\lambda_2^2}{2}Z_{24} + \frac{\lambda_1^4}{4}Z_{24}$, $\Xi_{12,17} = -T_1 + T_2^T$, $\Xi_{12,23} = Q_{12}^T D$, $\Xi_{13,11} = 36\lambda_2 e^{-2\alpha\lambda_2}Z_{18}$, $\Xi_{13,13} = -192e^{-2\alpha\lambda_2}Z_{18}$, $\Xi_{14,1} = -120e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{14,11} = 480e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{14,14} = -3600e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{14,16} = 8640e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{15,11} = -60\lambda_2 e^{-2\alpha\lambda_2}Z_{18}$, $\Xi_{15,13} = 360e^{-2\alpha\lambda_2}Z_{18}$, $\Xi_{15,15} = -720e^{-2\alpha\lambda_2}Z_{18}$, $\Xi_{16,1} = 360e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{16,11} = -1080e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{16,14} = 8640e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{16,16} = -21600e^{-2\alpha\lambda_2}Z_{22}$, $\Xi_{17,1} = Q_{14}^T A + Q_{14}^T B_1 + Z_2^T - Q_{13}$, $\Xi_{17,2} = Q_{14}^T B_2$, $\Xi_{17,4} = \Xi_{17,5} = \Xi_{17,6} = Q_{14}^T$, $\Xi_{17,7} = Q_{14}^T C$, $\Xi_{17,8} = -Q_{14}^T B_1$, $\Xi_{17,12} = -T_1^T + T_2$, $\Xi_{17,17} = -2Q_{14}^T + 2T_2$, $\Xi_{17,23} = Q_{14}^T D$, $\Xi_{18,2} = e^{-2\alpha\lambda_2}G_6 - e^{-2\alpha\lambda_2}F_6^T + e^{-2\alpha\lambda_2}F_7 + \lambda_2 F_9 - \lambda_1 F_9$, $\Xi_{18,18} = -e^{-2\alpha\lambda_2}Z_4 - e^{-2\alpha\lambda_2}G_6 + e^{-2\alpha\lambda_2}F_6 + e^{-4\alpha\lambda_2}F_6^T + \lambda_2 F_8 - \lambda_1 F_8 + e^{-2\alpha\lambda_2}Z_5$, $\Xi_{18,20} = -e^{-2\alpha\lambda_2}G_5$, $\Xi_{19,1} = 2e^{-4\alpha\lambda_2}Z_{23}^T$, $\Xi_{19,19} = -\lambda_1^2 e^{-2\alpha\lambda_1}Z_{13} - 2e^{-4\alpha\lambda_2}Z_{23}$, $\Xi_{20,2} = e^{-2\alpha\lambda_2}G_5$, $\Xi_{20,18} = -e^{-2\alpha\lambda_2}G_5$, $\Xi_{20,20} = -e^{-2\alpha\lambda_2}G_4 - e^{-2\alpha\lambda_2}Z_{14}$, $\Xi_{21,21} = \rho_d e^{-2\alpha\rho_2}Z_6$, $\Xi_{22,22} = -Z_7$, $\Xi_{23,1} = D^T Z_1^T + D^T Q_9$, $\Xi_{23,2} = D^T Q_{10}$, $\Xi_{23,8} = D^T Q_{11}$, $\Xi_{23,12} = D^T Q_{12}$, $\Xi_{23,17} = D^T Q_{14}$, $\Xi_{23,23} = -Z_8$, $\Xi_{24,24} = -Z_8 - Z_{10}$, $\Xi_{25,25} = -Z_9$, $\Xi_{26,26} = -Z_{10}$, $\Xi_{27,1} = \lambda_2 e^{-4\alpha\lambda_2}Z_{24} - \lambda_1 e^{-4\alpha\lambda_2}Z_{24}$, $\Xi_{27,27} = -e^{-4\alpha\lambda_2}Z_{24}$, $W = Z_1 E$, and other terms are 0.

Theorem 3.1. *For $\|C\| + \gamma < 1$, if there are symmetric matrices $Z_i > 0$, $i = 1, 2, ..., 26$, any appropriate dimensional matrices $S, Q_j, j = 1, 2, ..., 14, F_k, k = 1, 2, ..., 10$, $T_l, l = 1, 2, G_n, n = 1, 2, ..., 6$ and positive scalars $\eta, \rho, \gamma, \varepsilon_n$, $n = 1, 2, 3$, satisfying the following LMIs:*

$$\begin{bmatrix} Z_{15} & F_1 & F_2 \\ * & F_3 & F_4 \\ * & * & F_5 \end{bmatrix} \geq 0, \tag{3.2}$$

$$\begin{bmatrix} Z_{19} & F_6 & F_7 \\ * & F_8 & F_9 \\ * & * & F_{10} \end{bmatrix} \geq 0, \tag{3.3}$$

$$\begin{bmatrix} Z_{17} & S \\ * & Z_{17} \end{bmatrix} \geq 0, \tag{3.4}$$

$$\begin{bmatrix} G_1 & G_2 \\ * & G_3 \end{bmatrix} \geq 0, \tag{3.5}$$

$$\begin{bmatrix} G_4 & G_5 \\ * & G_6 \end{bmatrix} \geq 0, \tag{3.6}$$

$$\Xi < 0, \tag{3.7}$$

then the system (2.1) is exponentially stable with a decay rate $\alpha > 0$.

Proof. Define a Lyapunov-Krasovskii functional candidate for the system (2.11)-(2.13) as follow

$$V(t) = \sum_{i=1}^{10} V_i(t), \tag{3.8}$$

where

$$V_1(t) = x^T(t)Z_1x(t) = \beta_1^T(t)I_0\Psi_1\beta_1(t),$$

which $\beta_1(t) = \begin{bmatrix} x(t) \\ x(t-\lambda(t)) \\ \int_{t-\lambda(t)}^{t} \dot{x}(s)ds \\ \dot{x}(t) \end{bmatrix}$, $I_0 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\Psi_1 = \begin{bmatrix} Z_1 & 0 & 0 & 0 \\ Q_1 & Q_2 & Q_3 & Q_4 \\ Q_5 & Q_6 & Q_7 & Q_8 \\ Q_9 & Q_{10} & Q_{11} & Q_{12} \end{bmatrix}$,

$$\begin{aligned}
V_2(t) &= x^T(t)Z_2x(t) = \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}^T \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Z_2 & 0 \\ Q_{13} & Q_{14} \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}, \\
V_3(t) &= \int_{t-\lambda_2}^{t} e^{2\alpha(s-t)}x^T(s)Z_3x(s)ds + \int_{t-\lambda_1}^{t} e^{2\alpha(s-t)}x^T(s)Z_4x(s)ds \\
&\quad + \int_{t-\lambda_2}^{t-\lambda_1} e^{2\alpha(s-t)}x^T(s)Z_5x(s)ds, \\
V_4(t) &= \int_{t-\rho(t)}^{t} e^{2\alpha(s-t)}x^T(s)Z_6x(s)ds + \rho_2\int_{-\rho_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)Z_7x(\theta)d\theta ds \\
&\quad +\rho_2\int_{-\rho_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)Z_8x(\theta)d\theta ds \\
&\quad +\rho_1\int_{-\rho_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)Z_9x(\theta)d\theta ds \\
&\quad +(\rho_2-\rho_1)\int_{-\rho_2}^{-\rho_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)Z_{10}x(\theta)d\theta ds, \\
V_5(t) &= \lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)Z_{11}x(\theta)d\theta ds \\
&\quad +\lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)Z_{12}x(\theta)d\theta ds \\
&\quad +\lambda_1\int_{-\lambda_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)Z_{13}x(\theta)d\theta ds \\
&\quad +(\lambda_2-\lambda_1)\int_{-\lambda_2}^{-\lambda_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)Z_{14}x(\theta)d\theta ds,
\end{aligned} \tag{3.9}$$

$$
\begin{aligned}
V_6(t) &= \int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}\dot{x}^T(\theta)Z_{15}\dot{x}(\theta)d\theta ds \\
&\quad +\lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}\dot{x}^T(\theta)Z_{16}\dot{x}(\theta)d\theta ds \\
&\quad +\lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}\dot{x}^T(\theta)Z_{17}\dot{x}(\theta)d\theta ds \\
&\quad +\lambda_2\int_{t-\lambda_2}^{t}\int_{s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)Z_{18}x(\theta)d\theta ds \\
&\quad +\int_{-\lambda_2}^{-\lambda_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}\dot{x}^T(\theta)Z_{19}\dot{x}(\theta)d\theta ds,
\end{aligned}
$$

$$
\begin{aligned}
V_7(t) &= \lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}\begin{bmatrix} x(\theta) \\ \dot{x}(\theta)\end{bmatrix}^T\begin{bmatrix} G_1 & G_2 \\ * & G_3\end{bmatrix}\begin{bmatrix} x(\theta) \\ \dot{x}(\theta)\end{bmatrix} d\theta ds \\
&\quad +(\lambda_2-\lambda_1)\int_{-\lambda_2}^{-\lambda_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}\begin{bmatrix} x(\theta) \\ \dot{x}(\theta)\end{bmatrix}^T\begin{bmatrix} G_4 & G_5 \\ * & G_6\end{bmatrix}\begin{bmatrix} x(\theta) \\ \dot{x}(\theta)\end{bmatrix} d\theta ds, \\
V_8(t) &= \frac{(\lambda_2)^2}{2}\int_{-\lambda_2}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\alpha(u+\theta-t)}\dot{x}^T(u)Z_{20}\dot{x}(u)dud\theta ds \\
&\quad +(\lambda_2)^2\int_{-\lambda_2}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\alpha(u+\theta-t)}\dot{x}^T(u)Z_{21}\dot{x}(u)dud\theta ds \\
&\quad +\int_{t-\lambda_2}^{t}\int_{s}^{t}\int_{\theta}^{t} e^{2\alpha(u-t)}\dot{x}^T(u)Z_{22}\dot{x}(u)dud\theta ds \\
&\quad +(\lambda_1)^2\int_{-\lambda_1}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\alpha(u+\theta-t)}\dot{x}^T(u)Z_{23}\dot{x}(u)dud\theta ds \\
&\quad +\frac{(\lambda_2^2-\lambda_1^2)}{2}\int_{-\lambda_2}^{-\lambda_1}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\alpha(u+\theta-t)}\dot{x}^T(u)Z_{24}\dot{x}(u)dud\theta ds, \\
V_9(t) &= (\sigma_2-\sigma_1)\int_{t-\sigma(t)}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)Z_{25}\dot{x}(s)ds, \\
V_{10}(t) &= (\lambda_2-\lambda_1)\int_{t-\lambda(t)}^{t} e^{2\alpha(s-t)}x^T(s)Z_{26}x(s)ds.
\end{aligned}
$$

Taking the time derivative of $V(t)$ along the solution of (2.11) - (2.13)

$$
\dot{V}(t) = \sum_{i=1}^{10}\dot{V}_i(t). \tag{3.10}
$$

We compute $\dot{V}_1(t), \dot{V}_2(t)$ and $\dot{V}_3(t)$ as

$$
\begin{aligned}
\dot{V}_1(t) &= 2\begin{bmatrix} x(t) \\ x(t-\lambda(t)) \\ \int_{t-\lambda(t)}^{t}\dot{x}(s)ds \\ \dot{x}(t)\end{bmatrix}^T\begin{bmatrix} Z_1 & Q_1^T & Q_5^T & Q_9^T \\ 0 & Q_2^T & Q_6^T & Q_{10}^T \\ 0 & Q_3^T & Q_7^T & Q_{11}^T \\ 0 & Q_4^T & Q_8^T & Q_{12}^T\end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \beta_2(t) \\ \beta_2(t) \\ \beta_3(t)\end{bmatrix} \\
&\quad +2\alpha x^T(t)Z_1x(t)-2\alpha V_1(t),
\end{aligned}
$$

where $\beta_2(t) = x(t) - x(t-\lambda(t)) - \int_{t-\lambda(t)}^{t} \dot{x}(s)ds$
and $\beta_3(t) = -\dot{x}(t) + \big[A + B_1\big]x(t) + B_2 x(t-\lambda(t)) + C\dot{x}(t-\sigma(t)) + f_1(t, x(t))$
$+ f_2(t, x(t-\lambda(t))) + f_3(t, \dot{x}(t-\sigma(t))) - B_1 \int_{t-\lambda(t)}^{t} \dot{x}(s)ds + D \int_{t-\rho(t)}^{t} x(s)ds,$

$$\begin{aligned}
\dot{V}_2(t) &= 2x^T(t)Z_2\dot{x}(t) \\
&= 2x^T(t)Z_2 z(t) + 2x^T(t)Q_{13}\big[\dot{x}(t) - z(t)\big] + 2z^T(t)Q_{14}^T \\
&\quad \times\big[-z(t) + Ax(t) + B_1 x(t) + B_2 x(t-\lambda(t)) + C\dot{x}(t-\sigma(t)) \\
&\quad + f_1(t, x(t)) + f_2(t, x(t-\lambda(t))) + f_3(t, \dot{x}(t-\sigma(t))) \\
&\quad - B_1 \int_{t-\lambda(t)}^{t} \dot{x}(s)ds + D\int_{t-\rho(t)}^{t} x(s)ds\big] \\
&\quad + 2\alpha x^T(t)Z_2 x(t) - 2\alpha x^T(t)Z_2 x(t), \\
\dot{V}_3(t) &= x^T(t)(Z_3 + Z_4)x(t) - e^{-2\alpha\lambda_2}x^T(t-\lambda_2)(Z_3 + Z_5)x(t-\lambda_2) \\
&\quad - e^{-2\alpha\lambda_1}x^T(t-\lambda_1)(Z_4 - Z_5)x(t-\lambda_1) - 2\alpha V_3(t).
\end{aligned}$$

By Lemma 2.4 and Lemma 2.5, we obtain $\dot{V}_4(t), \dot{V}_5(t)$ as following

$$\begin{aligned}
\dot{V}_4(t) &\le x^T(t)Z_6 x(t) + (\rho_2)^2 x^T(t)Z_7 x(t) + (\rho_2)^2 x^T(t)Z_8 x(t) \\
&\quad + (\rho_1)^2 x^T(t)Z_9 x(t) + (\rho_d - 1)e^{-2\alpha\rho_2}x^T(t-\rho(t))Z_6 x(t-\rho(t)) \\
&\quad + (\rho_2 - \rho_1)^2 x^T(t)Z_{10}x(t) - 2\alpha V_4(t) \\
&\quad - \Big(\int_{t-\rho_2}^{t} x(s)ds\Big)^T Z_7 \Big(\int_{t-\rho_2}^{t} x(s)ds\Big) \\
&\quad - \Big(\int_{t-\rho_1}^{t} x(s)ds\Big)^T Z_9 \Big(\int_{t-\rho_1}^{t} x(s)ds\Big) \\
&\quad - \int_{t-\rho(t)}^{t} x^T(s)dsZ_8 \int_{t-\rho(t)}^{t} x(s)ds \\
&\quad - \int_{t-\rho_2}^{t-\rho(t)} x^T(s)dsZ_8 \int_{t-\rho_2}^{t-\rho(t)} x(s)ds \\
&\quad - \int_{t-\rho(t)}^{t-\rho_1} x^T(s)dsZ_{10} \int_{t-\rho(t)}^{t-\rho_1} x(s)ds \\
&\quad - \int_{t-\rho_2}^{t-\rho(t)} x^T(s)dsZ_{10} \int_{t-\rho_2}^{t-\rho(t)} x(s)ds, \\
\dot{V}_5(t) &\le (\lambda_2)^2 x^T(t)\big(Z_{11} + Z_{12}\big)x(t) + (\lambda_1)^2 x^T(t)Z_{13}x(t) \\
&\quad + (\lambda_2 - \lambda_1)^2 x^T(t)Z_{14}x(t) - 2\alpha V_5(t) \\
&\quad - e^{-2\alpha\lambda_2}\Big(\frac{1}{\lambda_2}\int_{t-\lambda_2}^{t} x^T(s)ds\Big)(\lambda_2)^2 Z_{11}\Big(\frac{1}{\lambda_2}\int_{t-\lambda_2}^{t} x(s)ds\Big) \\
&\quad - e^{-2\alpha\lambda_2}\Big(\int_{t-\lambda(t)}^{t} x^T(s)ds\Big)Z_{12}\Big(\int_{t-\lambda(t)}^{t} x(s)ds\Big) \\
&\quad - e^{-2\alpha\lambda_2}\Big(\int_{t-\lambda(t)}^{t-\lambda_1} x^T(s)ds\Big)Z_{14}\Big(\int_{t-\lambda(t)}^{t-\lambda_1} x(s)ds\Big)
\end{aligned}$$

$$-e^{-2\alpha\lambda_2}\Big(\int_{t-\lambda_2}^{t-\lambda(t)} x^T(s)ds\Big)(Z_{12}+Z_{14})\Big(\int_{t-\lambda_2}^{t-\lambda(t)} x(s)ds\Big)$$
$$-e^{-2\alpha\lambda_1}\Big(\frac{1}{\lambda_1}\int_{t-\lambda_1}^{t} x^T(s)ds\Big)(\lambda_1)^2 Z_{13}\Big(\frac{1}{\lambda_1}\int_{t-\lambda_1}^{t} x(s)ds\Big).$$

Applying Lemma 2.7, Lemma 2.8, Lemma 2.9 and Lemma 2.10, we obtain

$$\begin{aligned}\dot V_6(t) \quad \le \quad & \lambda_2\dot x^T(t)Z_{15}\dot x(t)+(\lambda_2)^2\dot x^T(t)\big(Z_{16}+Z_{17}\big)\dot x(t)\\ &+(\lambda_2-\lambda_1)\dot x^T(t)Z_{19}\dot x(t)+(\lambda_2)^2 x^T(t)Z_{18}x(t)\\ &+\lambda_2 e^{-2\alpha\lambda_2}\beta_4^T(t)\Psi_2\beta_4(t)+e^{-2\alpha\lambda_2}\beta_4^T(t)\Psi_3\beta_4(t)\\ &+e^{-2\alpha\lambda_2}\beta_6^T(t)\Psi_4\beta_6(t)+(\lambda_2-\lambda_1)\beta_5^T(t)\Psi_5\beta_5(t)-2\alpha V_6(t)\\ &+e^{-2\alpha\lambda_2}\beta_5^T(t)\Psi_6\beta_5(t)+e^{-2\alpha\lambda_2}\beta_4^T(t)\Psi_7\beta_4(t)-\beta_7^T(t)\Psi_8\beta_7(t),\end{aligned}$$

where $\beta_4(t)=\begin{bmatrix}x(t)\\ x(t-\lambda(t))\\ x(t-\lambda_2)\end{bmatrix}$, $\beta_5(t)=\begin{bmatrix}x(t-\lambda_1)\\ x(t-\lambda(t))\\ x(t-\lambda_2)\end{bmatrix}$, $\beta_6(t)=\begin{bmatrix}x(t)\\ x(t-\lambda_2)\\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t} x(s)ds\end{bmatrix}$,

$\beta_7(t)=\begin{bmatrix}\frac{1}{\lambda_2}\int_{t-\lambda_2}^{t} x(t)dt\\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\int_{t-\lambda_2}^{u} x(\theta)d\theta du\\ \frac{1}{\lambda_2{}^2}\int_{t-\lambda_2}^{t}\int_{t-\lambda_2}^{u}\int_{t-\lambda_2}^{s} x(\theta)d\theta ds du\end{bmatrix}$, $\Psi_2=\begin{bmatrix}F_3 & F_4 & 0\\ * & F_3+F_5 & F_4\\ * & * & F_5\end{bmatrix}$,

$\Psi_3=\begin{bmatrix}F_1+F_1^T & -F_1^T+F_2 & 0\\ * & F_1+F_1^T-F_2-F_2^T & -F_1^T+F_2\\ * & * & -F_2-F_2^T\end{bmatrix}$,

$\Psi_4=\begin{bmatrix}-4Z_{16} & -2Z_{16} & 6Z_{16}\\ * & -4Z_{16} & 6Z_{16}\\ * & * & -12Z_{16}\end{bmatrix}$, $\Psi_5=\begin{bmatrix}F_8 & F_9 & 0\\ * & F_8+F_{10} & F_9\\ * & * & F_{10}\end{bmatrix}$,

$\Psi_6=\begin{bmatrix}F_6+F_6^T & -F_6^T+F_7 & 0\\ * & F_6+F_6^T-F_7-F_7^T & -F_6^T+F_7\\ * & * & -F_7-F_7^T\end{bmatrix}$,

$\Psi_7=\begin{bmatrix}-Z_{17} & Z_{17}-S & S\\ * & -2Z_{17}+S+S^T & Z_{17}-S\\ * & * & -Z_{17}\end{bmatrix}$

and $\Psi_8=\begin{bmatrix}-9\lambda_2^2 e^{-2\alpha\lambda_2}Z_{18} & -36\lambda_2 e^{-2\alpha\lambda_2}Z_{18} & -60\lambda_2 e^{-2\alpha\lambda_2}Z_{18}\\ 36\lambda_2 e^{-2\alpha\lambda_2}Z_{18} & -192e^{-2\alpha\lambda_2}Z_{18} & 360e^{-2\alpha\lambda_2}Z_{18}\\ -60\lambda_2 e^{-2\alpha\lambda_2}Z_{18} & 360e^{-2\alpha\lambda_2}Z_{18} & -720e^{-2\alpha\lambda_2}Z_{18}\end{bmatrix}$.

From Lemma 2.6, we compute $\dot V_7(t)$ as

$$\begin{aligned}\dot V_7(t) \quad \le \quad & (\lambda_2)^2\begin{bmatrix}x(t)\\ \dot x(t)\end{bmatrix}^T\begin{bmatrix}G_1 & G_2\\ * & G_3\end{bmatrix}\begin{bmatrix}x(t)\\ \dot x(t)\end{bmatrix}\\ &+(\lambda_2-\lambda_1)^2\begin{bmatrix}x(t)\\ \dot x(t)\end{bmatrix}^T\begin{bmatrix}G_4 & G_5\\ * & G_6\end{bmatrix}\begin{bmatrix}x(t)\\ \dot x(t)\end{bmatrix}\\ &+e^{-2\alpha\lambda_2}\beta_8^T(t)\Psi_8\beta_8(t)+e^{-2\alpha\lambda_2}\beta_9^T(t)\Psi_9\beta_9(t)-2\alpha V_7(t),\end{aligned}$$

where $\beta_8(t) = \begin{bmatrix} x(t) \\ x(t-\lambda(t)) \\ x(t-\lambda_2) \\ \int_{t-\lambda(t)}^{t} x(s)ds \\ \int_{t-\lambda_2}^{t-\lambda(t)} x(s)ds \end{bmatrix}$, $\beta_9(t) = \begin{bmatrix} x(t-\lambda_1) \\ x(t-\lambda(t)) \\ x(t-\lambda_2) \\ \int_{t-\lambda(t)}^{t-\lambda_1} x(s)ds \\ \int_{t-\lambda_2}^{t-\lambda(t)} x(s)ds \end{bmatrix}$,

$$\Psi_8 = \begin{bmatrix} -G_3 & G_3 & 0 & -G_2^T & 0 \\ * & -G_3-G_3^T & G_3 & G_2^T & -G_2^T \\ * & * & -G_3 & 0 & G_2^T \\ * & * & * & -G_1 & 0 \\ * & * & * & * & -G_1 \end{bmatrix}$$

and $\Psi_9 = \begin{bmatrix} -G_6 & G_6 & 0 & -G_5^T & 0 \\ * & -G_6-G_6^T & G_6 & G_5^T & -G_5^T \\ * & * & -G_6 & 0 & G_5^T \\ * & * & * & -G_4 & 0 \\ * & * & * & * & -G_4 \end{bmatrix}$.

Using Lemma 2.3, Lemma 2.4 and Lemma 2.11, $\dot{V}_8(t)$ can be estimated as following

$$\begin{aligned} \dot{V}_8(t) \leq\ & \frac{(\lambda_2)^4}{4}\dot{x}^T(t)Z_{20}\dot{x}(t) + \frac{(\lambda_2)^4}{2}\dot{x}^T(t)Z_{21}\dot{x}(t) + \frac{(\lambda_2)^2}{2}\dot{x}^T(t)Z_{22}\dot{x}(t) \\ & + \frac{(\lambda_1)^4}{2}\dot{x}^T(t)Z_{23}\dot{x}(t) + \frac{(\lambda_2^2-\lambda_1^2)^2}{4}\dot{x}^T(t)Z_{24}\dot{x}(t) - 2\alpha V_8(t) \\ & -e^{-4\alpha\lambda_2}\left(\lambda_2 x(t) - \int_{t-\lambda_2}^{t} x(s)ds\right)^T Z_{20}\left(\lambda_2 x(t) - \int_{t-\lambda_2}^{t} x(s)ds\right) \\ & -e^{-4\alpha\lambda_2}\beta_{10}^T(t)Z_{24}\beta_{10}(t) + e^{-2\alpha\lambda_2}\beta_{11}^T(t)\Psi_{10}\beta_{11}(t) \\ & +{\lambda_2}^2 e^{-4\alpha\lambda_2}\begin{bmatrix} x(t) \\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} -2Z_{21} & 2Z_{21} \\ * & -2Z_{21} \end{bmatrix} \begin{bmatrix} x(t) \\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t} x(s)ds \end{bmatrix} \\ & +{\lambda_1}^2 e^{-4\alpha\lambda_2}\begin{bmatrix} x(t) \\ \frac{1}{\lambda_1}\int_{t-\lambda_1}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} -2Z_{23} & 2Z_{23} \\ * & -2Z_{23} \end{bmatrix} \begin{bmatrix} x(t) \\ \frac{1}{\lambda_1}\int_{t-\lambda_1}^{t} x(s)ds \end{bmatrix}, \end{aligned}$$

where $\beta_{10}(t) = (\lambda_2-\lambda_1)x(t) - \int_{t-\lambda_2}^{t-\lambda_1} x(s)ds,\ \beta_{11}(t) = \begin{bmatrix} x(t) \\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t} x(s)ds \\ \frac{1}{\lambda_2^2}\int_{t-\lambda_2}^{t}\int_{x}^{t} x(s)dsdu \\ \frac{1}{\lambda_2^3}\int_{t-\lambda_2}^{t}\int_{x}^{t}\int_{s}^{t} x(\theta)d\theta dsdu \end{bmatrix}$

and $\Psi_{10} = \begin{bmatrix} -12Z_{22} & 12Z_{22} & -120Z_{22} & 360Z_{22} \\ 12Z_{22} & -72Z_{22} & 480Z_{22} & -1080Z_{22} \\ -120Z_{22} & 480Z_{22} & -3600Z_{22} & 8640Z_{22} \\ 360Z_{22} & -1080Z_{22} & 8640Z_{22} & -21600Z_{22} \end{bmatrix}$.

Differentiating $V_9(t)$ and $V_{10}(t)$, we have

$$\begin{aligned} \dot{V}_9(t) \leq\ & (\sigma_2-\sigma_1)\dot{x}^T(t)Z_{25}\dot{x}(t) - 2\alpha V_9(t) \\ & -(\sigma_2-\sigma_1)(1-\sigma_d)e^{-2\alpha\sigma_2}\dot{x}^T(t-\sigma(t))Z_{25}\dot{x}(t-\sigma(t)), \\ \dot{V}_{10}(t) \leq\ & (\lambda_2-\lambda_1)x^T(t)Z_{26}x(t) - 2\alpha V_{10}(t) \\ & -(\lambda_2-\lambda_1)(1-\lambda_d)e^{-2\alpha\lambda_2}x^T(t-\lambda(t))Z_{26}x(t-\lambda(t)). \end{aligned}$$

From (2.5) - (2.7), for any positive scalars ε_1, ε_2 and ε_3, it can checked that the following inequalities hold:

$$\varepsilon_1(\eta^2 x^T(t)x(t) - \delta_1^T(t)\delta_1(t)) \geq 0, \tag{3.11}$$

$$\varepsilon_2(\rho^2 x^T(t-\lambda(t))x(t-\lambda(t)) - \delta_2^T(t)\delta_2(t)) \geq 0, \tag{3.12}$$

$$\varepsilon_3(\gamma^3 \dot{x}^T(t-\sigma(t))\dot{x}(t-\sigma(t)) - \delta_3^T(t)\delta_3(t)) \geq 0. \tag{3.13}$$

From (2.12), we have

$$2\dot{x}^T(t)T_1\dot{x}(t) - 2\dot{x}^T(t)T_1 z(t) = 0, \tag{3.14}$$

$$2z^T(t)T_2\dot{x}(t) - 2z^T(t)T_2 z(t) = 0. \tag{3.15}$$

We can conclude the following inequality by (3.10) - (3.15)

$$\dot{V}(t) + 2\alpha V(t) \quad \leq \quad \xi^T(t)\Xi\xi(t),$$

where $\xi(t) = \big[x(t),\ x(t-\lambda(t)),\ x(t-\lambda_2),\ f_1(t,x(t)),\ f_2(t,x(t-\lambda(t))),$ $f_3(t,\dot{x}(t-\sigma(t))),\ \dot{x}(t-\sigma(t)),\ \int_{t-\lambda(t)}^{t}\dot{x}(s)ds,\ \int_{t-\lambda(t)}^{t}x(s)ds,\ \int_{t-\lambda_2}^{t-\lambda(t)}x(s)ds,$ $\frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}x(s)ds, \dot{x}(t),\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\int_{t-\lambda_2}^{u}x(\theta)d\theta du,\ \frac{1}{\lambda_2^2}\int_{t-\lambda_2}^{t}\int_{u}^{t}x(s)dsdu,$ $\frac{1}{\lambda_2^2}\int_{t-\lambda_2}^{t}\int_{t-\lambda_2}^{u}\int_{t-\lambda_2}^{s}x(\theta)d\theta dsdu,\ \frac{1}{\lambda_2^3}\int_{t-\lambda_2}^{t}\int_{u}^{t}\int_{s}^{t}x(\theta)d\theta dsdu,\ z(t),\ x(t-\lambda_1),$ $\frac{1}{\lambda_1}\int_{t-\lambda_1}^{t}x(s)ds,\ \int_{t-\lambda(t)}^{t-\lambda_1}x(s)ds,\ x(t-\rho(t)),\ \int_{t-\rho_2}^{t}x(s)ds,\ \int_{t-\rho(t)}^{t}x(s)ds,$ $\int_{t-\rho_2}^{t-\rho(t)}x(s)ds,\ \int_{t-\rho_1}^{t}x(s)ds,\ \int_{t-\rho(t)}^{t-\rho_1}x(s)ds,\ \int_{t-\lambda_2}^{t-\lambda_1}x(s)ds\big]$, and Ξ is defined in (3.1). If conditions (3.2)-(3.7) hold, then

$$\dot{V}(t) + 2\alpha V(t) \leq 0, \quad \forall t \in R^+. \tag{3.16}$$

From (3.16), we obtain

$$\|x(t,\phi)\| \leq M\|\phi\|e^{-\alpha t}, \quad t \in R^+,$$

where $M, \alpha \in R^+$. Hence, system (2.1) is exponentially stable. ∎

Now, the delay-range-dependent exponential stability criterion of equation (2.1) is demonstrated where D is a zero matrix. We define a new parameter

$$\hat{\Xi} = \big[\hat{\Xi}_{(i,j)}\big]_{20\times 20}, \tag{3.17}$$

where $\hat{\Xi}_{(i,j)} = \Xi_{(i,j)}$.

Corollary 3.2. *For $\|C\| + \gamma < 1$, if there are symmetric matrices $Z_i > 0$, $i = 1,2,...,18$, any appropriate dimensional matrices $S, Q_j, j = 1,2,...,14$, $F_k, k = 1,2,...,10$, $T_l, l = 1,2, G_n, n = 1,2,...,6$ and positive real constants ε_n, $n = 1,2,3$, satisfying the following*

LMIs:

$$\begin{bmatrix} Z_{15} & F_1 & F_2 \\ * & F_3 & F_4 \\ * & * & F_5 \end{bmatrix} \geq 0, \tag{3.18}$$

$$\begin{bmatrix} Z_{19} & F_6 & F_7 \\ * & F_8 & F_9 \\ * & * & F_{10} \end{bmatrix} \geq 0, \tag{3.19}$$

$$\begin{bmatrix} Z_{17} & S \\ * & Z_{17} \end{bmatrix} \geq 0, \tag{3.20}$$

$$\begin{bmatrix} G_1 & G_2 \\ * & G_3 \end{bmatrix} \geq 0, \tag{3.21}$$

$$\begin{bmatrix} G_4 & G_5 \\ * & G_6 \end{bmatrix} \geq 0, \tag{3.22}$$

$$\hat{\Xi} < 0. \tag{3.23}$$

Then the system (2.1) with interval time-varying delays (2.2)-(2.4) is exponentially stable with a decay rate α when D is a zero matrix.

4. Numerical examples

We give two numerical examples to present the improvement and performance of our stability criteria by comparing the least upper bounds of the parameter λ_2 and considering the rate of convergence α for guaranteeing exponential stability.

Example 4.1. The neutral system:

$$\begin{aligned} \dot{x}(t) \;=\; & \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix} x(t-\lambda(t)) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \dot{x}(t-\sigma(t)) \\ & + f_1(t, x(t)) + f_2(t, x(t-\lambda(t))) + f_3(t, \dot{x}(t-\rho(t))) \\ & + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \int_{t-\rho(t)}^{t} x(s)ds. \end{aligned} \tag{4.1}$$

When $\eta = 0.1$, $\rho = \gamma = 0.05$, $\alpha = 0.5$, $\lambda(t) = 0.3 + \dfrac{\sin(t)}{5}$, $\sigma(t) = 0.4 + \dfrac{\cos(t)}{5}$ and $\rho(t) = 0.5 + \dfrac{\cos(t)}{5}$. We separate matrix B as $B = B_1 + B_2$, where

$$B_1 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}.$$

By using the linear matrix inequalities (3.2)-(3.7) in theorem 3.1, the least upper bounds of λ_2 that guarantee the exponential stability for this example are presented in Table 1 for various values of λ_1 and α. Table 2 represents the least upper bounds of α of this example with different values of λ_1 and λ_2.

Example 4.2. The neutral system:

$$\begin{aligned} \dot{x}(t) \;=\; & \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix} x(t-\lambda(t)) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \dot{x}(t-\sigma(t)) \\ & + f_1(t, x(t)) + f_2(t, x(t-\lambda(t))) + f_3(t, \dot{x}(t-\rho(t))). \end{aligned} \tag{4.2}$$

TABLE 1. The least upper bounds of λ_2 for Example 4.1 with different values of α and λ_1 when $\eta = 0.1$, $\rho = \gamma = 0.05$, $\lambda_d = 0.7$, $\sigma_1 = 0.3$, $\sigma_2 = 0.5$, $\sigma_d = 0.1$, $\rho_1 = 0.3$, $\rho_2 = 0.4$, $\rho_d = 0.4$.

λ_1	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$
0.2	1.3426×10^4	67.2152	26.0551	15.4044	10.7194
0.8	1.3426×10^4	64.0632	25.4109	14.5405	9.0356
1.0	1.2999×10^4	64.0520	25.0769	14.3710	9.0530
5.0	1.1435×10^4	63.1158	24.2267	14.4175	9.0000
10.0	1.0999×10^4	61.1467	24.2057	13.9840	10.3017

TABLE 2. The least upper bounds of α for Example 4.1 with different values of λ_1 and λ_2 when $\eta = 0.1$, $\rho = \gamma = 0.05$, $\lambda_d = 0.7$ $\sigma_1 = 0.3$, $\sigma_2 = 0.5$, $\sigma_d = 0.1$, $\rho_1 = 0.3$, $\rho_2 = 0.4$, $\rho_d = 0.4$.

λ_1	$\lambda_2 = 5.0$	$\lambda_2 = 6.0$	$\lambda_2 = 7.0$	$\lambda_2 = 8.0$
1.0	1.6082	1.3585	0.7331	0.6559
2.0	1.6003	1.3535	0.7257	0.6551
3.0	1.5003	1.3510	0.7053	0.6410
4.0	1.4738	1.4580	0.8030	0.5501

When $\lambda_1 = 0$, $\eta = 0.1$, $\rho = 0.05$, $\gamma = 0.05$, $\lambda_2 = \sigma_2$. Separate the matrix $B = B_1 + B_2$ as

$$B_1 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}.$$

By solving the linear matrix inequalities (3.18)-(3.23) in Corollary 3.2, the comparison for the least upper bounds of λ_2 that ensure the exponential stability of equation (4.2) are represented in Table 3.

TABLE 3. The least upper bounds of λ_2 for Example 4.2.

α	0.1	0.3	0.5	0.7	0.9
($\lambda_d = \sigma_d = 0$) [1]	10.2180	2.9471	1.4126	0.7232	0.3045
($\lambda_d = \sigma_d = 0$) [9]	12.2475	3.7460	1.9563	1.1015	0.5957
This paper	63.8665	24.6105	14.9035	10.1982	9.1202
($\lambda_d = \sigma_d = 0.5$) [1]	6.7523	1.7922	0.7308	0.3580	0.1027
($\lambda_d = \sigma_d = 0.5$) [9]	10.8211	3.3202	1.7390	0.9662	0.4857
This paper	65.3508	22.1301	16.5754	12.2947	8.4637

5. CONCLUSIONS

New criterion for the exponential stability of neutral system with interval time-varying discrete, neutral and distributed delays, and nonlinear uncertainties have been established. Moreover, we presented the improved delay-range-dependent exponential stability criterion for neutral system with interval time-varying discrete and neutral delays, and nonlinear uncertainties. The result has been obtained by using Jensen's integral inequality, Wirtinger-base integral inequality, Leibniz-Newton fomula, Peng-Park's integral inequality, mixed model transformation, utilization of zero equation, decomposition matrix technique and the appropriate Lyapunov-Krasovskii functional (LKF). The exponential stability criteria are expressed in term of LMIs. The effectiveness of the theoretical results has been demonstrated by two numerical examples.

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